Modelling gravitational waves from compact binaries

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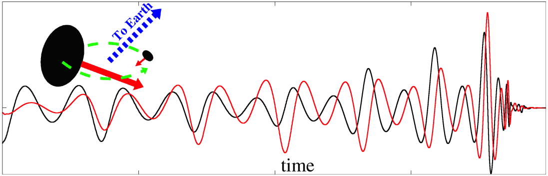
Simulating eXtreme Spacetimes (SXS) Collaboration

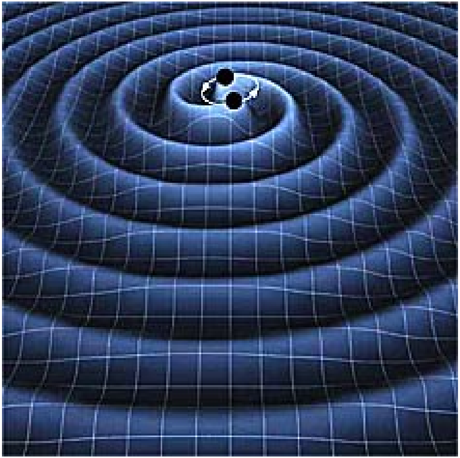
Beyond the First Century of General Relativity
30 May 2015, Rochester Institute of Technology



Why model compact binaries?

- Two-body problem is a fundamental problem in gravity
- No known exact solution in theory of general relativity
- One of the most promising sources of detectable gravitational waves
- Want large compactness $\frac{GM}{Rc^2}$ to probe strong gravity
  - Sun $\sim 10^{-6}$
  - White Dwarf $\sim 10^{-4}$
  - Neutron star ~ 0.1
  - Black hole ~ 1

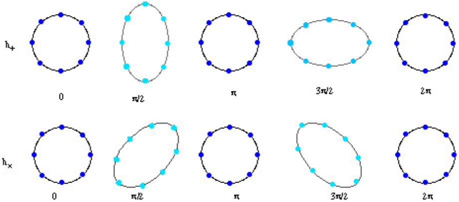
Gravitational Waves (GWs)

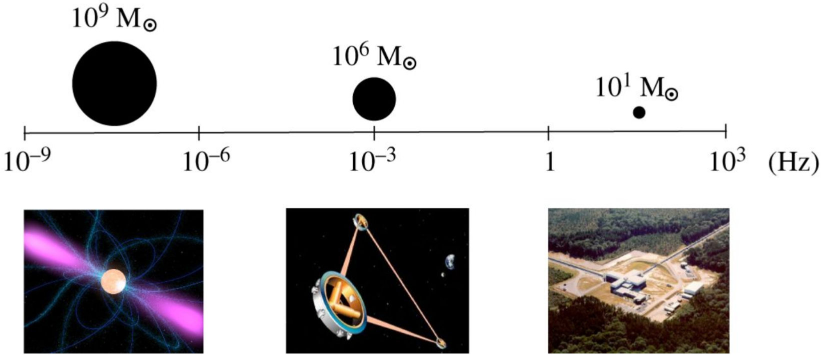


- Ripples in spacetime traveling at speed of light
- Generated by time-varying quadrupole moment
  $h \sim \frac{Gm}{Dc^2} \ddot{\mathcal{Q}}$
- Travel unimpeded across universe (but can be lensed)
- Gravity is weak, need astrophysical sources moving at relativistic speeds

Detecting gravitational waves

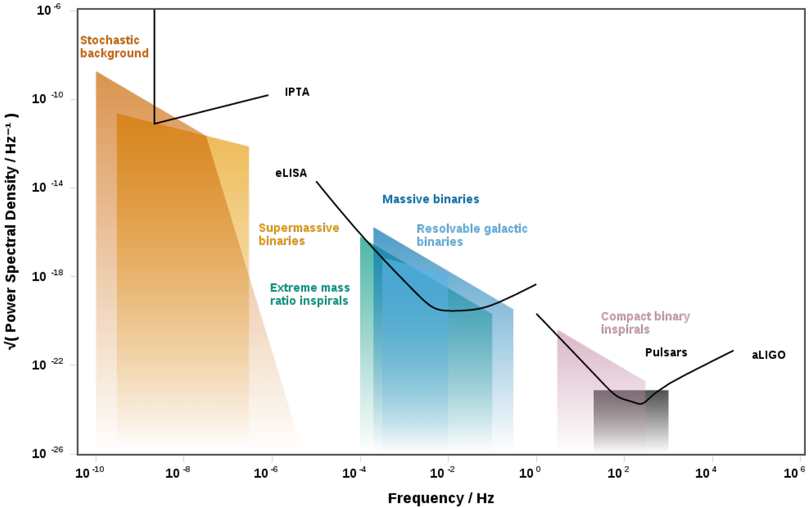
- Stretch and shrink distances
- Wide spectrum of sources lead to different detection strategies





[Cornish; 1204.2000]

Compact binary systems

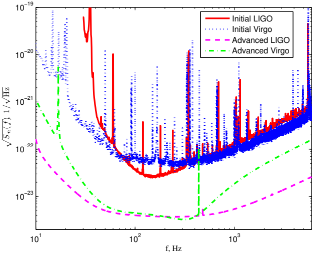


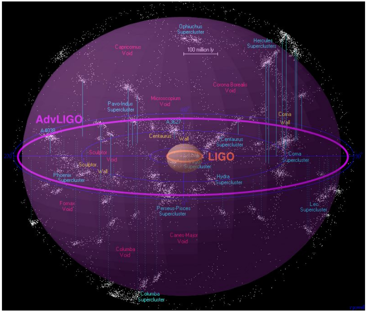
[Moore, Cole, Berry; 1408.0740]

Compact binary systems: event rates (per year)

[LIGO-VIRGO collaboration; 1003.2480]

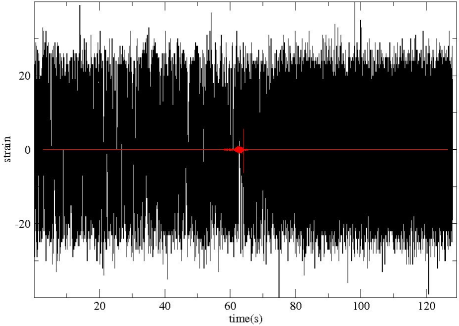
Interferometer	Source	low	realistic	high
Initial	NS-NS	2×10^{-4}	0.02	0.2
	NS-BH	7×10^{-5}	0.004	0.1
	BH-BH	2×10^{-4}	0.007	0.5
Advanced	NS-NS	0.4	40	400
	NS-BH	0.2	10	300
	BH-BH	0.4	20	1000





Detecting gravitational waves: matched filtering

- need to integrate model waveform template with noisy data
- need different template for each set of binary parameters m_i, $\vec{S}_i$, e, ...



[http://www.astro.cardiff.ac.uk/research/gravity/tutorial/]

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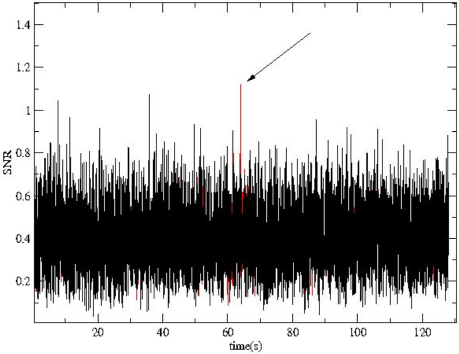
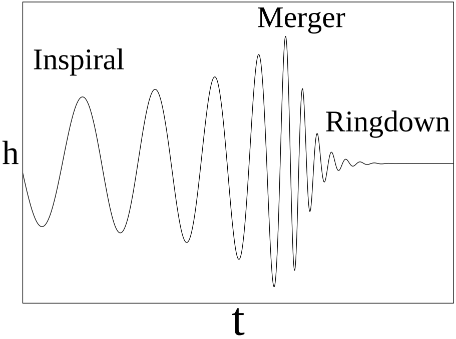


[http://www.astro.cardiff.ac.uk/research/gravity/tutorial/]

Modelling gravitational waves for templates



- Inspiral
  - Post-Newtonian expansion when $v \ll c$
  - Numerical relativity when v/c is large
- Merger
  - Numerical relativity
- Ringdown
  - BH perturbation theory
  - Numerical relativity

Numerical relativity needs to:

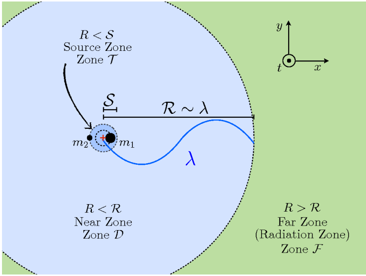
- simulate the “late” inspiral and merger
- determine what “late” means

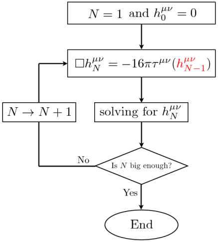
Post-Newtonian approximation

[Blanchet, 1310.1528v2]

- Problems to solve
  - Wave generation problem
  - Wave propagation problem
  - Orbital and spin dynamics problem
  - Radiation reaction problem
- Tools used
  - Post-Minkowski expansion
  - Post-Newtonian expansion
  - Multipolar expansion
  - Matched asymptotic expansions
- Slow motion, weak field approximation

  $\epsilon \sim \frac{Gm}{rc^2} \sim \left(\frac{v}{c}\right)^2$





[Mirshekari; 1308.5240]

Post-Newtonian Approximation

- Equations of motion
  - Acceleration $\vec{a}$
  - "Constants" of the motion E, $\vec{J}$, $\vec{P}$, $\vec{X_{COM}}$
  - Spin-orbit effects first enter at $\epsilon^{3/2}$
  - Spin-spin effects at ϵ^{2}.
  - Spin precession equations $\frac{d\vec{S}_a}{dt}$
  - Radiation reaction first enters at $\epsilon^{5/2}$
  - Tidal effects first enter at ϵ^{5}
- Wave generation
  - Gravitational wave fluxes $\mathcal{L} = \frac{dE}{dt}$, $\frac{d\vec{J}}{dt}$, $\frac{d\vec{P}}{dt}$
  - Polarization waveforms h_{+} and $h_{\times}$
  - Gravitational wave modes $h_{\ell m}$
- To produce accurate templates, assume:
  - Adiabatic inspiral of quasi-circular orbits
  - Energy balance

Adiabatic Inspiral of Quasicircular Orbits

- Emission of gravitational radiation circularizes the orbit
- Can express E, $\mathcal{L}$, h_+, and $h_\times$ in terms of an orbital frequency parameter $x \equiv (\frac{Gm\omega_{orb}}{c^3})^{2/3}$
- For an equal mass, non-spinning binary

$$E = -\frac{mc^2}{8}x\left[1 - \frac{37}{48}x - \frac{1069}{384}x^2 + \left(\frac{1427365}{331776} - \frac{205}{384}\pi^2\right)x^3\right]$$

$$\begin{aligned}\mathcal{L} \;&=\; \frac{2c^5}{5G}x^5\left\{1 - \frac{373}{84}x + 4\pi x^{3/2} - \frac{59}{567}x^2 - \frac{767}{42}\pi x^{5/2}\right.\\ &\left.+\left[\frac{18608019757}{209563200} + \frac{355}{64}\pi^2 - \frac{1712}{105}\gamma - \frac{856}{105}\ln(16x)\right]x^3 + \frac{16655}{6048}\pi x^{7/2}\right\}\end{aligned}$$

PN waveforms: polarization waveforms and modes

Decompose polarization waveforms into spin-weighted spherical harmonics:

$$h_+ - ih_\times = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} h^{\ell m}\,{}_{-2}Y^{\ell m}(\Theta, \Phi)$$

Polarization waveforms combine multiple modes

$$\begin{aligned} h_{ij}^{TT} &= \frac{4G}{c^2 R} \Pi_{ijmn} \sum_{\ell=2}^{\infty} \left\{ \frac{1}{c^\ell \ell!} \mathcal{U}_{mnL-2}(T_R) N_{L-2} \right. \\ &\quad \left. + \frac{2\ell}{c^{\ell+1}(\ell+1)!} \epsilon_{pq(m} \mathcal{V}_{n)pL-2}(T_R) N_{qL-2} \right\}. \end{aligned}$$

Can compute modes directly from multipole moments, some modes can be computed to higher order than the highest order of the polarization waveform.

$$h^{\ell m} = \frac{G}{\sqrt{2} R c^{\ell+2}} \left(U^{\ell m}(T_R) - \frac{i}{c} V^{\ell m}(T_R) \right)$$

PN waveforms: modes to full known order

- For an equal mass, non-spinning binary

$$
\begin{aligned}
(h_+ - ih_\times)^{(2,2)} \quad = \quad & -2\sqrt{\frac{\pi}{5}}\frac{Gm}{c^2R}e^{-2i\Phi}x\left\{1-\frac{373}{168}x+2\pi x^{3/2}-\frac{62653}{24192}x^2\right. \\
& -\left[\frac{197}{42}\pi+6i\right]x^{5/2}+\left[\frac{43876092677}{1117670400}+\frac{99}{128}\pi^2\right. \\
& \left.\left.-\frac{428}{105}\ln x-\frac{856}{105}\gamma-\frac{1712}{105}+\frac{428}{105}i\pi\right]x^3\right\}
\end{aligned}
$$

[Kidder; 0710.0614]

Energy Balance

$$\frac{dE}{dt} = -\mathcal{L}$$

- Given $E(x)$, $\mathcal{L}(x)$ and $h(x)$ need to choose:
  - Order k_Φ through which terms in E and $\mathcal{L}$ are kept
  - Order k_A through which terms in h are kept
  - How to use energy balance to obtain $x(t)$ and $\Phi(t)$

$$\frac{dx}{dt} = -\frac{\mathcal{L}}{(dE/dx)}$$

PN Taylor Approximants

[Damour, Iyer, Sathyaprakash; gr-qc/0010009]

- TaylorT1: Numerically integrate $\frac{dx}{dt} = -\frac{\mathcal{L}}{(dE/dx)}$ and $\frac{d\Phi}{dt} = \frac{x^{3/2}c^3}{Gm}$
- TaylorT2: Analytically integrate $t(x) = t_0 + \int_x^{x_0} dx \frac{(dE/dx)}{\mathcal{L}}$ and $\Phi(x) = \Phi_0 + \int_x^{x_0} dx \frac{x^{3/2}c^3}{Gm}\frac{(dE/dx)}{\mathcal{L}}$
- TaylorT3: Introduce $\tau \equiv \frac{\nu c^3}{5Gm}(t_0 - t)$ where $\tau^{-1/4} = O(c^{-2})$ and invert the TaylorT2 expression for $t(x)$ to obtain $x(t)$ and then $\Phi(t)$

[Buonanno, Cook, Pretorius; gr-qc/0610122]

- TaylorT4: Expand $\mathcal{F}(x) = -\frac{\mathcal{L}}{(dE/dx)}$ as a single Taylor series and numerically integrate $\frac{dx}{dt} = \mathcal{F}$ and $\frac{d\Phi}{dt} = \frac{x^{3/2}c^3}{Gm}$

PN history

PN order	Equations of Motion	Gravitational radiation
N	Newton	Landau, Lifshitz
1PN	Lorentz, Droste (1917) Einstein et al (1938) Fock (1939)	Wagoner, Will (1976) Blanchet, Schäfer (1989)
1.5PN-SO	Barker, O'Connell (1975)	Kidder (1995)
1.5PN-T		Wiseman (1993) Blanchet, Schäfer (1993)
2PN	Ohta et al (1974)	Blanchet et al (1995) Will, Wiseman (1996)
2PN-SS	Barker, O'Connell (1975)	Kidder (1995)
2.5PN	Damour, Deruelle (1981) Damour, Schäfer (1985) Kopeikin (1985)	Blanchet (1996)
2.5PN-SO	Tagoshi et al (2001)	Blanchet et al (2006)

PN: Currently known terms

- Equations of motion
    - non-spinning 4PN
    - spin-orbit 3.5PN
    - spin-spin 4PN
    - spin-spin-spin 3.5PN
- Gravitational radiation
    - non-spinning 3.5PN
    - spin-orbit 3.5PN
    - spin-spin 3PN
    - spin-spin-spin 3.5PN

Importance of PN terms

Number of cycles in bandwidth of LIGO-VIRGO from $10 Hz$ to “merger”

PN order	$1.4\,M_\odot + 1.4\,M_\odot$	$10\,M_\odot + 1.4\,M_\odot$	$10\,M_\odot + 10\,M_\odot$
N	15952.6	3558.9	598.8
1PN	439.5	212.4	59.1
1.5PN-T	-210.3	-180.9	-51.2
1.5PN-SO	$65.6 + 65.6$	$114.0 + 11.7$	$16.0 + 16.0$
2PN	9.9	9.8	4.0
2.5PN-T	-11.7	-20.0	-7.1
2.5PN-SO	$9.3 + 9.3$	$33.8 + 2.9$	$5.7 + 5.7$
3PN	2.6	2.3	2.2
3PN-SO	$-3.2 - 3.2$	$-13.2 - 1.3$	$-2.6 - 2.6$
3.5PN-T	-0.9	-1.8	-0.8
3.5PN-SO	$1.9 + 1.9$	$11.1 + 0.8$	$1.7 + 1.7$
4PN-SOT	$-1.5 - 1.5$	$-8.0 - 0.7$	$-1.5 - 1.5$

[Blanchet, 1310.1528v2]

Ingredients of a numerical simulation

- Slice spacetime into spatial hypersurfaces
- Initial conditions: solve elliptic constraint equations
- Evolution equations: Generalized harmonic, BSSN
- Numerical method: Finite difference, spectral methods
- Computational domain: deal with singularity, artificial outer boundary
- Coordinate conditions
- Boundary conditions: excision, no incoming radiation
- Control of constraint violations

Brief history of numerical relativity for binary black holes

- 1964 – First attempt: 2D head-on equal-mass

  [Hahn, Lundquist; Ann. Phys. 29, 304 (1964)]
- mid-1970s – First success: 2D head-on equal-mass

  [Smarr, Cadez, DeWitt, Eppley; PRD 14, 2443 (1976)]
- mid-1990s – Computational Grand Challenge
- Formulations of Einstein's equations, conquering constraint growth
- 2005 – First successful simulation (unique methods)

  [Pretorius; gr-qc/0507014]
- Moving punctures

  [Campanelli, Lousto, Marronetti, Zlochower; gr-qc/0511048]

  [Baker, Centrella, Choi, Koppitz, van Meter; gr-qc/0511103]

  Adopted by many groups: RIT, GSFC, GaTech, Illinois, FAU, LSU, Maryland, AEI, Jena, Vienna, Palma, …
- Spectral Einstein Code, 15 orbits, high accuracy

  [Boyle et al; 0710.0158] [Scheel et al; 0810.1767]

BBH: Kicks

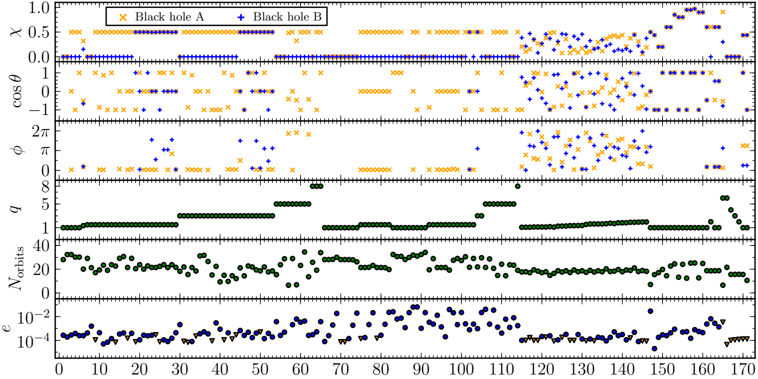
- Anisotropic GW emission leads to net linear momentum flux
- Final black hole is kicked with respect to initial center of mass
- Non-spinning BHs $v_{max} \approx 175 \pm 11 \frac{km}{s}$ at mass-ratio 2.77
  [Gonzalez et al; gr-qc/0610154]
- Spinning BHs give higher kicks
  - Aligned spins $v_{max} \approx 500 \frac{km}{s}$
    [Herrmann et al; gr-qc/0701143] [Koppitz et al; gr-qc/0701163]
  - Super kicks for spins equal and opposite in orbital plane
    v_{max} up to $4000 \frac{km}{s}$! [Campanelli et al; gr-qc/0701164]
  - Hangup kicks, generic orientation v_{max} up to $5000 \frac{km}{s}$!
    [Lousto, Zlochower; 1108.2009]
- $v_{recoil} > 1000 \frac{km}{s}$ is between $0.1 - 17\%$
  [Zlochower and Lousto; 1503.07536]

SpEC and waveform catalog

[Mroue et al, 1304.6077]

Publicly available at `www.black-holes.org/waveforms`

- 8-dimensional parameter space: mass-ratio, spins, eccentricity
- Currently 201 simulations, with another 100+ coming soon.



Effective-one-body model

[Buonanno, Damour; gr-qc/9811091]

[Damour; 1212.3169]

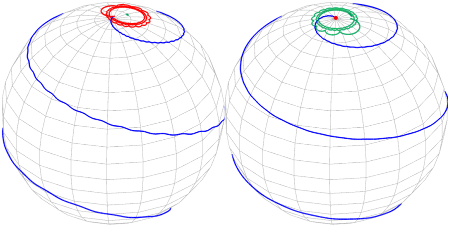
- Conservative Hamiltonian dynamics
  $H = \mu\sqrt{p_r^2 + A(r)[1 + \frac{p_r^2}{r^2} + 2(4-3\nu)\nu\frac{p_r^4}{r^2}}, \quad A(r) = \sum_{k=0}^{4}\frac{a_k(\nu)}{r^k} + \frac{a_5(\nu)}{r^5}$
- Radiation-reaction force from flux
  $\frac{dp_r}{dt} = -\frac{\partial H}{\partial p_r} + a_{RR}^4 \frac{\dot{r}}{r^2\Omega}\hat{\mathcal{F}}_\phi$
  $\frac{dp_\phi}{dt} = -\frac{v_\Omega^3}{\nu V_\phi^6} F_4^4(V_\phi;\nu, v_{pole})$, using 4-PN term $\mathcal{F}_{8,\nu=0} + \nu A_8$
- Description of GW waveform
  $h_{\ell m}^F = h_{\ell m}^{N,\epsilon)}\hat{S}_{eff}^{(\epsilon)} T_{\ell m} e^{i\delta_{\ell m}}(\rho_{\ell m})^\ell$
- Match BH ringdown modes
- Fit free parameters to NR simulations

  [Taracchini et al; 1311.2544]

  [Damour, Nagar; 1406.6913]
- Incorporate results from perturbation theory (i.e. some terms through linear order in ν)

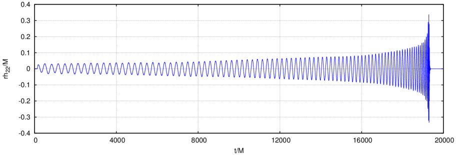
Recent Results: Flip Flop

- 48 orbits, 3 precession cycles
- $\hat{L}$, $\hat{J}$, $\hat{S}$

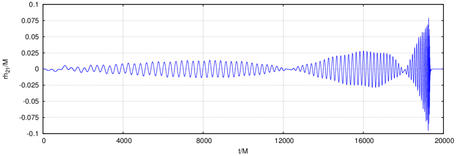


[Lousto and Healy; 1410.3830]

- h_{22}

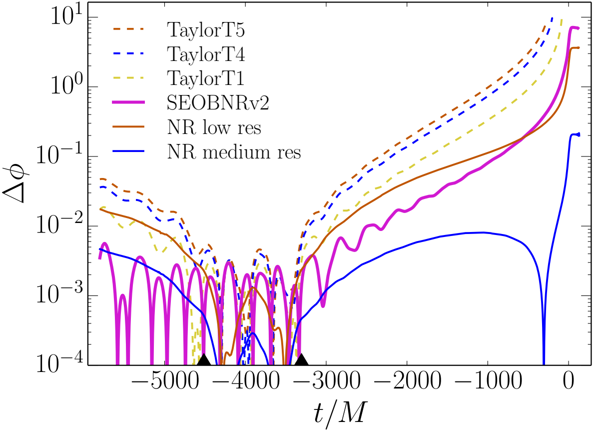


- h_{21}



Recent Results: Spin 0.994

Waveform phase error:

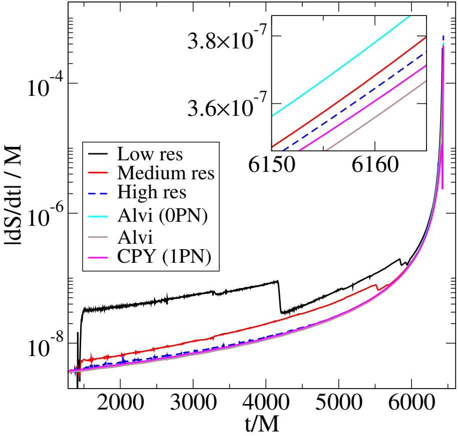


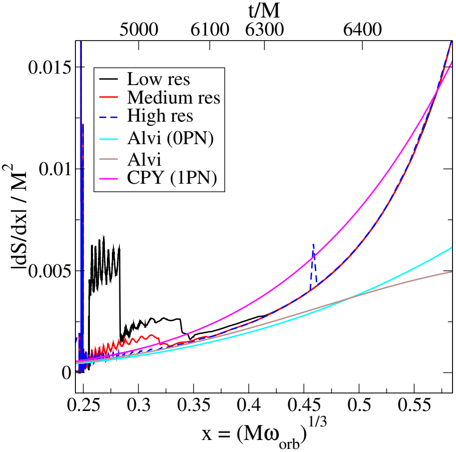
[Scheel et al; 1412.1803]

- Equal mass, aligned spins, $\chi_a = \frac{S_a}{m_a^2} = 0.994$, 25.4 orbits.
- $\chi_f = 0.949931(5)$
- $E_{rad} = 11.351(5)$

Recent Results: Spin 0.994

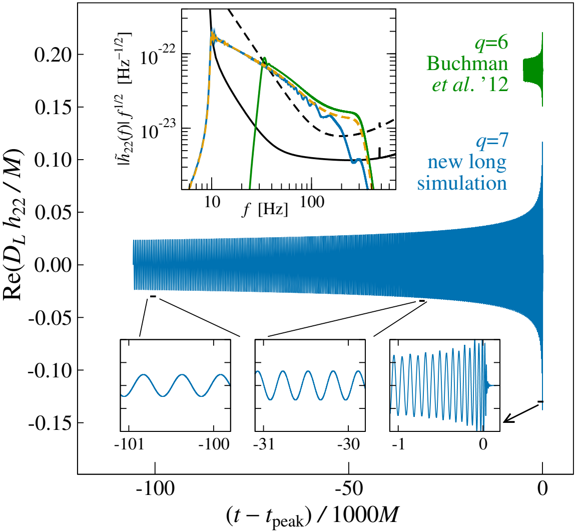
Examine tidal effects on spin magnitude, compared to PN predictions





[Scheel et al; 1412.1803]

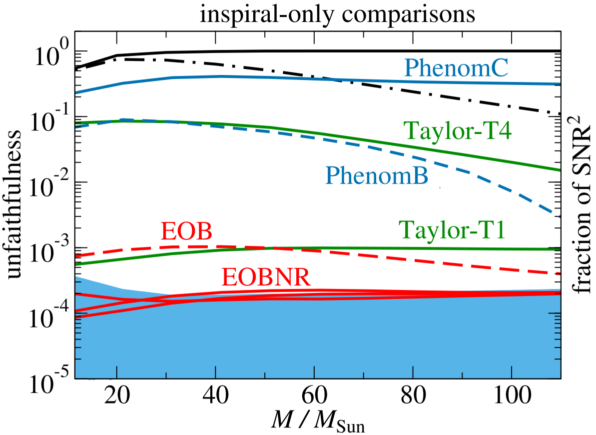
Recent Results: BBH of 170 Orbits



- 3× frequency range
- $40M_\odot$ entire Advanced LIGO spectrum

[Szilagyi et al; 1502.04953]

Waveform models vs 170 Orbits NR

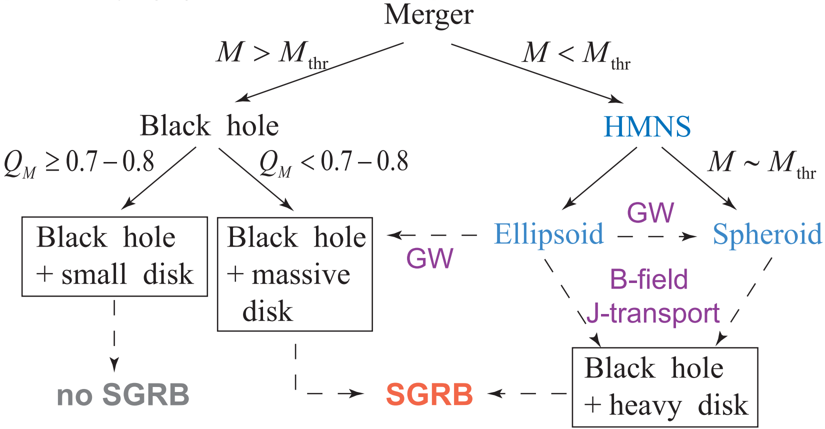


[Szilagyi et al; 1502.04953]

- Standard PN $\sim$ 10%
- Phenom $\sim$ 30%
- uncalibrated EOB $\sim$ 0.1%
- calibrated EOBNR $\sim$ 0.02%

Binary neutron stars

[Shibata, Uryū; gr-qc/9911058] **Review:** [Faber, Rasio; 1204.3858]



Merger

$M > M_{\text{thr}}$

$M < M_{\text{thr}}$

Black hole

HMNS

$Q_M \geq 0.7 - 0.8$

$Q_M < 0.7 - 0.8$

$M \sim M_{\text{thr}}$

Black hole + small disk

Black hole + massive disk

GW

Ellipsoid

GW

Spheroid

B-field

J-transport

no SGRB

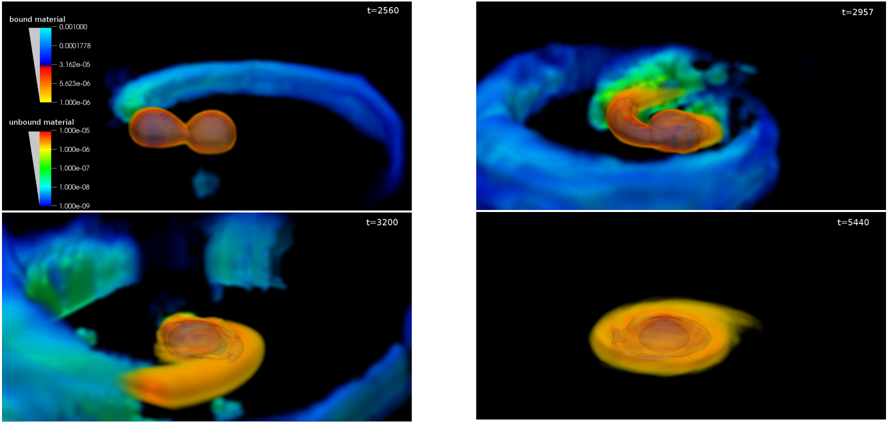
SGRB

Black hole + heavy disk

- small: $<< 0.01 M_\odot$, massive: $0.01 - 0.03 M_\odot$, heavy: $> 0.05 M_\odot$.

[Shibata, Taniguchi; astro-ph/0603145]

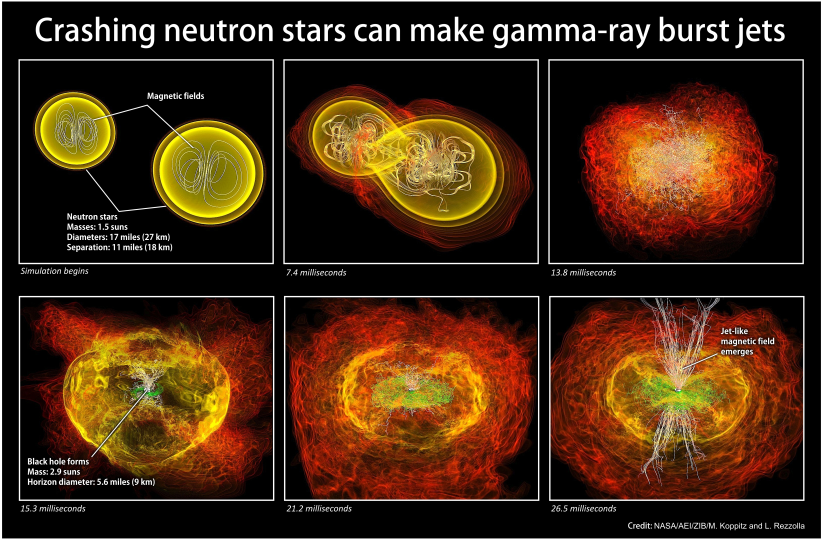
Binary neutron stars



[Dietrich et al; 1504.0126]

Binary neutron stars: Jets

[Rezzolla et al; 1101.4298]



Crashing neutron stars can make gamma-ray burst jets

Simulation begins

7.4 milliseconds

13.8 milliseconds

15.3 milliseconds

21.2 milliseconds

26.5 milliseconds

Credit: NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

[Dionysopoulou et al; 1502.0202]

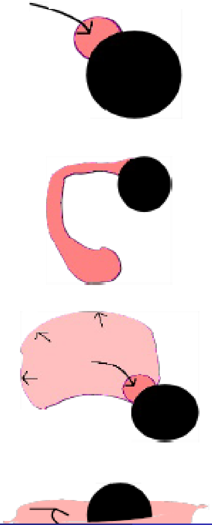
Black hole - neutron star binaries

[Shibata, Uryū; astro-ph/0611522]

Review: [Shibata, Taniguchi; Living Rev. Relativity, 14, (2011), 6]

- Numerical challenges of both BHs and NSs
- Want to learn:
    - When does NS disrupt?
    - How big is the the disk?
- Potential engine for short GRB
- Potential source for heaviest elements
- Also can form jets

  [Paschalidis, Ruiz, Shapiro; 1410.7392]



Future of Post-Newtonian Expansion: 4 PN

- Equations of motion known in ADM-TT gauge
  new effect: radiation-reaction from tails

  [Damour, Jaranowski, Schäfer; 1401.4548]
- EOM soon in harmonic coordinates

  [Marsat, Bohe, Faye, Blanchet, Buonanno]
- 2016? 4 PN energy flux, gravitational wave modes
- Need higher-order spin terms in fluxes and modes.

Future of binary black holes

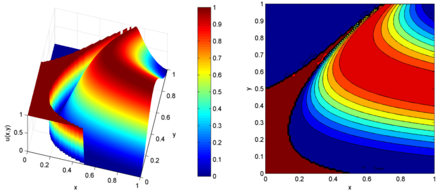
- Push parameter space, longer simulations
- Nail down the details
  - better initial data
  - subtle issues with extracting gravitational waveforms
  - extracting gauge invariant information
- Explore possible EM counterparts (e.g. circum-binary disk)
- Alternative theories of gravity?

Future of BH-NS and NS-NS simulations

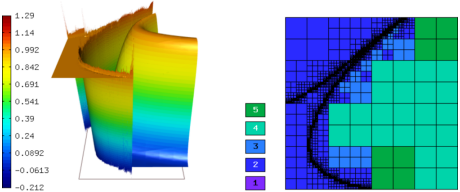
- Fully explore fate as a function of parameters
  - BH-NS: disruption vs plunge
  - NS-NS: prompt collapse vs delayed collapse
  - size, mass, and composition of accretion disk
  - mass and composition of ejecta
- More realistic descriptions of the neutron star
  - Equations of state that satisfy astrophysical and laboratory constraints
  - Model the crust
- Multi-messenger astronomy
  - Gravitational wave signature
  - Electromagnetic signature
    - magnetic fields
    - radiation transport
    - r-process nucleosynthesis
  - Neutrino signature
- Want higher accuracy!

Future of Numerical Relativity: Discontinuous Galerkin Methods

2-d Advection-reaction problem, exact solution:



h-p adaptive DG solution:



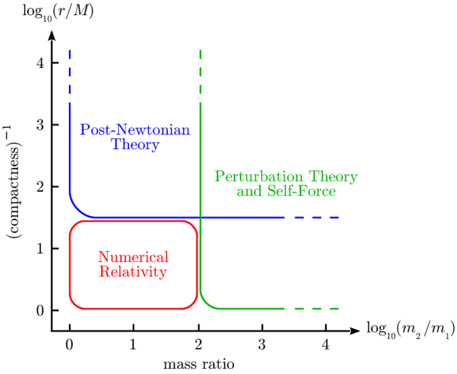
Solution

Mesh and polynomial orders (85220 DOF)

[www.hpfem.org/hermes]

- Finite element method in which solution is allowed to be discontinuous across element boundaries.
- High-order method that recover spectral accuracy in smooth regions
- Advantages of high-resolution shock-capturing methods
- Well suited for h-p adaptivity
- Local method well suited for massive parallelism

Conclusions



[Le Tiec; 1408.5505]

- Compact binaries can be modelled with a variety of methods.
- Pushing post-Newtonian methods to higher order was needed to accurately model the late inspiral.
- Numerical relativity was critical in bridging the gap from late inspiral through ringdown.
- The methods agree within their expected errors.
- Still work to do for spacetimes with matter.