Interpreting gravitational wave measurements as constraints on binary evolution?

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Syracuse, May 29 2009

Outline I : Internal

- * background
  - GW measurements of binaries: review slides (lots available)
  - Expected rates & therefore number (conservative assumptions to be described later)
  - What will we get out… put into context
- Formation model
  - Stellar ev (?) and binary ev (?)
  - Uncertainties I: Model itself
    - Evolution (CE2); SN kicks; winds (strength + character)
  - Uncertainties II: Input uncertainties
    - SFR
    - Z distribution (detection-weighted); Z evolution

- Mass distribution constraints
  - Speculation re generic constraints
- Conclusions
  - Lots of info…but hard to break degeneracies of metallicity
  - Spin alignment helpful…probably distinguishable

Outline

- GW measurements of binaries
- Predictions and uncertainties: Binary evolution
- Constraints: Mass distribution only
- Spin and alignment?
- Conclusions

Binary sources

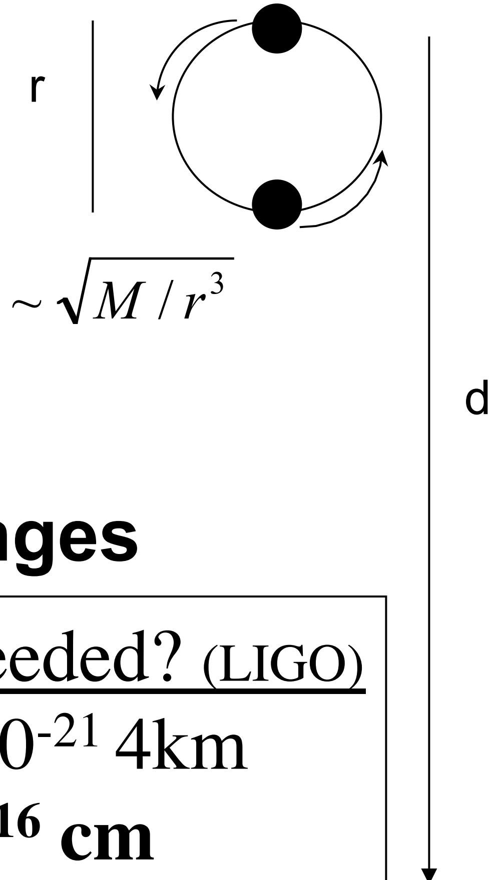
- **Example**:

  Two black holes

  Newtonian circular orbit

$$f = 2 f_{orb} = 2(\Omega / \pi)$$

$$f = 10^3\, Hz\, (M / 8M_o)^{-1} (r / 6M)^{-3/2}$$



$$\Omega \sim \sqrt{M / r^3}$$

- **Characteristic** relative **length changes**

  ~ (kinetic energy)/(distance)

$$h \sim \frac{1}{d} \frac{d^2 I}{dt^2} \sim \frac{Mv^2}{d} \sim \frac{M}{d} (M$$

$$h \sim 10^{-21} (M / 8M_0)^{5/3} (d / 3$$

<u>Sensitivity needed? (LIGO)</u>

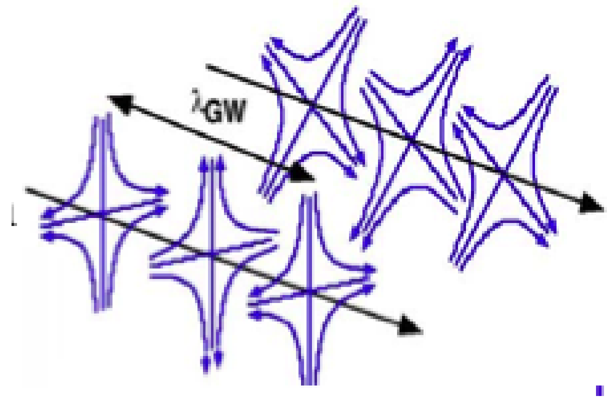
ΔL ~ h L ~ 10^{-21} 4km

~ 4 x 10^{-16} cm

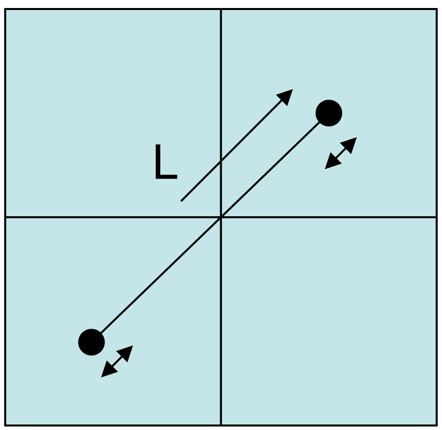
laser light	~ 10^{-4}cm
atom	~ 10^{-8}cm
proton	~ 10^{-13}cm

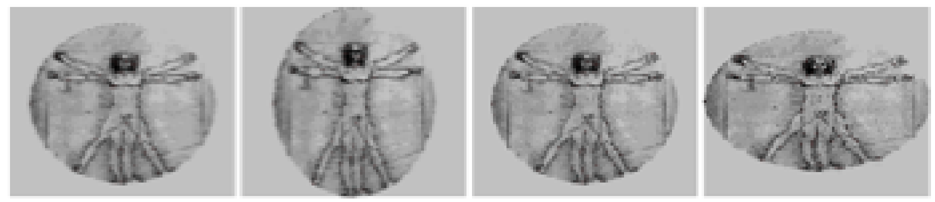
Gravitational plane waves

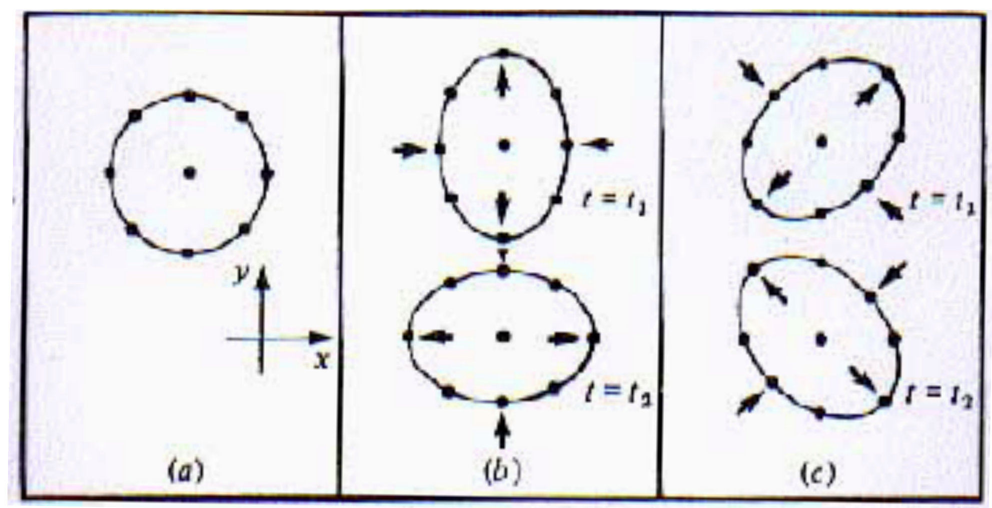
- Stretching and squeezing
  Perpendicular to propagation
- Two spin-2 (tensor) polarizations



$$h \sim \Delta L / L$$







$h_{xx} = -h_{yy}$ $h_{xy} = -h_{yx}$

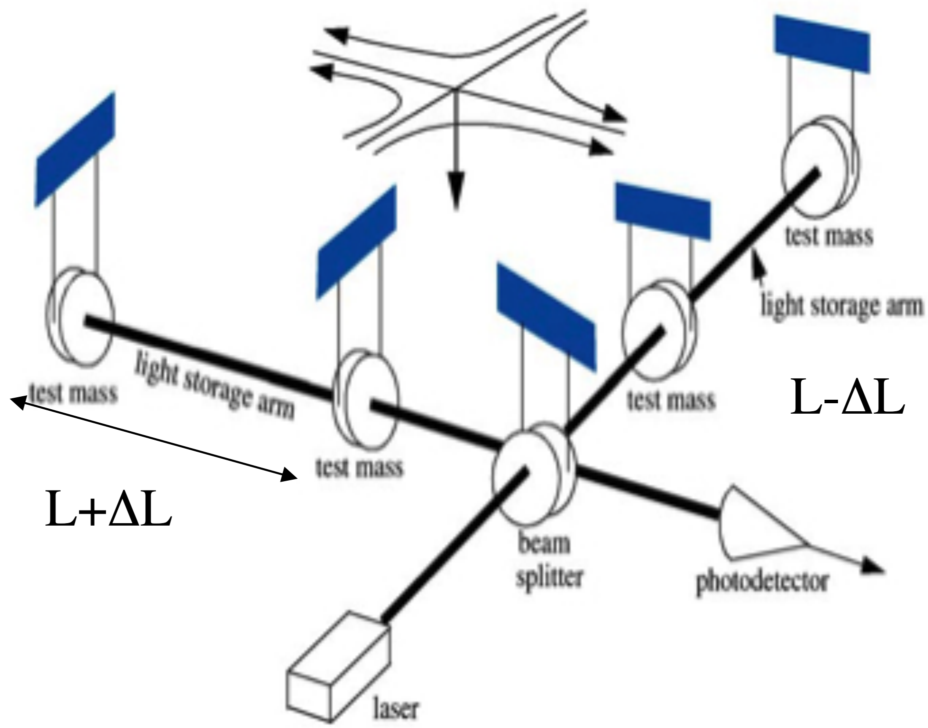
Detecting gravitational waves

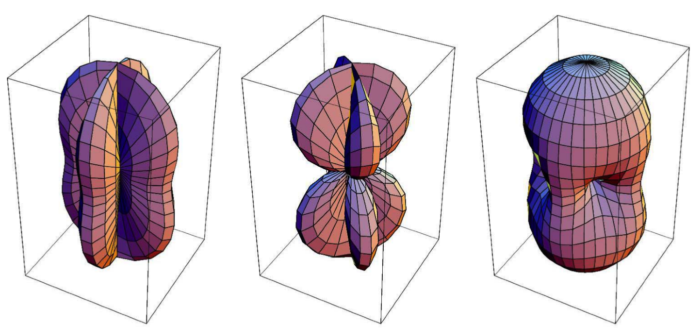
- Interferometer:
  - Compares two distances
  - Sensitive to

$$f \approx 1/t_{store}$$

[tunable]

  - Each interferometer = (weakly) directional antenna





+ polarization

× polarization

averaged

Jay Marx, Texas symposium 2006

GW measurements of binaries

- <u>Mass</u>
  Must match!
  df/dt -> mass
  [mass *ratio* : fine structure]

- <u>Distance</u>

$$SNR \propto \frac{M^{5/6}}{d}$$

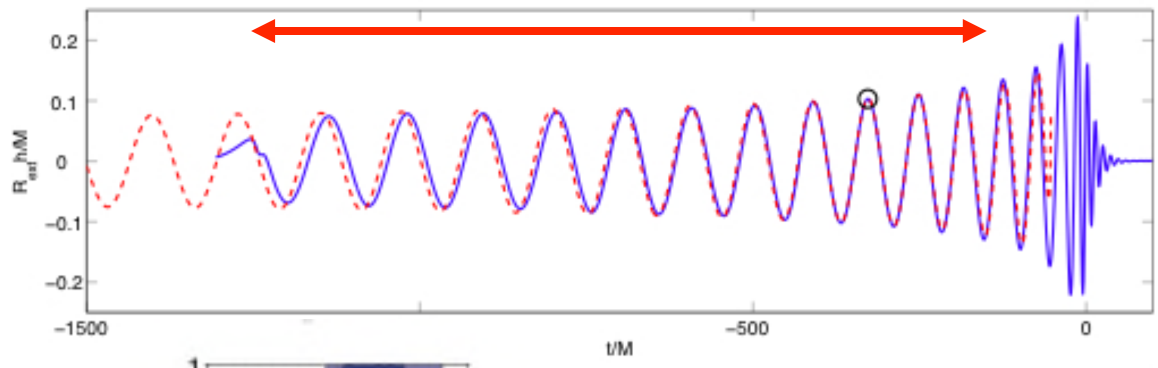
- <u>Orbit orientation</u>:
  Measure beaming?…but
  – Distance-inclination degeneracy

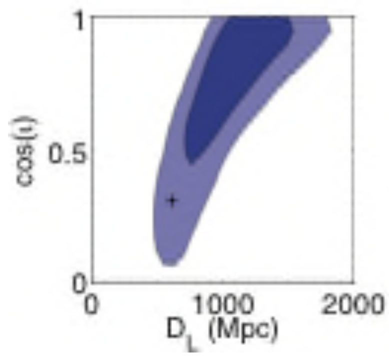
$$\delta X/X \simeq O(1)/\rho$$

  significant vs beaming angle

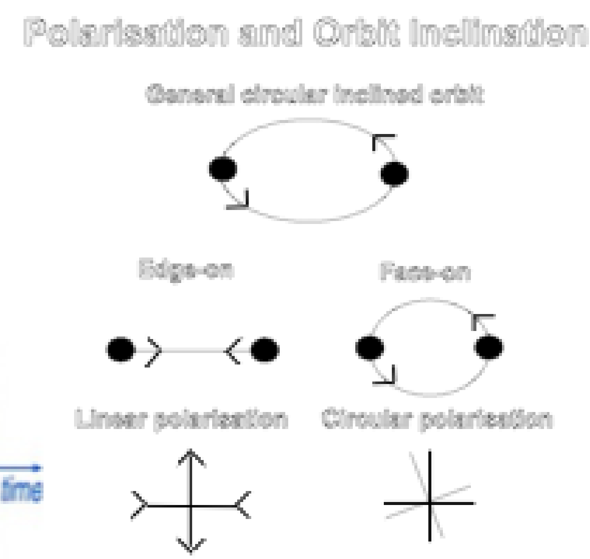
- <u>(Black hole) spin</u>
  Precession
  Only if extreme

  Alignment test => cluster origin test?
  – Possible
  – Must correct
    - mild intrinsic bias for alignment
    - Significant search strategy bias against arbitrary spin



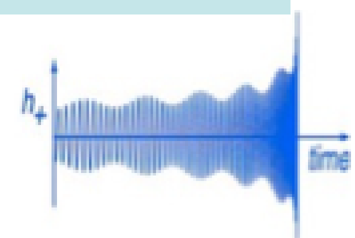


Nissanke et al 0904.1017



Beamed, polarized emission

Spin-orbit coupling



GW: Binary parameters

Rule of thumb:

$$\delta X/X \simeq O(1)/\rho$$

Real calculation:

Van der Sluys et al 0710.1897

a=0.5,Θ=20°

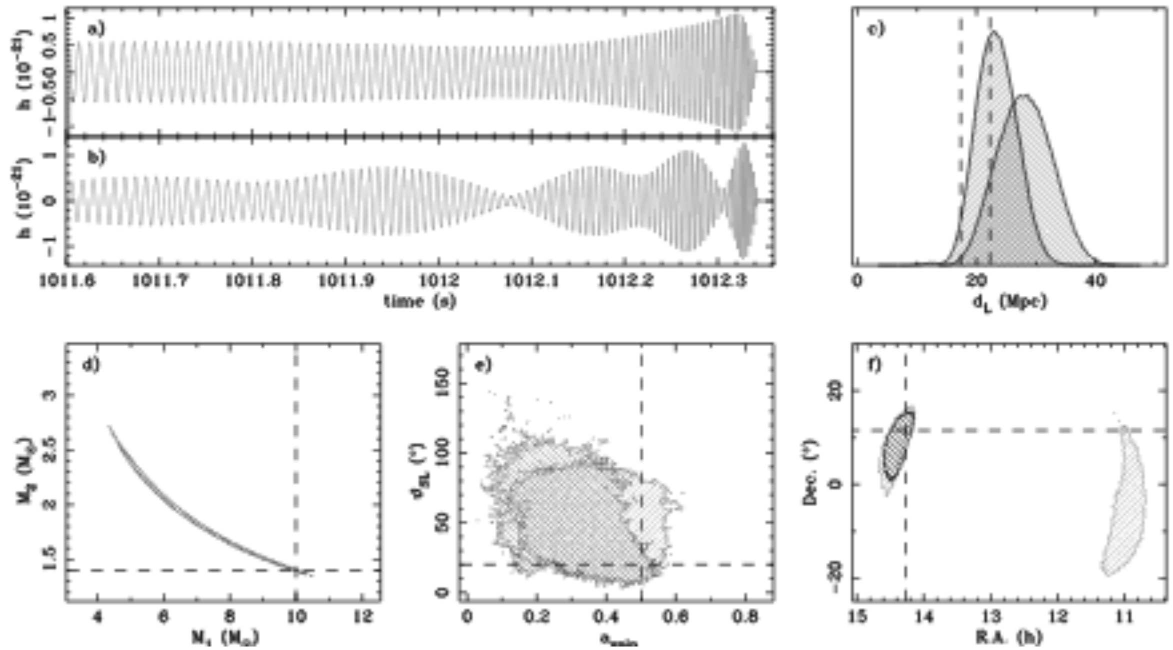


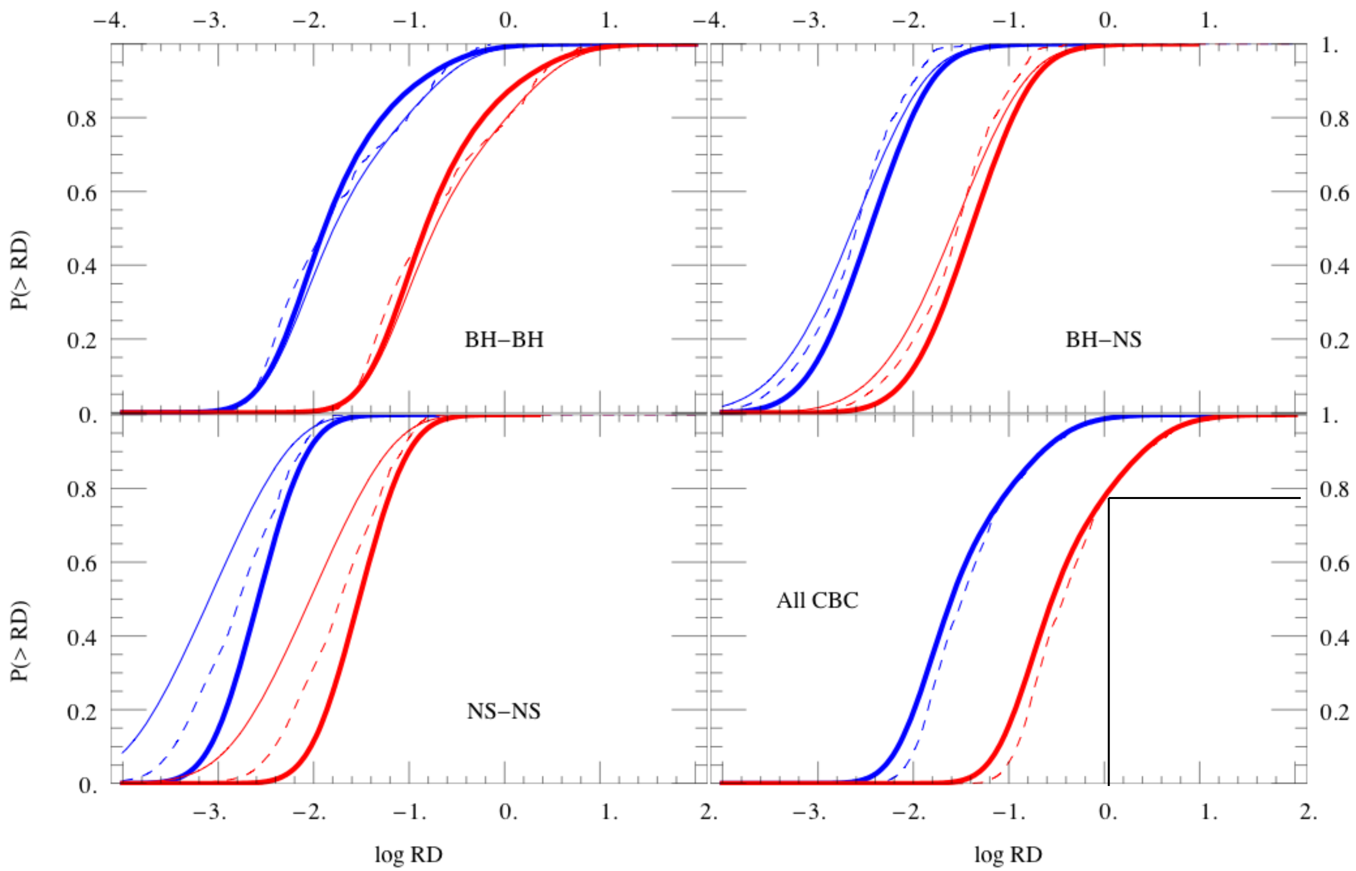
Table (SNR 17, 2-detector)

TABLE 1
INJECTION DETAILS AND WIDTHS OF THE 90%-PROBABILITY INTERVALS OF THE MCMC RUNS DESCRIBED IN THE TEXT

n_{det}	a_{spin}	θ_{SL} (°)	d_L (Mpc)	M_1 (%)	M_2 (%)	$\mathcal{M}$ (%)	η (%)	t_c (ms)	d_L (%)	a_{spin}	θ_{SL} (°)	ϕ_c (°)	α_c (°)	Pos. (°^{2})	Ori. (°^{2})
2	0.0	0	16.0	95	83	2.6	138	18	86	0.63	—	323	—	537	19095
2	0.1	20	16.4	102	85	1.2	90	10	91	0.91	169	324	326[a]	406	16653
2	0.1	55	16.7	51	38	0.88	59	7.9	58	0.32	115	322	326	212	3749
2	0.5	20	17.4	53[b]	42[a]	0.90	50[b]	5.4	46[a]	0.26	56	330	301[b]	111[a]	3467[a]
2	0.5	55	17.3	31	24	0.62	41	4.9	21	0.12	24	323	269[a]	19.8	178[a]
2	0.8	20	17.9	54[a]	42[a]	0.86[a]	54[a]	6.0	56	0.16	25[a]	325	319	104[a]	1540
2	0.8	55	17.9	21	16	0.66	29	4.7	22	0.15	15	320	323	22.8	182[a]

Roever et al gr-qc/0609131
Cutler and Flanagan
Van den Broeck and Sengupta
Bose and Ajith 0901.4936

Expected measurements



Key
Blue : D_{bns} =15 Mpc
Red : D_{bns} =27 Mpc

Heavy : best
(errors+ constraints)

Dashed :
raw simulation data

Thin :
no PSR constraints

Net detection probability (slightly out of date):

$$P_{\text{detect}} = 0.34 + 0.64 \log \frac{VT}{V_c \text{yr}}$$

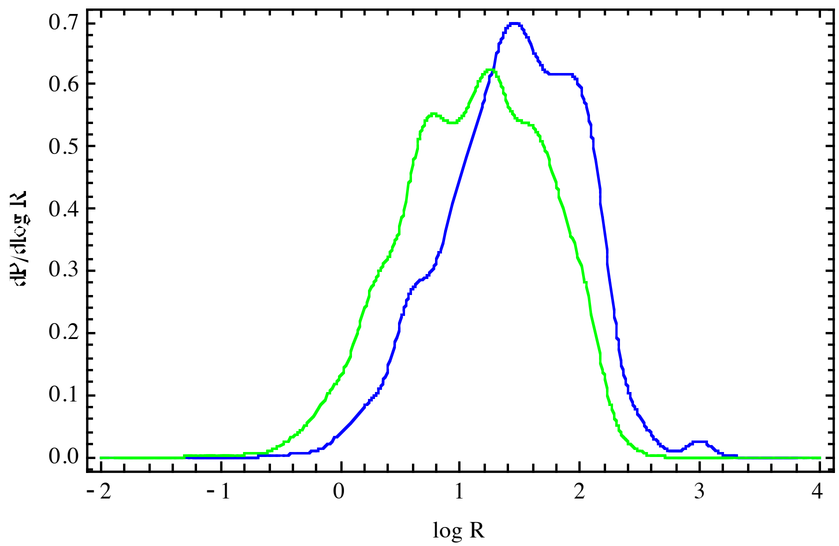
Expected measurements

Advanced detectors

Isolated binary evolution:O(3-100/yr)

aLIGO network with **blind search**, SNR 8

SNR range: 8 (min) -> $8n^{1/3} \lesssim 17$



O' Shaughnessy et al 2009, in prep

sGRB coincident signals?

Overall: O(70-200/yr) all sky (above BATSE/Swift photon count cut cut)

Estimate: Roughly uniform in z

$$R_{GRB+GW} \simeq D_{LIGO} H_o \frac{R_{GRB}}{\Delta z}$$
$$\simeq 0.1 R_{GRB} \simeq O(7-20/\text{yr})$$
$$\simeq 0.2 R_{GRB} \simeq O(14-40/\text{yr})$$

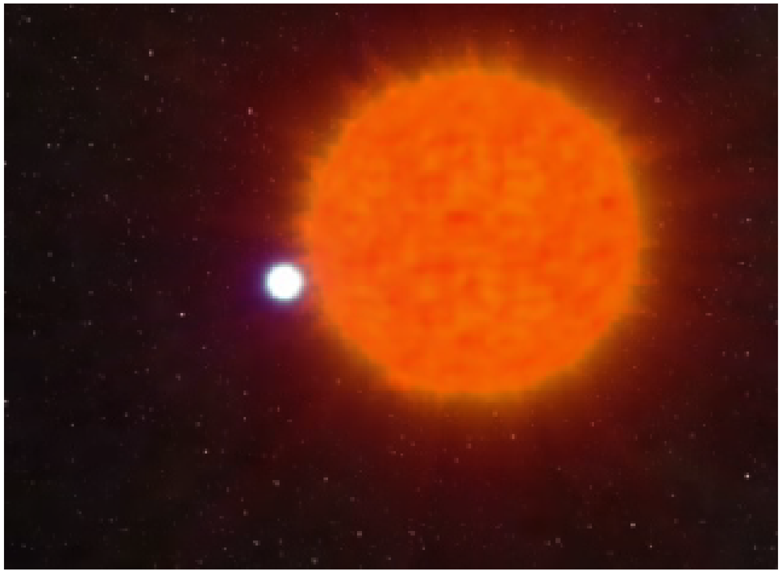
cf Dietz 0904.0347
Beware short-distance/
low-L extrapolation

Formation model

Isolated binary evolution

Outline of typical evolution

- Evolve and **expand**
- Mass transfer (perhaps)
- Supernovae #1
- Mass transfer (perhaps)
- Supernovae #2



Movie: John Rowe

Model uncertainties

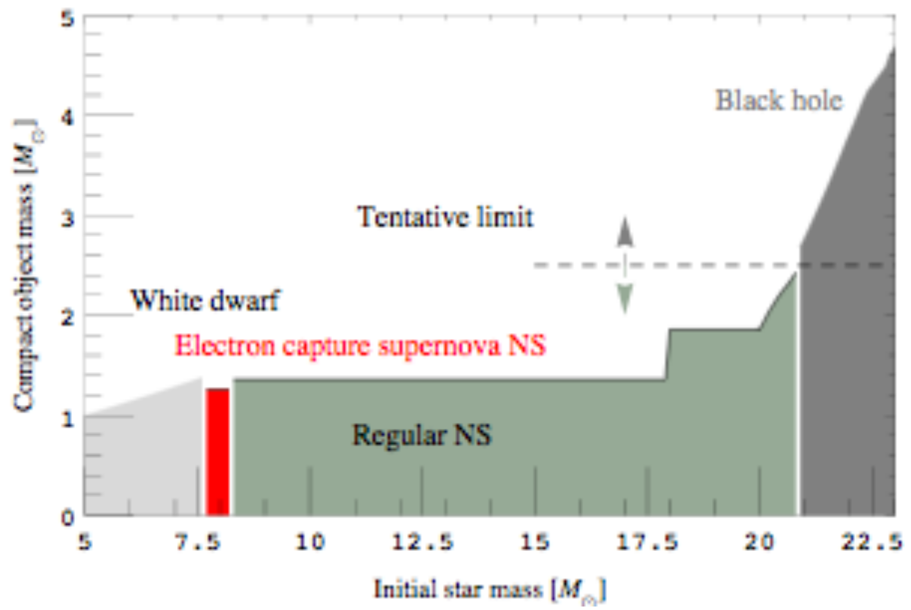
- <u>Evolution model</u>

  Hertzprung gap merger

<u>Others…</u>

Bondi accn rate (& AIC)

NS maximum mass



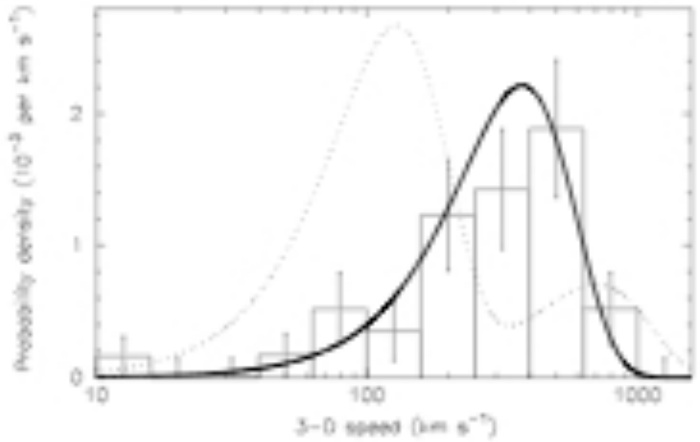
Model uncertainties

- Evolution model
- <u>Supernova kicks</u>

Isotropic kicks?

Hobbs vs Arzoumanian

<u>Group: explore all</u>



Hobbs et al

Polar?

Motivation: Spin-kick alignment?

(e.g., neutrino/B/.. kick)

For: obs claims (Lai et al 2001; Wang; Ng Romani Kaplan et al 2008);

Against: Willems et al 2008 (low kicks required to fit PSR-NS e; high kicks seem required for others)

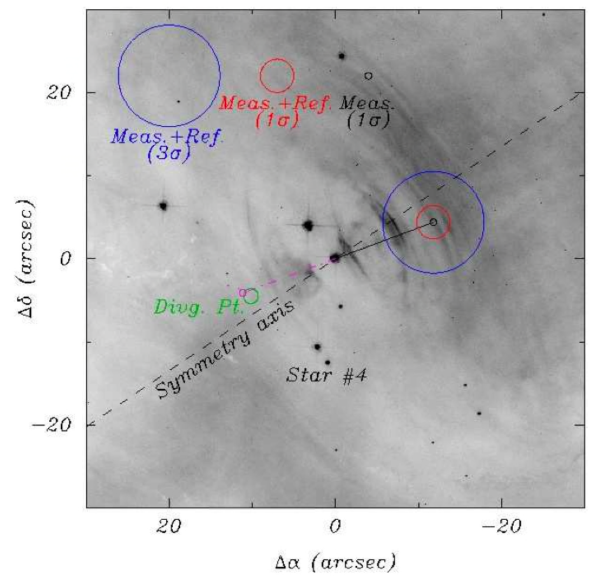
Impact for us:

huge rate reduction b/c never "kicking closer"

Kuranov et al 0901.1055; Postnov & Kuranov 0710.4465

<u>Group</u>:<u>not explored extensively now</u>; could be

Crab motion

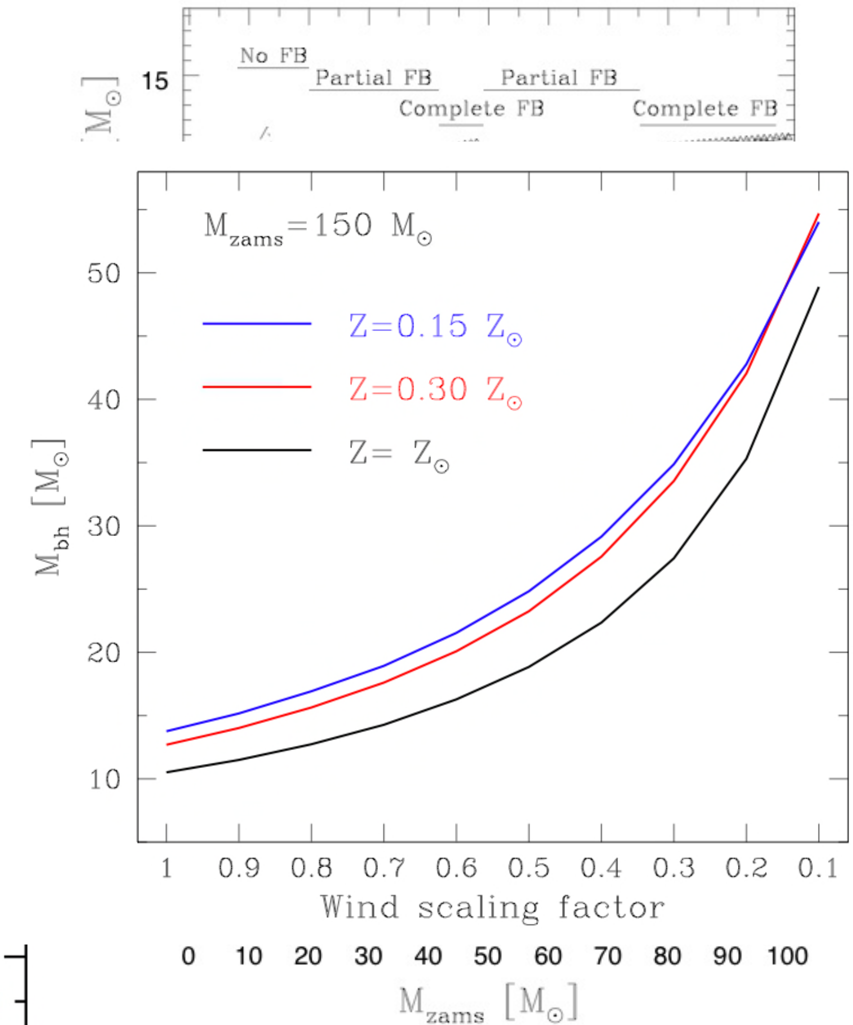


Model uncertainties

- Evolution model
- Supernova kicks
- <u>Winds</u>

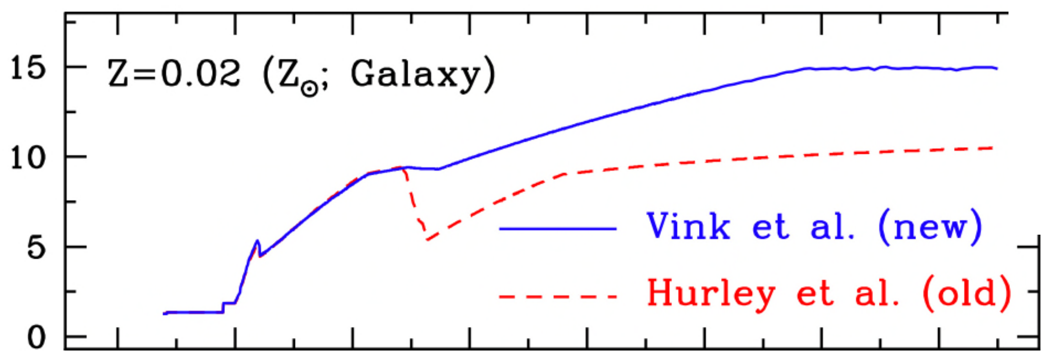
  Strong effect on star->BH mass

  Recent update



Belczynski et al 2002

"original" winds + scale factor



Belczynski et al 2009

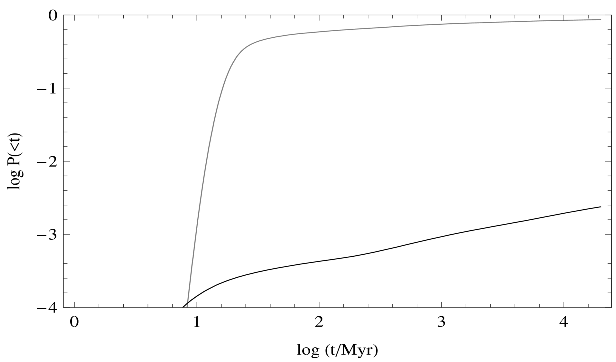
"revised" winds

Input uncertainties

Star formation history

- Normalization nearby
- Normalization at z ~ 1

+ long merger delays



log [P(<t)] (cumulative)

NS-NS : Gray

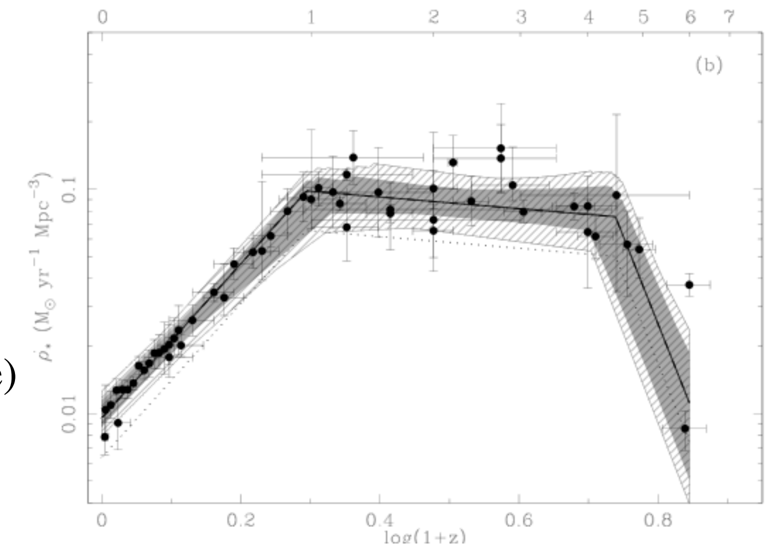
- More from **short** delays

  (extremely short in example)

BH-BH : Black

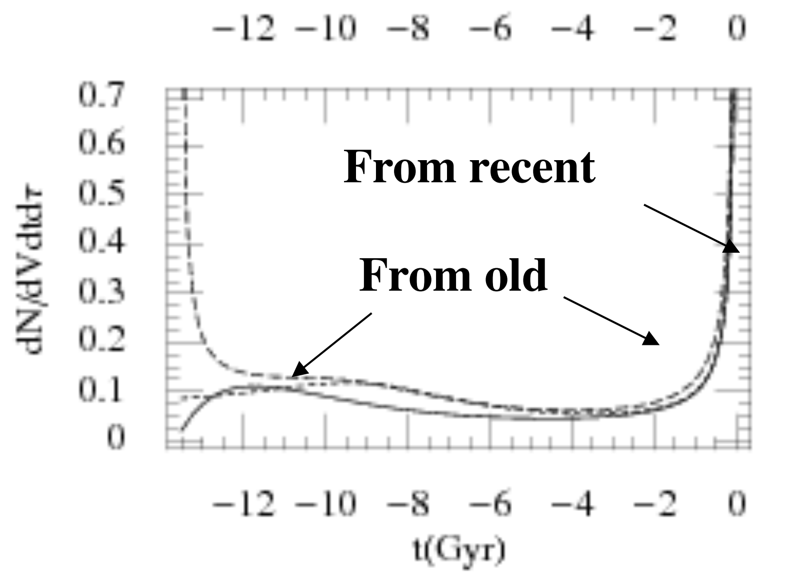
- mostly from **long** delays (Gyr)

  (note *log* scale)



Hopkins & Beacom ApJ 651 142 2006 (astro-ph/0601463): Fig. 4

Plot:
Birth time for present-day mergers



Binary fraction

Abt 1983; Duquennoy and Mayor 1991;

Lada 2006

Input uncertainties

Metallicity

- BH-BH progenitor observed in low-Z enviroment (IC 10 X-1)

  (Bulik et al 0803.3516)

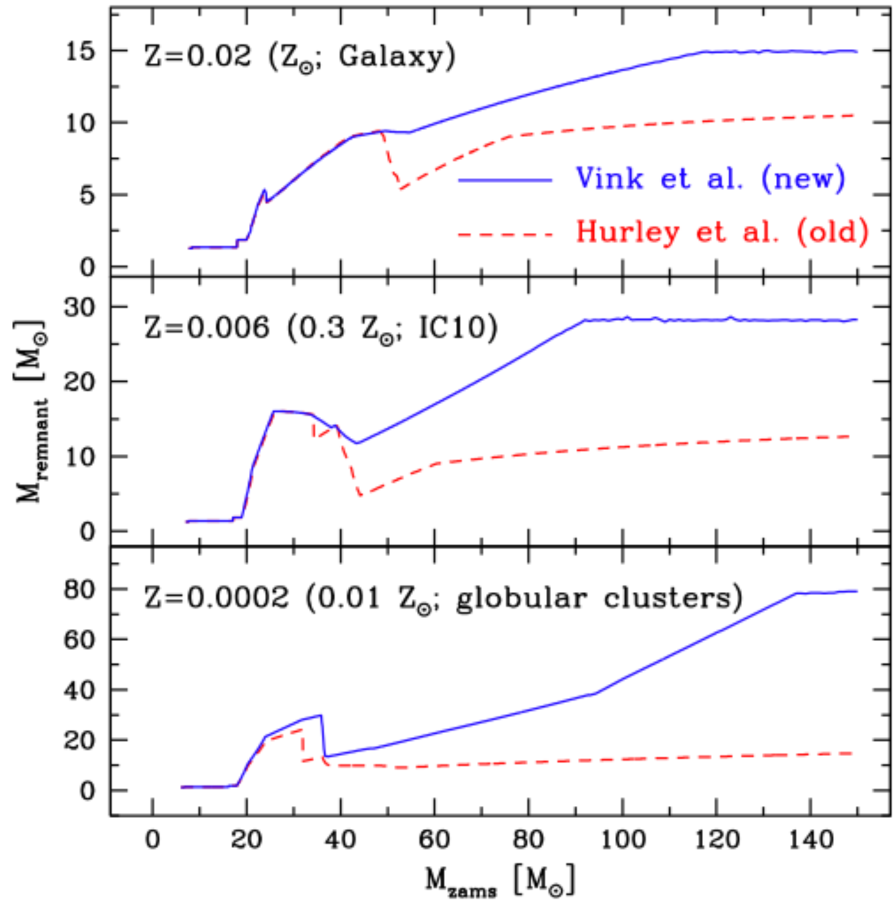
- BH mass, via winds, sensitive to Z

  (e.g., Belczynski et al 0904.2784)

Metallicity evolves significantly over time + long merger delays

and metallicity distribution even now

….expect atypical/low-Z environments to dominate detection rate?

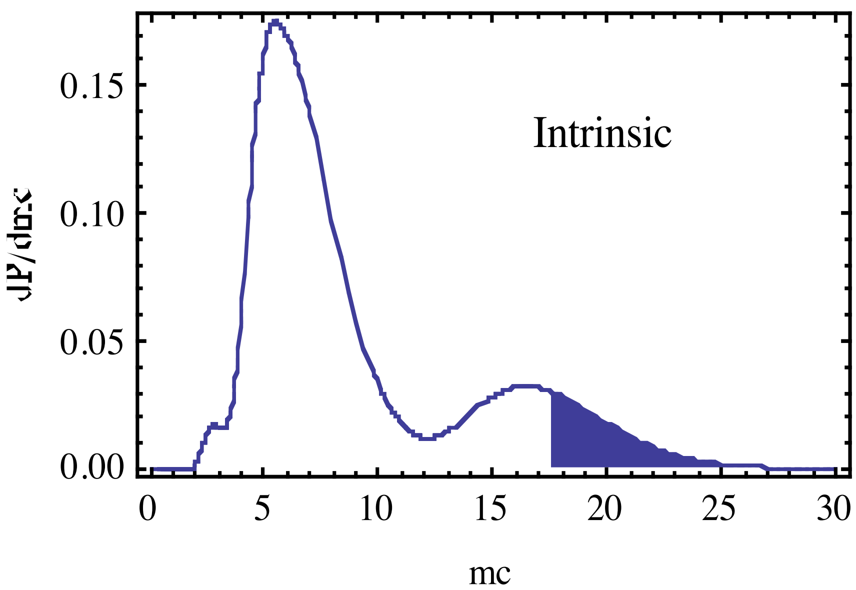


Belczynski et al 0904.2784

Practical challenges

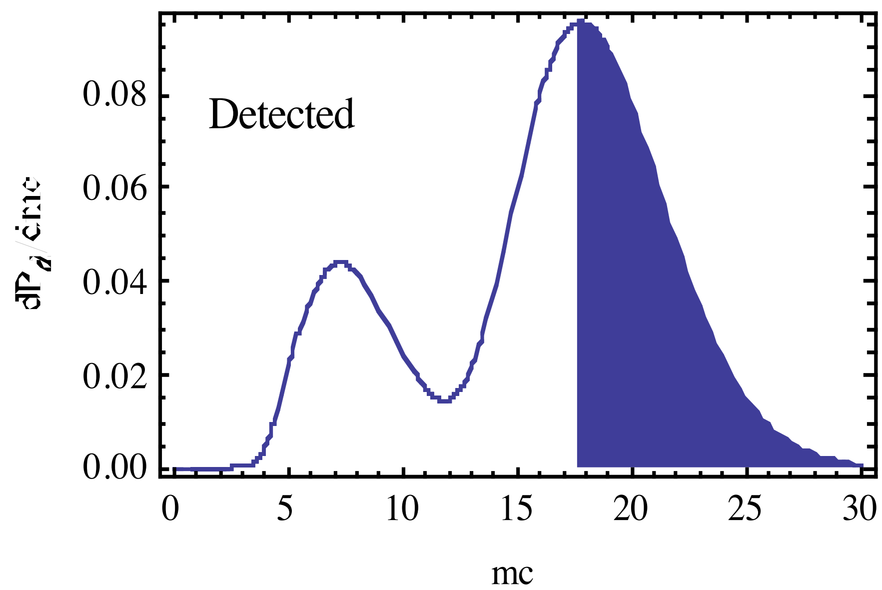
Detection-weighted bias…

Intrinsic



High mass: 10%

Detected



High mass: 50%

…makes BH-BH important, but …

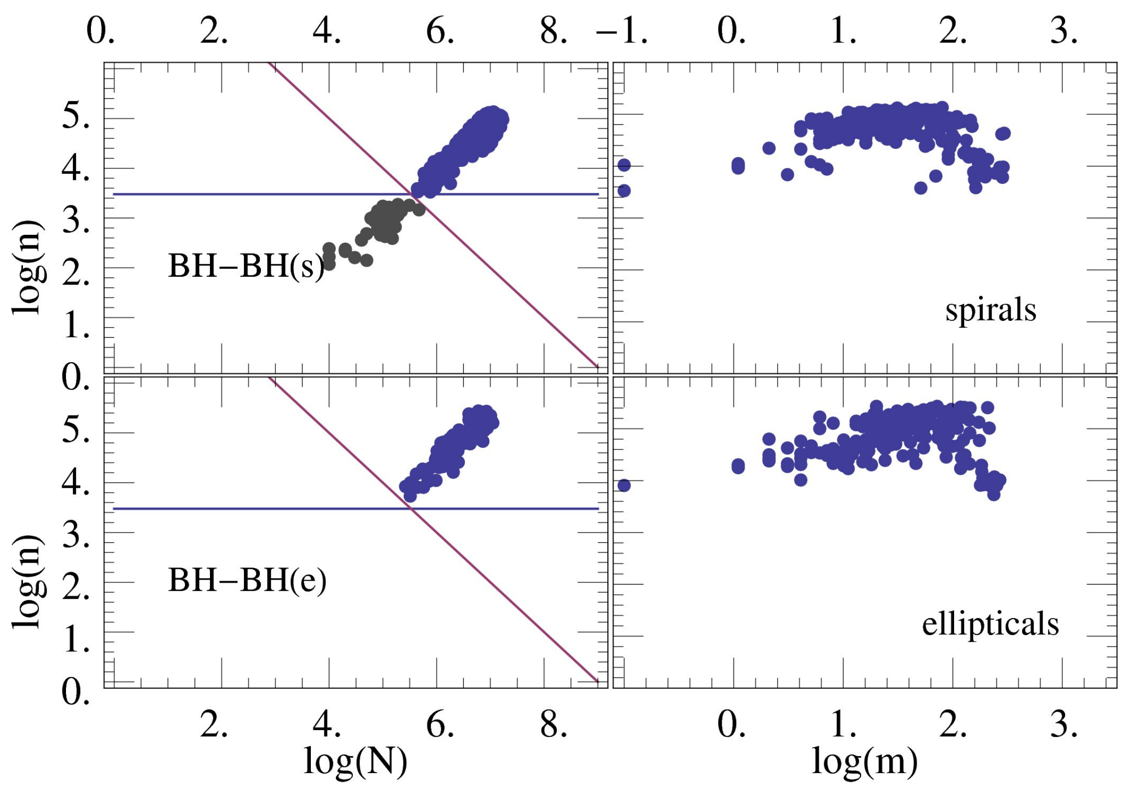
Practical challenges

BH-BH statistics require long computations

IMF -> rare, but

Don't want to cut off low mass
a priori, usually

1200 bin/hour/CPU -ish



Model-data comparisons

Number only

Method:

- Try each model H_k (= many trials of ‘n’)
- Ambiguity function
  - P(k|q) : poisson-ish

Outputs:

- *Model fraction left*: F(k) (90% probability)

  Intuition: F ~ $n^{-1/2}$

  Really:

  - accuracy **+ how often simulations occur**
  - very high, low rates excluded

Comments:

- Many uncertainties not added here:

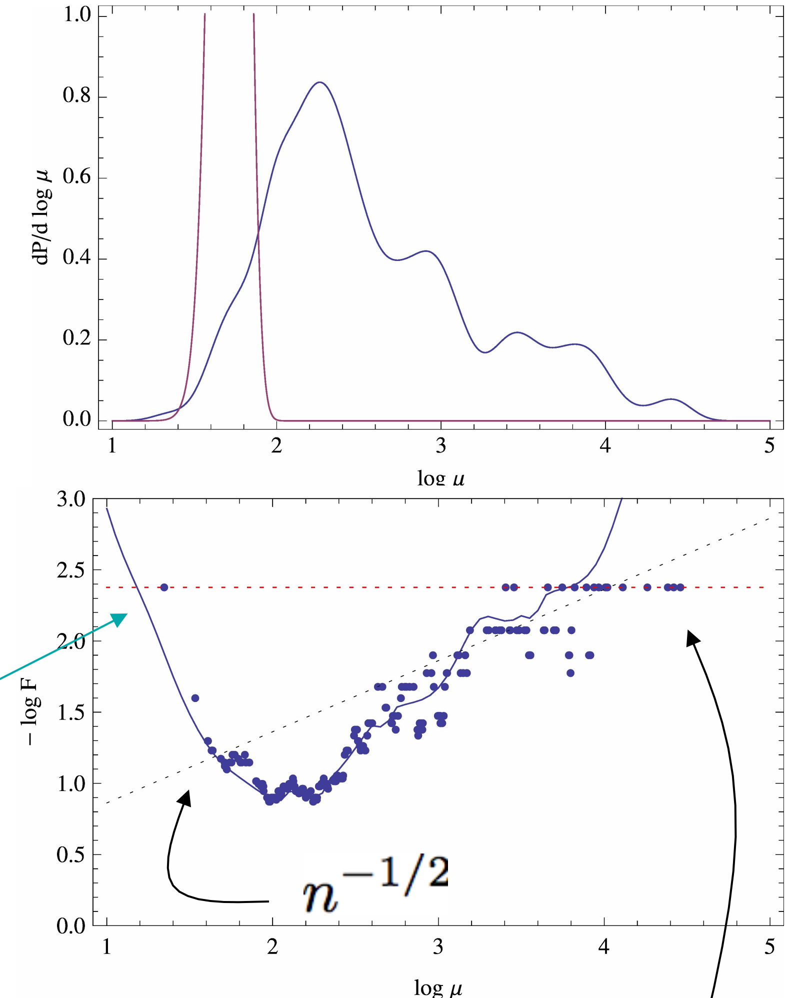
  simulation accuracy (monte carlo);

  star formation rate; metallicity;

  “fuzzy” detection surface

  realistic detection issues

  (waveform model systematics; calibration; …)



Direct tests

Limited by

1/(# of simulations)

Model-data comparisons

Expected improvement?

- <u>No model constraints?</u>:

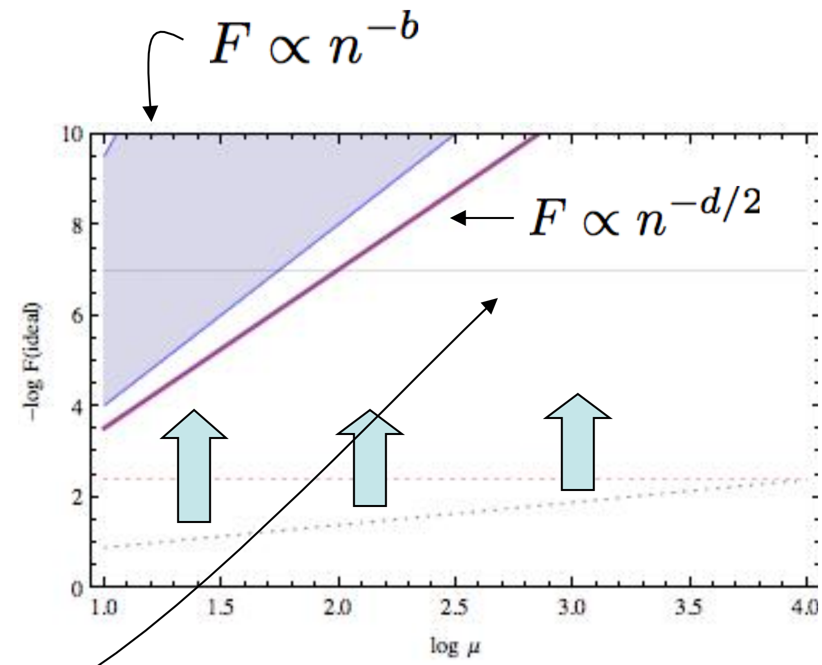
  Each detection specifies ~ b “new” bits

$$b = -\int p(M_c)\log_2[p(M_c)\sigma_m\sqrt{2\pi e}]$$
$$\simeq 3.5 - 9$$

  …. **but** : predicted mass distributions
  very similar ; overcounting “new” bits

- <u>Local likelihood: fisher matrix?</u>:

  …. **but** : weak constraints;
  partial localization usual until
  F ~ $(0.1)^d$ -ish
  (depending on natural scale in L(x))



$$\delta L \simeq -\frac{1}{2} n \Gamma_{ab}\delta x^a \delta x^b$$

$$|\delta L| < \Delta \rightarrow [\delta x]^d \propto n^{-d/2}$$

- <u>Effective dimension!</u>:
  - Count of “currently measured” params
  - Increases ~ monotonically

$$d_{eff} = -2 d\log F / d\log L$$

…**but** : unless many simulations,
bound below

Model-data comparisons

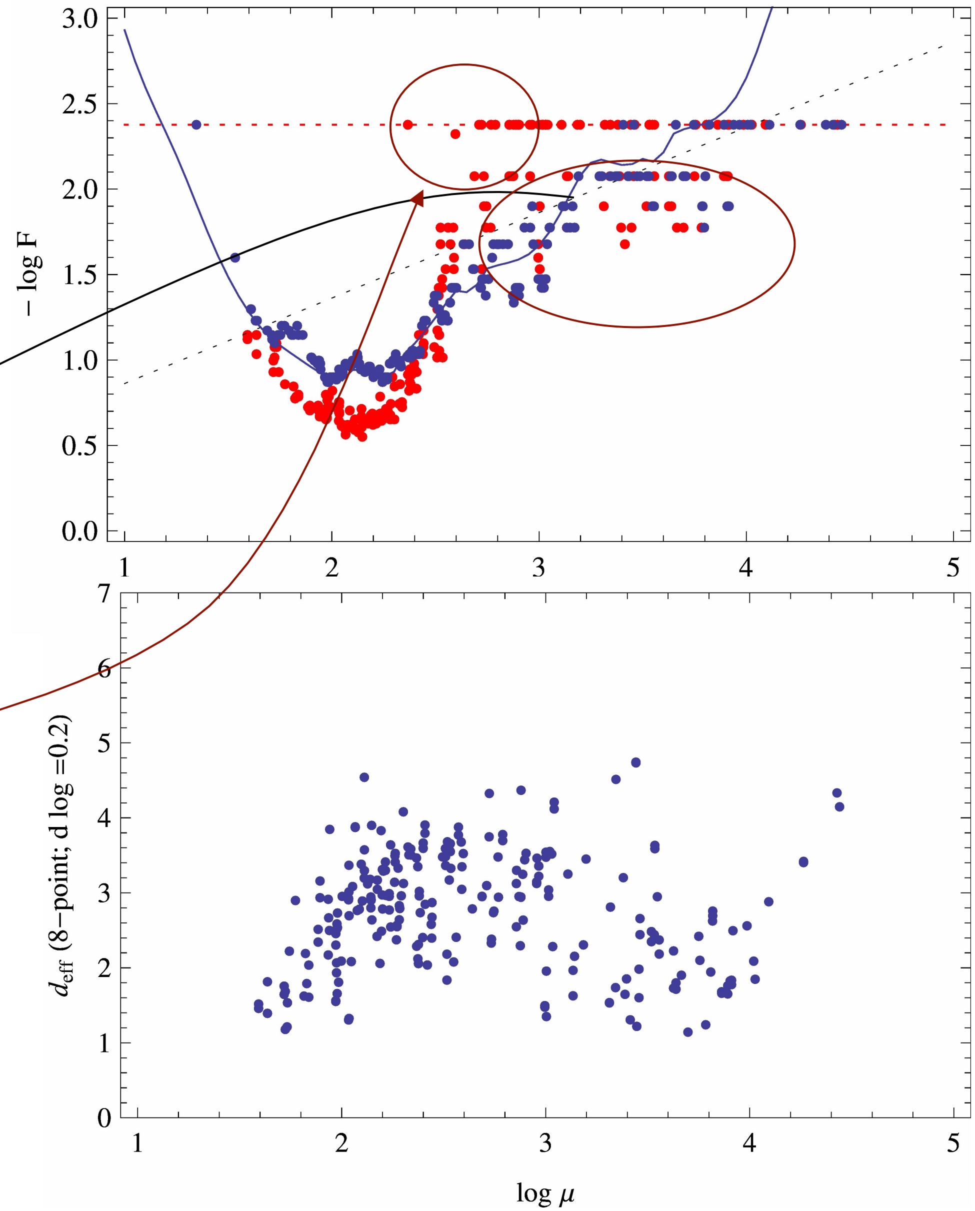
Rate + shape

- Low n:
  - Rate discrimination most important
  - Slight "scatter" :
    small mass distribution effects

- Large n: **not unique** scaling
  - Similar distributions:
    Little gain from mass info
    Rate dominates: scales as sqrt(n)
    Not localized in model space:
    no ~ quadratic degree of freedom
  - Less typical distributions:
    Possibly localized?
    Higher convergence

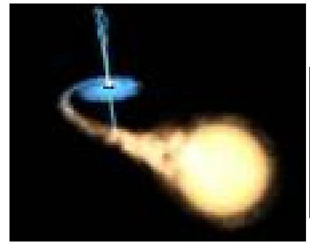
Estimating effective dimension:

- Local fits to likelihood
- Attainable .. **if** BNS range (-> n) higher



Spin?

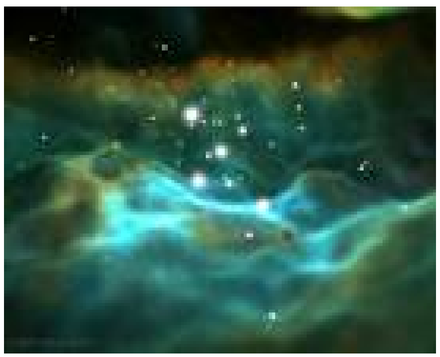
Alignment = signature!



Isolated binaries
Aligned spins

References include
•Belczynski, Kalogera, Bulik 2002; Belczynski
•O' Shaughnessy et al. in prep

+ astro-ph/0610076; 0609465; 0504479



Star forming gas

Interacting clusters' stellar mass binaries
Random spin alignment

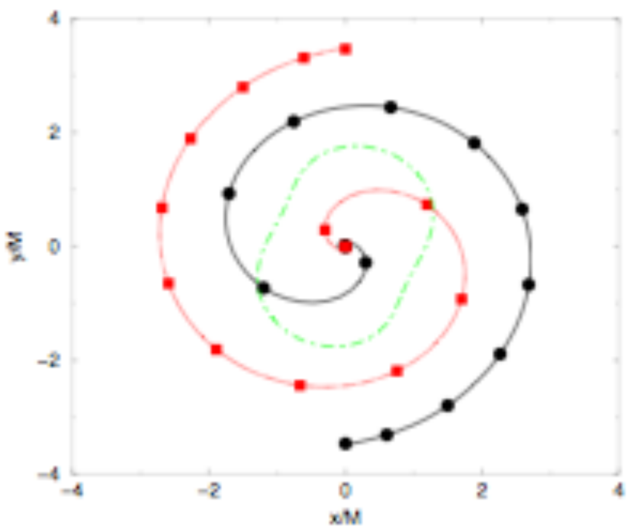
References include
• Sadowski et al 2008
•O' Shaughnessy et al PRD 76 061504
O' Leary et al astro-ph/0508224

Spin alignment test

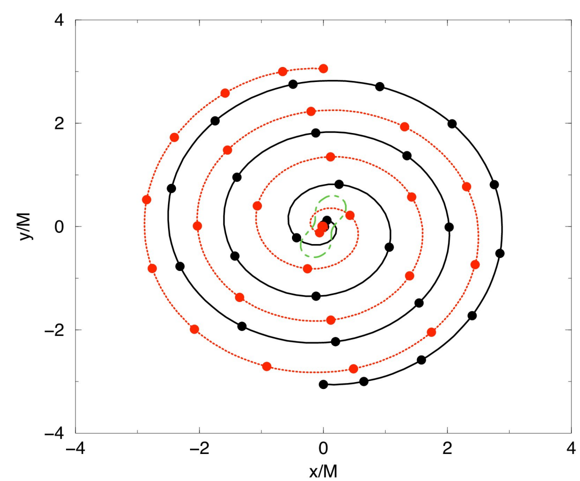
<u>Qualitatively</u>: **Duration**

Longer waveform <-> longer hangup <-> spins aligned

Campanelli et al gr-qc/0604012



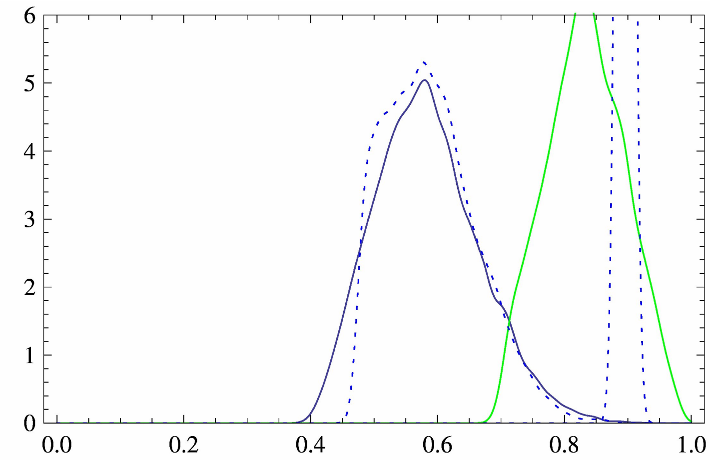
Both down

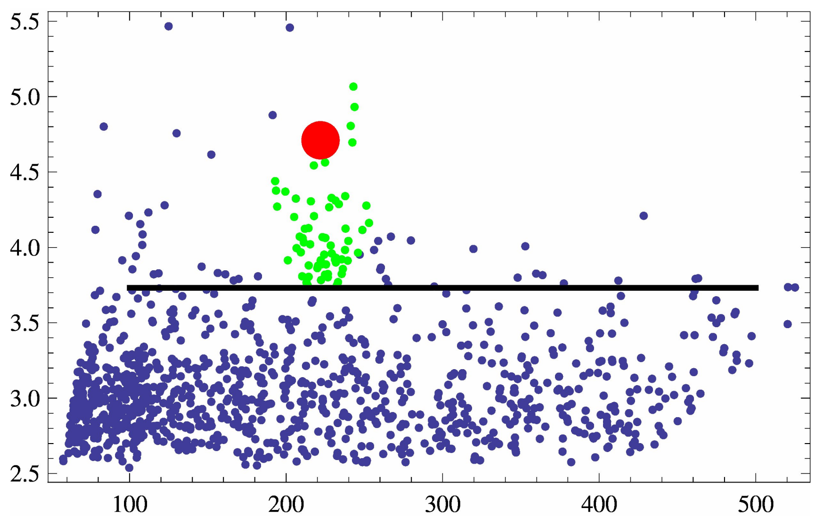


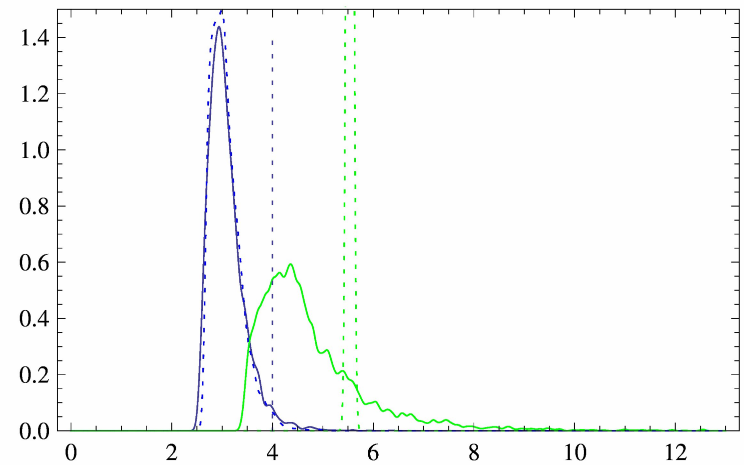
Both up

Spin alignment test?

Example: Ringdown







Spin alignment test?

Example: Inspiral waves (beta, sigma)

$$\begin{aligned}
\psi_f(t_f) &= 2\pi f t_{\rm ref} - \phi_{\rm ref} + \psi_N \sum_{k=0}^{5} \psi_k (\pi m f)^{(k-5)/3} \\
\psi_N &= \frac{3}{128\eta}, \quad \psi_0 = 1, \quad \psi_1 = 0, \\
\psi_2 &= \frac{5}{9}\left(\frac{743}{84} + 11\eta\right), \quad \psi_3 = -16\pi, \\
\psi_4 &= \frac{5}{72}\left(\frac{3058673}{7056} + \frac{5429}{7}\eta + 617\eta^2\right), \\
\psi_5 &= \frac{5}{3}\left(\frac{7729}{252} + \eta\right)\pi + \frac{8}{3}\left(\frac{38645}{672} + \frac{15}{8}\eta\right)\ln\left(\frac{v}{v_{\rm ref}}\right)\pi.
\end{aligned}$$

$$\begin{aligned}
v &= (\pi M f)^{1/3} \\
\Psi(f) &= 2\pi f t_c - \phi_c - \pi/4 \\
&+ \frac{3}{128}(\pi M_c f)^{-5/3}\Bigg[1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\eta\right)v^2 \\
&-4(4\pi - \beta)v^3 \\
&10\left(\frac{3058673}{1016064} + \frac{5429}{1008}\eta + \frac{617}{144}\eta^2 - \sigma\right)v^4 \\
&+\left(\frac{38645\pi}{252} - \frac{65}{3}\eta\right)\ln v \\
&+\left(\frac{11583231236531}{4694215680} - \frac{640\pi^2}{3} - \frac{6848\gamma}{21}\right)v^6 \\
&+\eta\left(\frac{15335597827}{3048192} + \frac{2255\pi^2}{12} + \frac{47324}{63} - \frac{7948}{9}\right)v^6 \\
&+\left(\frac{76055}{1728}\eta^2 - \frac{127825}{1296}\eta^3 - \frac{6848}{21}\ln 4v\right)v^6 \\
&+\pi\left(\frac{77096675}{254016} + \frac{378515}{1512}\eta - \frac{74045}{756}\eta^2\right)v^7\Bigg]
\end{aligned}$$

….A. Lundgren says?

$$\begin{aligned}
\beta &= \frac{\hat{L}}{M^2}\cdot[(\frac{113}{12} + \frac{25m_2}{4m_1})S_1 + (\frac{113}{12} + \frac{25m_1}{4m_2})S_2 \\
&= \frac{1}{12}[(113(m_1/M)^2 + 75\eta)\hat{L}\cdot\hat{a}_1 + (1\leftrightarrow 2)] \\
\sigma &= \frac{\eta}{48}[-247\hat{a}_1\cdot\hat{a}_2 + 721(\hat{L}\cdot\hat{a}_1)(\hat{L}\cdot\hat{a}_2)]
\end{aligned}$$

Summary

Even in best-known case (bin ev only),

large model space to constrain…

Model uncertainty: winds, kicks, NS mass, channels

Input uncertainty : dominant metallicity; f_b

…but measured distributions can do it, eventually…

"Infinite" number of DOF

Some features (BH mass peak; spin alignment) particularly helpful

…**if** we can correct for selection biases

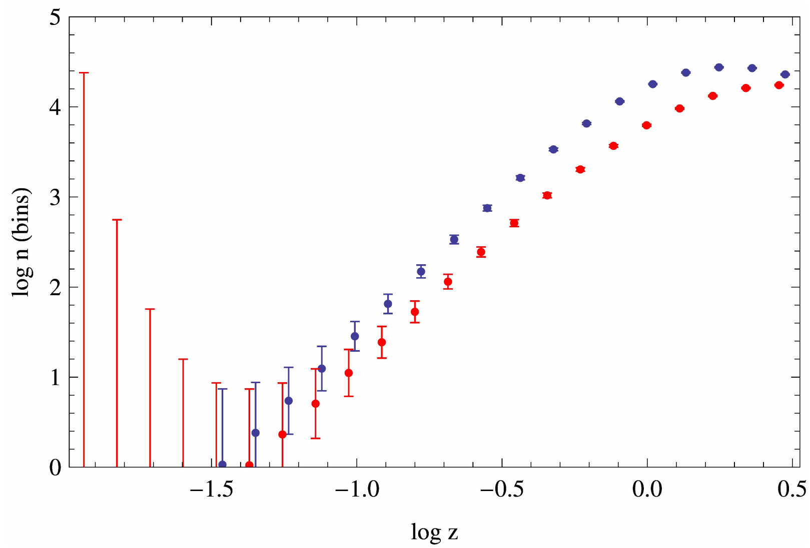
Intrinsic : e.g., aligned easier than nonaligned

Implementation : **report** 8d (s_1,s_2,m_1,m_2) efficiency/SNR threshold?

Bonus: Rate versus redshift

Key points:

- Use: High rate (NS-NS, BH-NS)
  Large lags
- Method(s): Binned vs predicted (χ^2)
  or smoothing; likelihood (like mass); …
- How to use?:
  - **Input** uncertainties (SFR; metallicity)
    +…many models similar up to scale
  - Large source model space, small variations



Conclusion:

High precision measurement of *something*