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Numerical models of black hole accretion flows

Charles F. Gammie [a,*], Scott Noble [b], Po Kin Leung [c]

[a] Physics Department and Astronomy Department, University of Illinois at Urbana-Champaign, USA
[b] Department of Physics and Astronomy, Johns Hopkins University, Baltimore, USA
[c] Astronomy Department, University of Illinois at Urbana-Champaign, USA

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Abstract

Black hole accretion flows are the hot, magnetized, turbulent plasma that surround black holes in astrophysical settings and in some cases make them the most luminous objects in the universe. I review recent developments in the numerical modeling of these plasmas. For weakly radiating flows it is now possible to follow the dynamics of the plasma by integrating the equations of general relativistic magnetohydrodynamics using numerical methods borrowed from nonrelativistic hydrodynamics. Ray tracing, along the curved trajectories followed by light close to the black hole event horizon, can then be used to create simulated observations of these weakly radiating flows. These models are relevant to the faint source at the galactic center, which may soon be directly imaged by millimeter interferometry.

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1. Introduction: Closing in on Sgr A*

The center of our galaxy is now the center of interest for many astronomers. It contains a compact object whose mass, $\approx 3.6 \times 10^6 M_\odot$, can be determined by patient observation of the orbits of nearby stars over a period of years. This is small compared to the compact objects found at the centers of most nearby galaxies, which can be as much as a thousand times larger. It is associated with a $\sim 10^{2.5} L_\odot$ electromagnetic source called Sgr A*. Most of this energy emerges at millimeter and submillimeter wavelengths. Sgr A* is faint for such a massive object; some galactic nuclei are bright enough to outshine their parent galaxy!

The Sgr A* millimeter source has been spatially resolved using a technique called very long baseline interferometry (VLBI) which patches together simultaneous observations taken at sites around the globe. VLBI constrains the source to be small, of order an astronomical unit (1.5×10^{13} cm) in size. The most conservative theory of Sgr A* has the millimeter photons generated by a hot ($\sim 10^{12}$ K), nearly transparent plasma that is gradually sinking into a black hole. The size is about right: the event horizon is expected to be $\approx 2GM_{BH}/c^2 \simeq 10^{12}$ cm $\simeq 0.07$ AU in radius.

Almost all large galaxies like the Milky Way are believed to harbor massive dark objects at their nuclei, but the putative black hole in Sgr A* is special because it is the largest on the sky, with an angular diameter of $9\text{–}18 \times 10^{-6}$ seconds of arc on the sky, not counting gravitational lensing effects that make it appear even larger (the next largest is a thousand times more massive, but a thousand times further away). This makes Sgr A* a prime candidate for future, higher resolution VLBI experiments that may provide a resolved snapshot of the accreting gas [3], and therefore the object of intense theoretical interest.

What will future VLBI experiments see? To answer this we must understand the flow of a hot, magnetized plasma deep in the gravitational potential of a black hole. Preliminary work [1] suggested that submillimeter imaging may show a ring of emission surrounding a dark spot: the event horizon of the black hole. This would go a long way toward confirming that Sgr A* is indeed a black hole. But to do more, to probe the relativistic gravitational field of the hole, will require robust, sophisticated, and predictive models. Because the accretion flow is expected to be turbulent, computational models are needed to enter this era of *precision bothrology*.

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[*] Corresponding author.
E-mail address: gammie@uiuc.edu (C.F. Gammie).

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2. Magnetohydrodynamic models in full general relativity

As a first step toward more realistic models of the galactic center, one can model the plasma as both nonradiating and a fluid. The fluid approximation sharply reduces the dimensionality of the problem, as does the absence of radiation. The fluid can then be integrated using standard techniques borrowed from nonrelativistic hydrodynamics [2,5,7,8]. The nonradiative approximation is excellent for Sgr A*. Usually it is assumed that the resistivity, viscosity, and thermal conductivity of the plasma are negligible. The magnetohydrodynamic approximation is used to evolve the electromagnetic field, and the fluid and field are evolved together in a relativistically correct way, without approximation.

These relativistic fluid models are evolved in the Kerr metric, which describes the gravitational field near an isolated black hole. The Kerr metric has only two parameters: the mass M and the spin of the black hole (spin angular momentum $J = a_* GM^2/c$, and $0 < a_* < 1$ according to general relativity). Charged black holes are possible, but implausible in astrophysics since the charge would be rapidly neutralized by the surrounding plasma. The Kerr metric is an approximation in the sense that it neglects the gravitational field of the accreting plasma; this is an excellent approximation for Sgr A*.

Most computational models start with a donut of centrifugally supported plasma surrounding the black hole. Initially the fluid is placed in a stable equilibrium. It is then threaded by a weak ($\beta \sim 10^2$) poloidal field, which triggers the *magnetorotational instability* and leads to the development of turbulence. Angular momentum evolution, which is impossible in axisymmetric hydrodynamics, can then occur via the magnetic field, producing an effective "turbulent viscosity". Matter in the inner disk loses angular momentum and accretes onto the black hole; matter in the outer disk moves slightly outward so as to conserve angular momentum. The models are run for $\sim 10^3 GM/c^3$, or about 5 hours when the models are scaled to fit the galactic center.

The structure of the turbulent state is shown in Fig. 1. Notice the empty regions over the poles, and the dense equatorial disk. Within the disk conditions vary sharply in space and time. But a typical number density $n \sim 10^7$ protons per cubic centimeter, a typical magnetic field $B \sim 10^2$ gauss, and a typical temperature $T \sim 10^{11}$ Kelvin. As seen by an observer at large distance the plasma speeds approach the speed of light. The corresponding accretion rate is about 10^{17} grams per second, or in astronomically relevant units $4 \times 10^{-9} M_\odot\ \mathrm{yr}^{-1}$.

3. Results of dynamical models

One of the key results to emerge from the dynamical modeling has been that matter accretes onto the black hole with lower specific angular momentum than one would naively expect [6].

The naive expectations are based on the classical "thin disk" model, which consists of a cool fluid circulating about the black hole on circular orbits. In the model, matter spirals in very gradually through the disk until it reaches an innermost stable circular orbit (ISCO; located at $r = 6GM/c^2$ for a nonrotating black hole). At this point circular orbits are marginally stable; further angular momentum loss causes the fluid to enter a "plunging region" where circular orbits are unstable. The fluid then plunges into the black hole, conserving its specific angular momentum along the way. The specific angular momentum of accreted material is therefore the specific angular momentum of a particle on the ISCO.

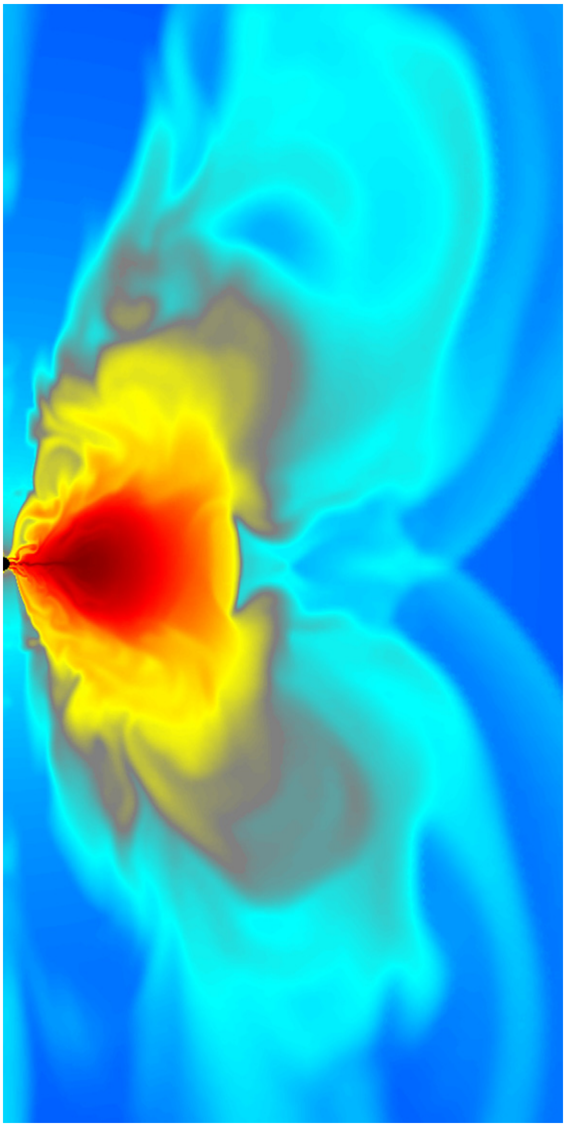

Fig. 1. Meridional slice through rest-mass density in an axisymmetric integration of accretion onto a $a_* = 0.93$ black hole, at the middle left of the figure. The color maps to the logarithm of rest-mass density, and the total density contrast is about 10^6 between the equator and the pole. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In the numerical models one can simply measure the mean specific angular momentum of the accreted matter, and it turns out to be smaller by 10–20% than the thin disk estimates. This can be understood through an analytic model in which a magnetized plasma forms a steady flow between the ISCO and the event horizon [4]. The field links the flow back to the accretion disk, radiating away (magnetohydrodynamically) the angular momentum of the infalling material. By the time the matter

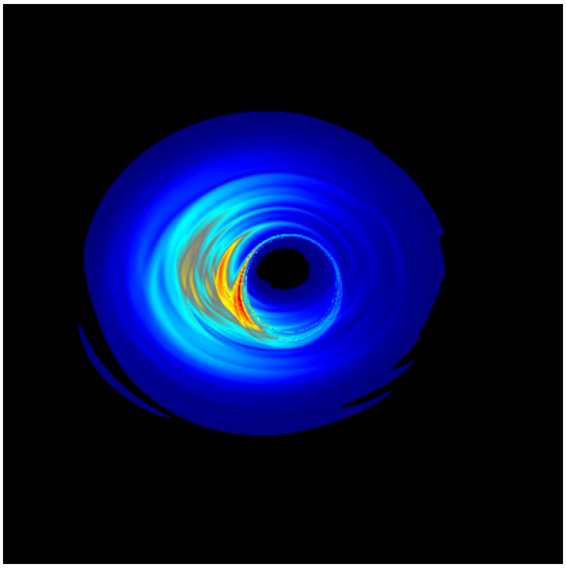

Fig. 2. 1 mm map of the galactic center, using a thermal synchrotron emission model. The black hole has spin $a_* \simeq 0.94$ and the inclination of the spin axis to the line of sight is 45 deg.

reaches the event horizon it has lost a significant fraction of its angular momentum.

Why does this matter? Because the spin of a black hole determines the structure of the accretion flow close to the event horizon, where most of the accretion energy is released. The spin of the black hole, in turn, is determined by the angular momentum of the matter that falls into it. If a black hole acquires most of its mass by accretion (as is likely for most supermassive black holes in galactic nuclei) then the specific angular momentum of the accreted material determines the spin. Our results suggest that spin equilibrium is reached at $a_* \simeq 0.93$ for accretion through a thick disk [6].

4. Radiative models

The spin of black holes is, at present, difficult to measure with confidence (but see, e.g., [9–11]). To bring our numerical models into contact with observations, we need to include the radiative physics that we initially neglected. As a first step we can neglect the back-reaction of the radiation on the flow, and calculate the observational appearance of the accretion flow after the simulation is over.

Fig. 2 shows the result of a calculation like this. What you are seeing is a model of the galactic center at a wavelength of 1 mm. Most of the emission comes from electrons (assumed to lie in a single-temperature thermal distribution function) spiraling around the magnetic field and producing synchrotron radiation. These electrons have typical Lorentz factors (in the fluid rest frame) of $\sim 10^2$.

The emission is determined by integrating the radiative transfer equation (really, the photon Boltzmann equation) along geodesics. An interesting feature of this calculation is that the geodesics are quite complicated—some light rays that pass close to the black hole circle the hole several times. This produces some of the rings of emission that are visible in the figure, because the rays make several passes through the optically thin emitting plasma. The model has two key unknown parameters: the spin of the hole and the inclination of the spin axis to the line of sight. This particular model assumes that $a_* \simeq 0.93$ and that the inclination is 45 deg.

Near future observations of the galactic center will have only a few resolution elements across this image, so comparisons with theory must rest on a few, large-scale features of the image (such as the hole in the middle—these are light rays that end on the event horizon), its time-variability, and its polarization properties.

5. Summary

We have created numerical dynamical and radiative transfer models of accretion flows onto black holes. These models touch on classical problems in computational physics, (magneto)hydrodynamics and transport phenomena, but they do so in an exotic astrophysical context. In spite of some conceptually (but not technically) minor complications caused by general relativity (e.g., light trajectories are curved), standard techniques can be used to obtain a solution. For the hydrodynamics, we use a standard shock-capturing, conservative technique. For the radiative transfer, we use ray-tracing.

Future models of Sgr A* will attempt to eliminate approximations and reduce modeling uncertainty. In particular, they must grapple with the approximation of treating the electron distribution function as thermal, with a temperature equal to the proton temperature. This is likely incorrect in two respects: first, a power law tail of nonthermal electrons is likely present that dominates emission at shorter wavelength, and second, anisotropies in the distribution function will change the emissivity by of order unity along some rays. For time variability studies, it will also be important to dispense with the “frozen fluid” approximation we use in our ray-tracing calculations, where our models use fluid variables from a single time slice. That this approximation is inadequate can be seen by noting that it eliminates Lorentz contraction! Also, much of the time variability is probably related to orbital motion of nonaxisymmetric bright spots in the flow; it is of course impossible to see this in an axisymmetric calculation, so future calculations must go to 3D (as has already been done by [2]).

While the back-reaction of the photons on the fluid is unimportant for Sgr A*, it is important for systems with higher accretion rates. There, one must solve the coupled general relativistic radiation magnetohydrodynamics problem. This will pose significant numerical challenges, but an attack on this problem is strongly motivated by the possibility of using the observed near-horizon radiation to diagnose the space–time and perhaps, one day, test general relativity.

Acknowledgements

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