Critical Phenomena in Velocity-Induced Perfect Fluid Collapse

Scott Noble[1,2]

Matt Choptuik[1]

[1]Dept. of Physics & Astronomy, University of British Columbia, Vancouver, B.C., Canada

[2]Center for Relativity, Dept. of Physics, University of Texas at Austin, Austin, TX, USA

Supported by CIAR, NSERC, CFI/BCKDF

Motivation

- J. Novak (2001):
  - "Ideal-gas" EOS: $P = (\Gamma - 1)\,\rho_\circ \epsilon$, $\Gamma = 2$
  - Tuning star's init. vel. $\rightarrow$ Type-II critical behavior;
  - $M_{BH} \propto |p - p^*|^\gamma$ with $\gamma \simeq 0.52$
- Neilsen and Choptuik (2000), Brady et al. (2002)
  - Studied ultra-relativistic fluid collapse;
  - A limit of "ideal-gas" case where $\rho \equiv (1 + \epsilon)\,\rho_\circ \simeq \rho_\circ \epsilon$
  - $P = (\Gamma - 1)\,\rho$, only EOS to admit CSS soln's;
  - For $\Gamma = 2$, $\gamma \simeq 0.95 \pm 0.02$
- Neilsen and Choptuik (2000)
  - For $\Gamma = 1.4$: Ideal-gas Type-II Sol'n. $=$ Ultra-rel. Type-II Sol'n.

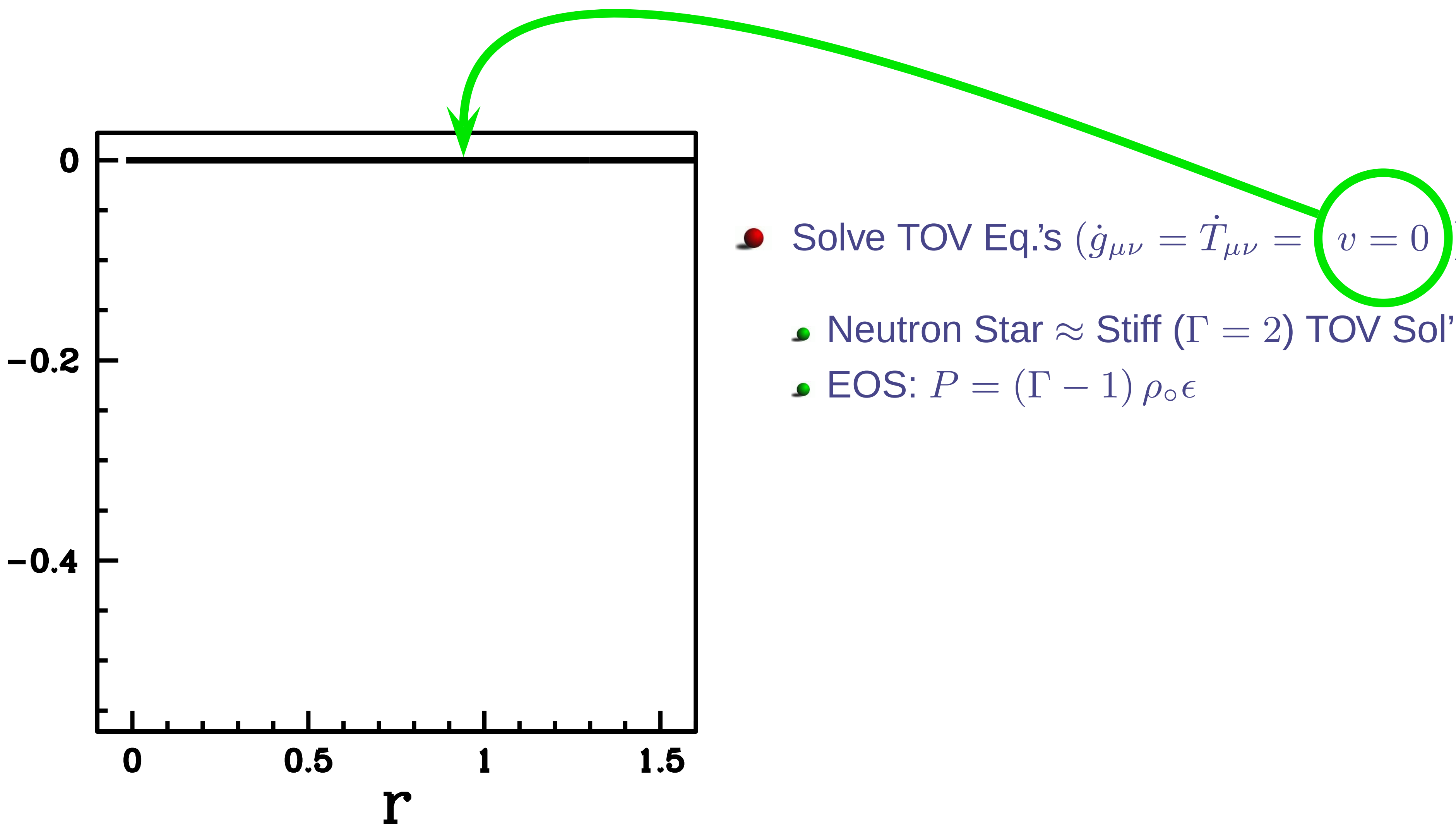
Motivation

- J. Novak (2001):
  - "Ideal-gas" EOS: $P = (\Gamma - 1)\, \rho_\circ \epsilon$, $\Gamma = 2$
  - Tuning star's init. vel. $\rightarrow$ Type-II critical behavior;
  - $M_{BH} \propto |p - p^*|^\gamma$ with $\boxed{\gamma \simeq 0.52}$
- Neilsen and Choptuik (2000), Brady et al. (2002)
  - Studied ultra-relativistic fluid collapse;
  - A limit of "ideal-gas" case where $\rho \equiv (1 + \epsilon)\, \rho_\circ \simeq \rho_\circ \epsilon$
  - $P = (\Gamma - 1)\, \rho$, only EOS to admit CSS soln's;
  - For $\Gamma = 2$, $\boxed{\gamma \simeq 0.95 \pm 0.02}$

??

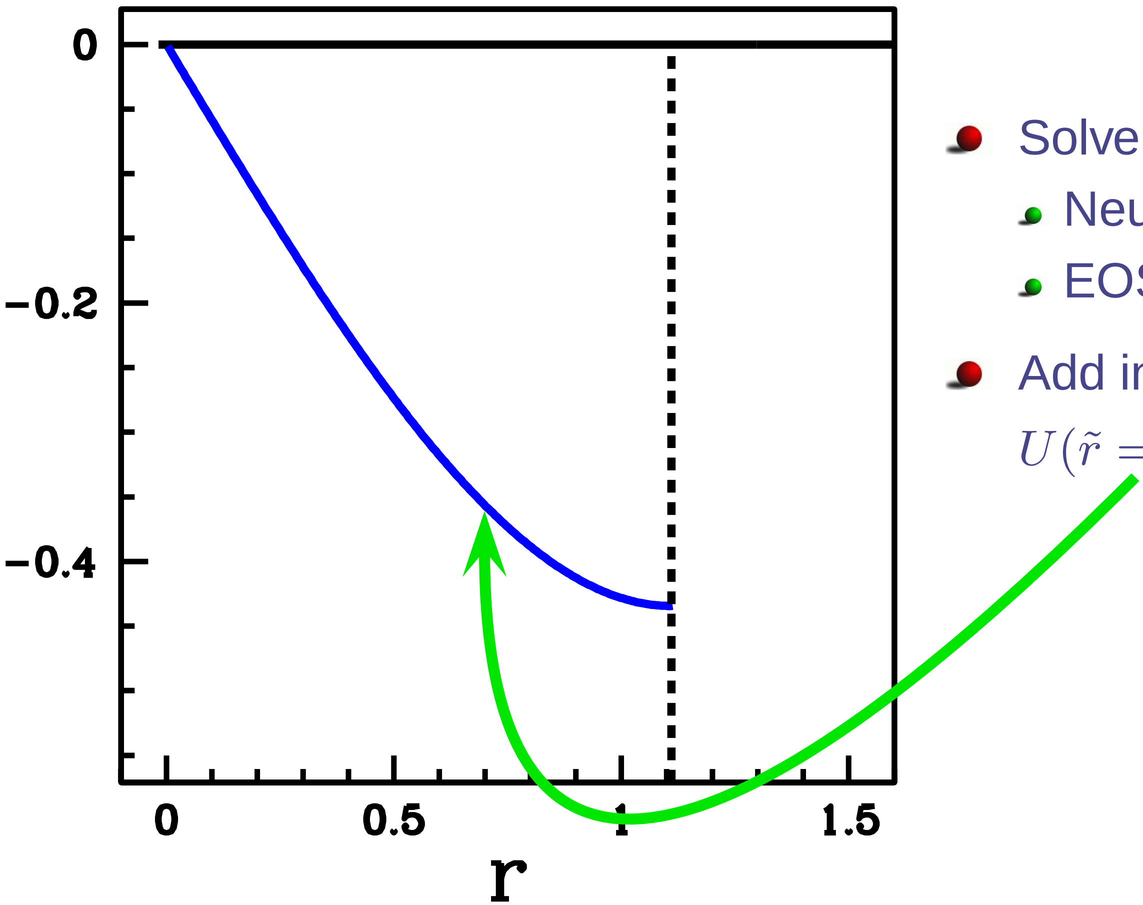
- Neilsen and Choptuik (2000)
  - For $\Gamma = 1.4$: Ideal-gas Type-II Sol'n. = Ultra-rel. Type-II Sol'n.

Initial Data : Prescription



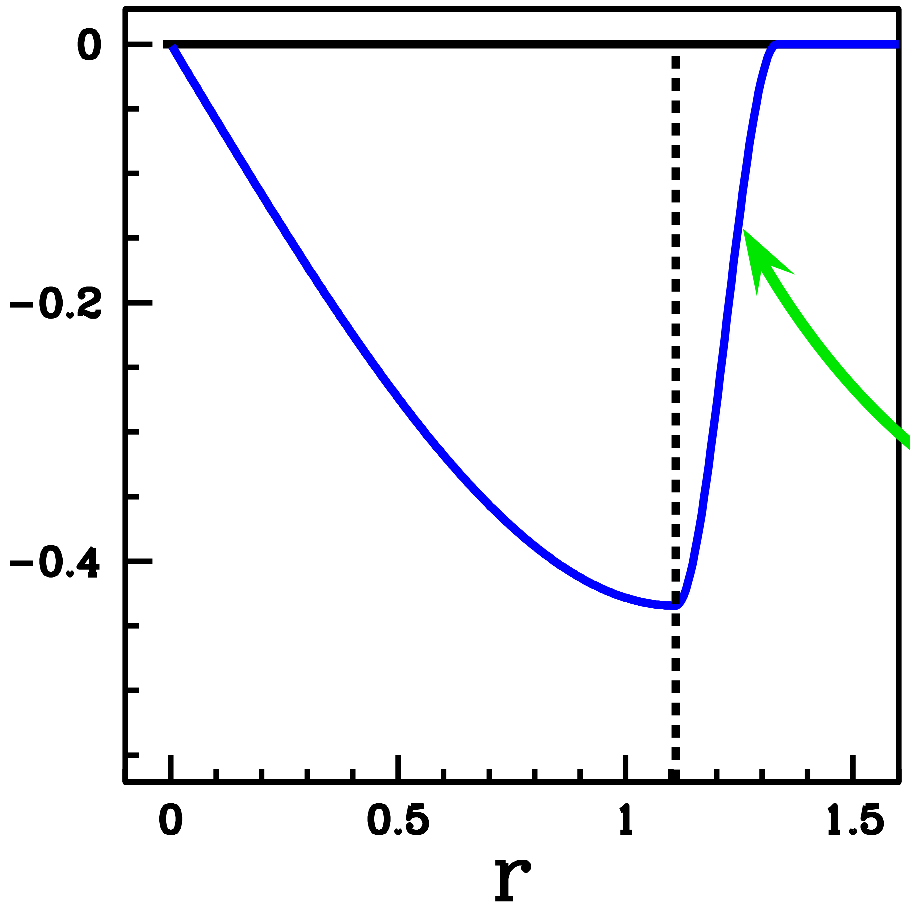
- Solve TOV Eq.'s ($\dot{g}_{\mu\nu} = \dot{T}_{\mu\nu} = v = 0$)
  - Neutron Star $\approx$ Stiff ($\Gamma = 2$) TOV Sol'n;
  - EOS: $P = (\Gamma - 1)\,\rho_\circ \epsilon$

Initial Data : Prescription



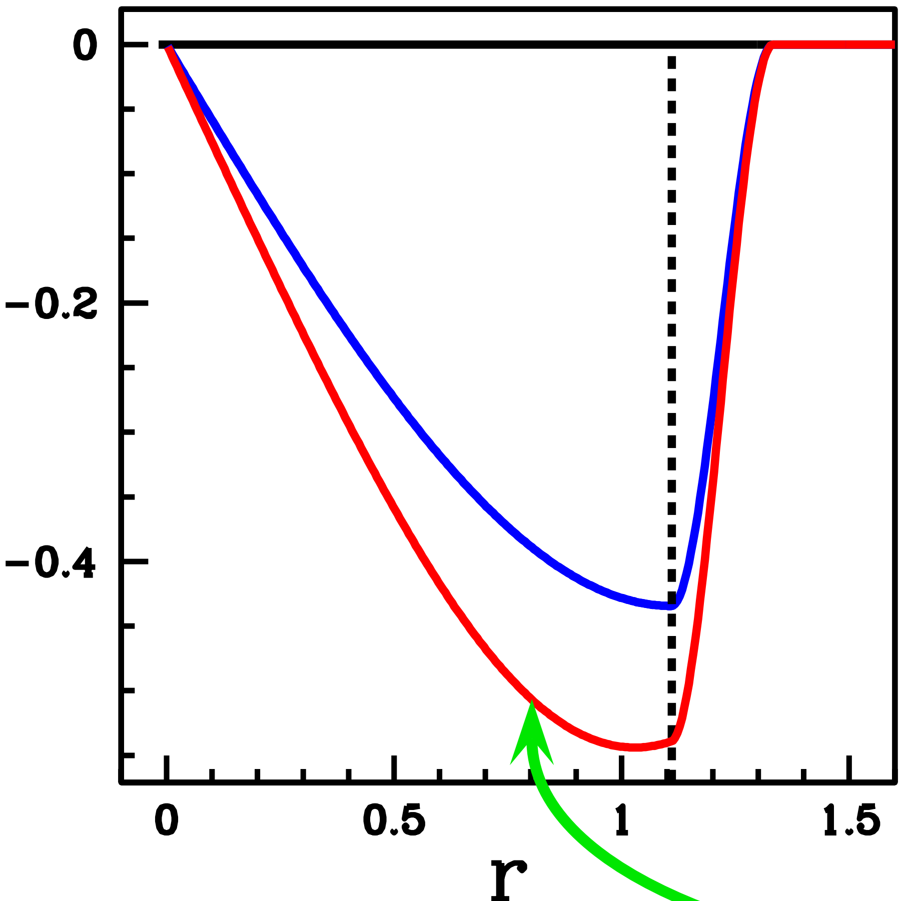
- Solve TOV Eq.'s $(\dot{g}_{\mu\nu} = \dot{T}_{\mu\nu} = v = 0)$
  - Neutron Star ≈ Stiff ($\Gamma = 2$) TOV Sol'n;
  - EOS: $P = (\Gamma - 1)\,\rho_\circ \epsilon$
- Add in-going coordinate velocity:
  $U(\tilde{r} = r/R_*) = \frac{u^r}{u^t} = p\,\frac{\tilde{r}}{2}\left[\tilde{r}^2 - 3\right]$

Initial Data : Prescription



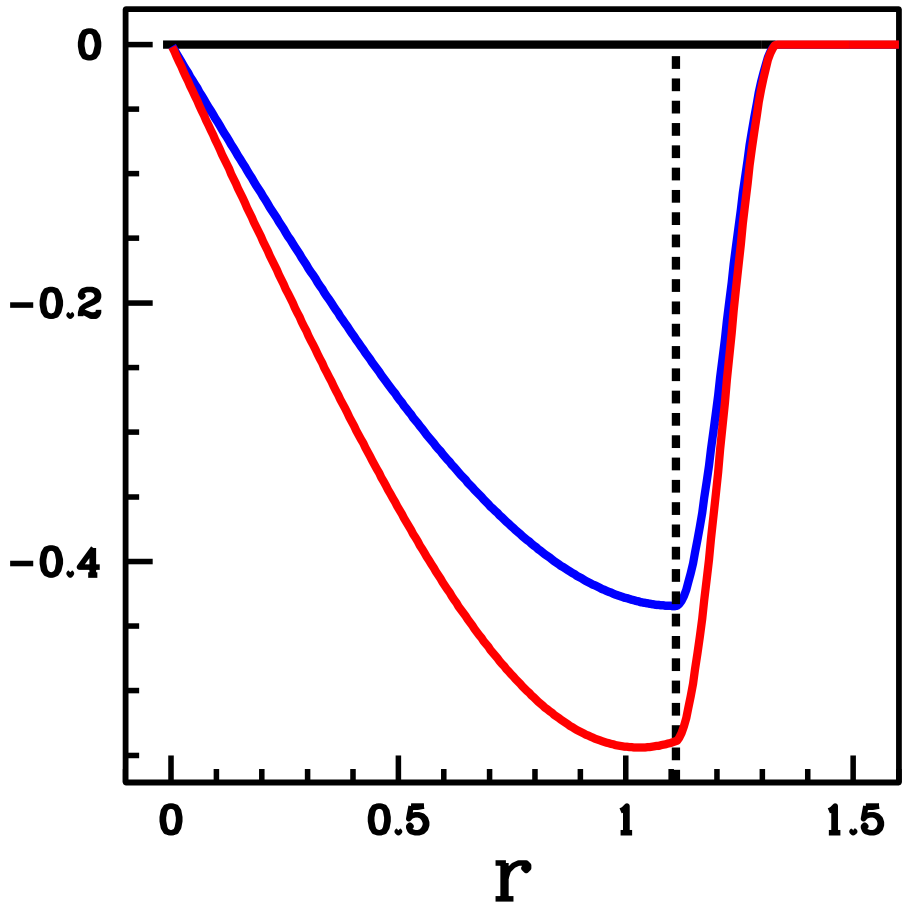
- Solve TOV Eq.'s $(\dot{g}_{\mu\nu} = \dot{T}_{\mu\nu} = v = 0)$
  - Neutron Star ≈ Stiff ($\Gamma = 2$) TOV Sol'n;
  - EOS: $P = (\Gamma - 1)\,\rho_\circ \epsilon$
- Add in-going coordinate velocity:
  $U(\tilde{r} = r/R_*) = \frac{u^r}{u^t} = p\,\frac{\tilde{r}}{2}\left[\tilde{r}^2 - 3\right]$
- Match to U = 0

Initial Data : Prescription



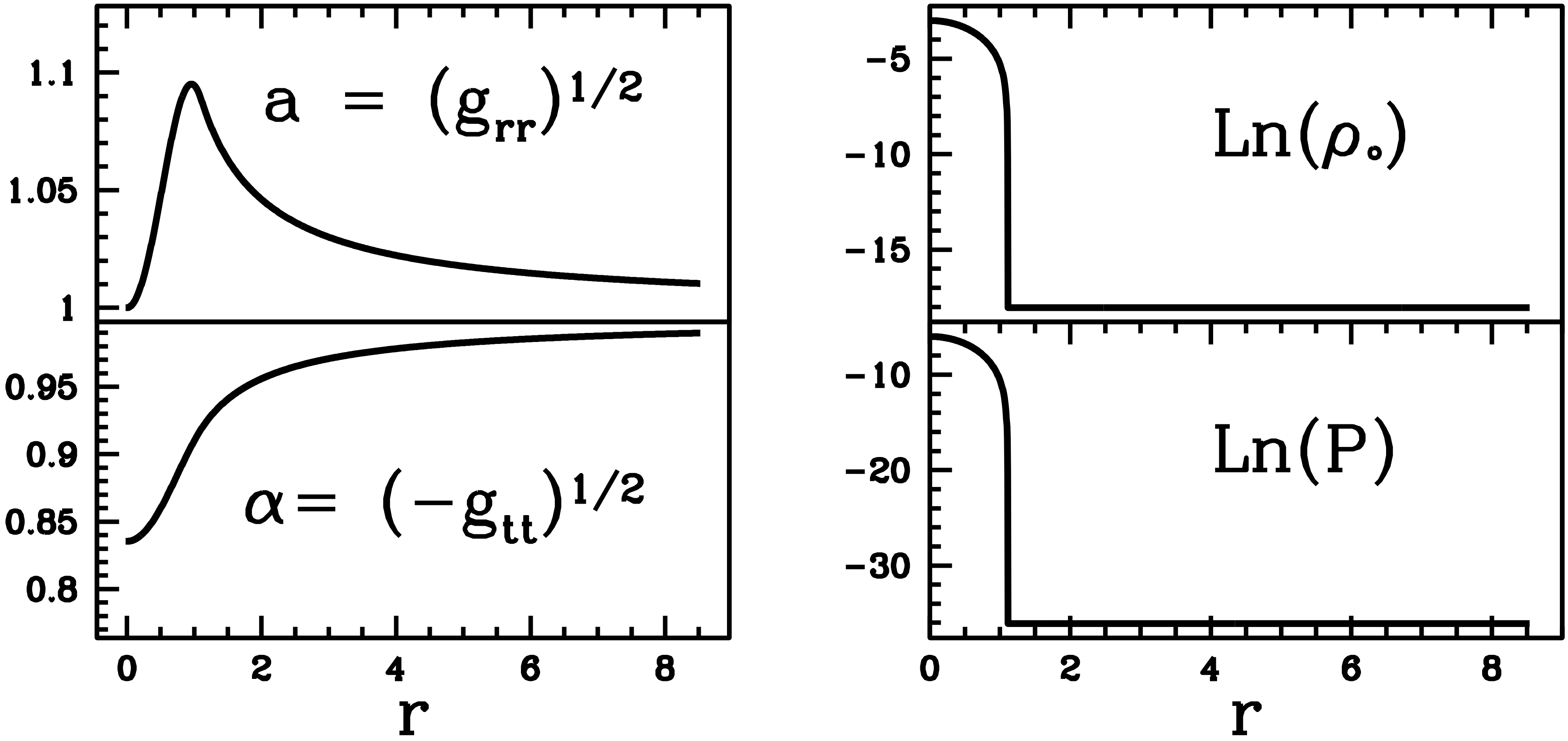
- Solve TOV Eq.'s $(\dot{g}_{\mu\nu} = \dot{T}_{\mu\nu} = v = 0)$
  - Neutron Star ≈ Stiff ($\Gamma = 2$) TOV Sol'n;
  - EOS: $P = (\Gamma - 1)\, \rho_\circ \epsilon$
- Add in-going coordinate velocity:
  $U(\tilde{r} = r/R_*) = \frac{u^r}{u^t} = p\, \frac{\tilde{r}}{2} \left[\tilde{r}^2 - 3\right]$
- Match to U = 0
- Solve ($\alpha' = \ldots$) and ($a' = \ldots$)
  and find $v = aU/\alpha$

Initial Data : Prescription

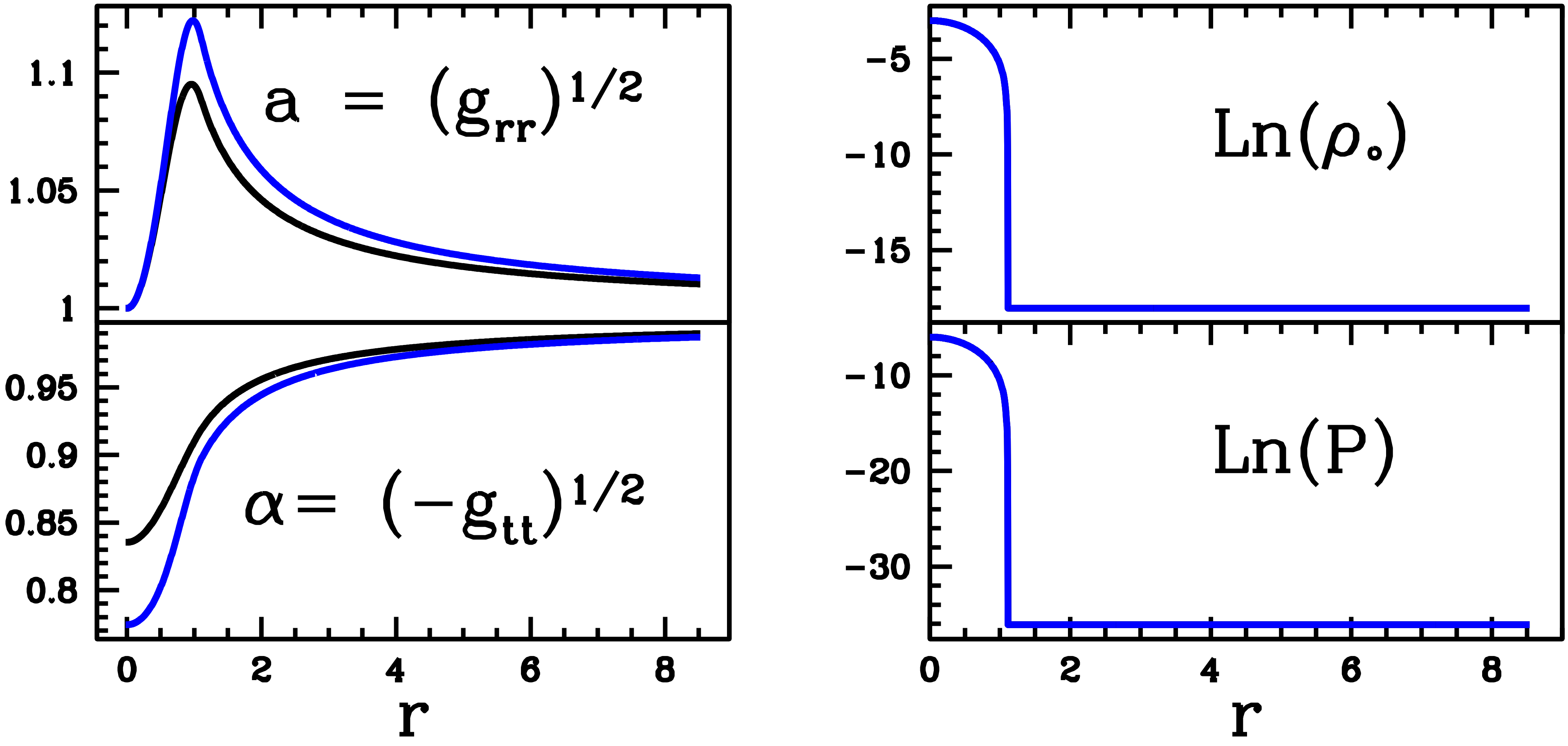


- Solve TOV Eq.'s $(\dot{g}_{\mu\nu} = \dot{T}_{\mu\nu} = v = 0)$
  - Neutron Star ≈ Stiff ($\Gamma = 2$) TOV Sol'n;
  - EOS: $P = (\Gamma - 1)\,\rho_\circ \epsilon$
- Add in-going coordinate velocity:
  $U(\tilde{r} = r/R_*) = \dfrac{u^r}{u^t} = p\,\dfrac{\tilde{r}}{2}\left[\tilde{r}^2 - 3\right]$
- Match to U = 0
- Solve ($\alpha' = \ldots$) and ($a' = \ldots$) and find $v = aU/\alpha$
- Tune to threshold of black hole formation

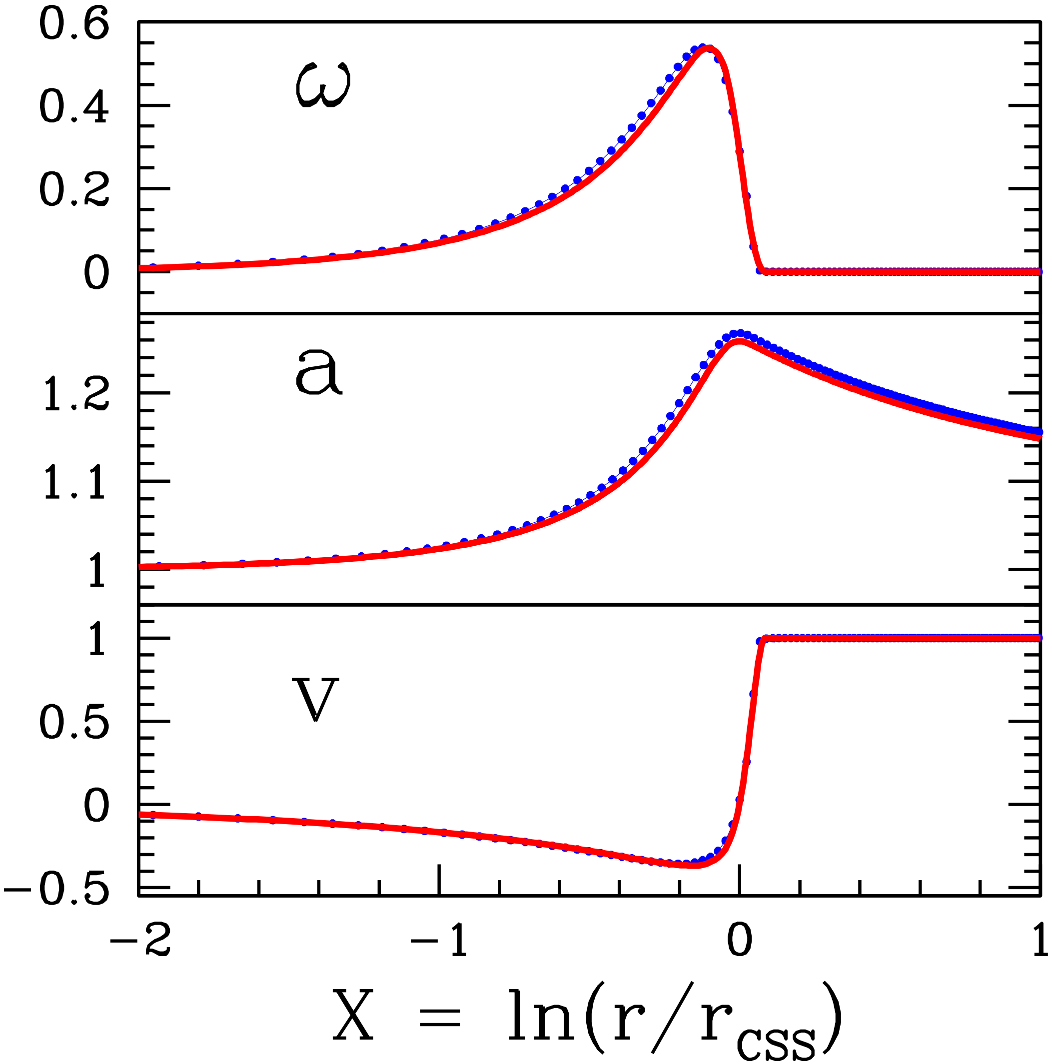
Initial Data : TOV Solution



Initial Data : TOV + In-going Velocity



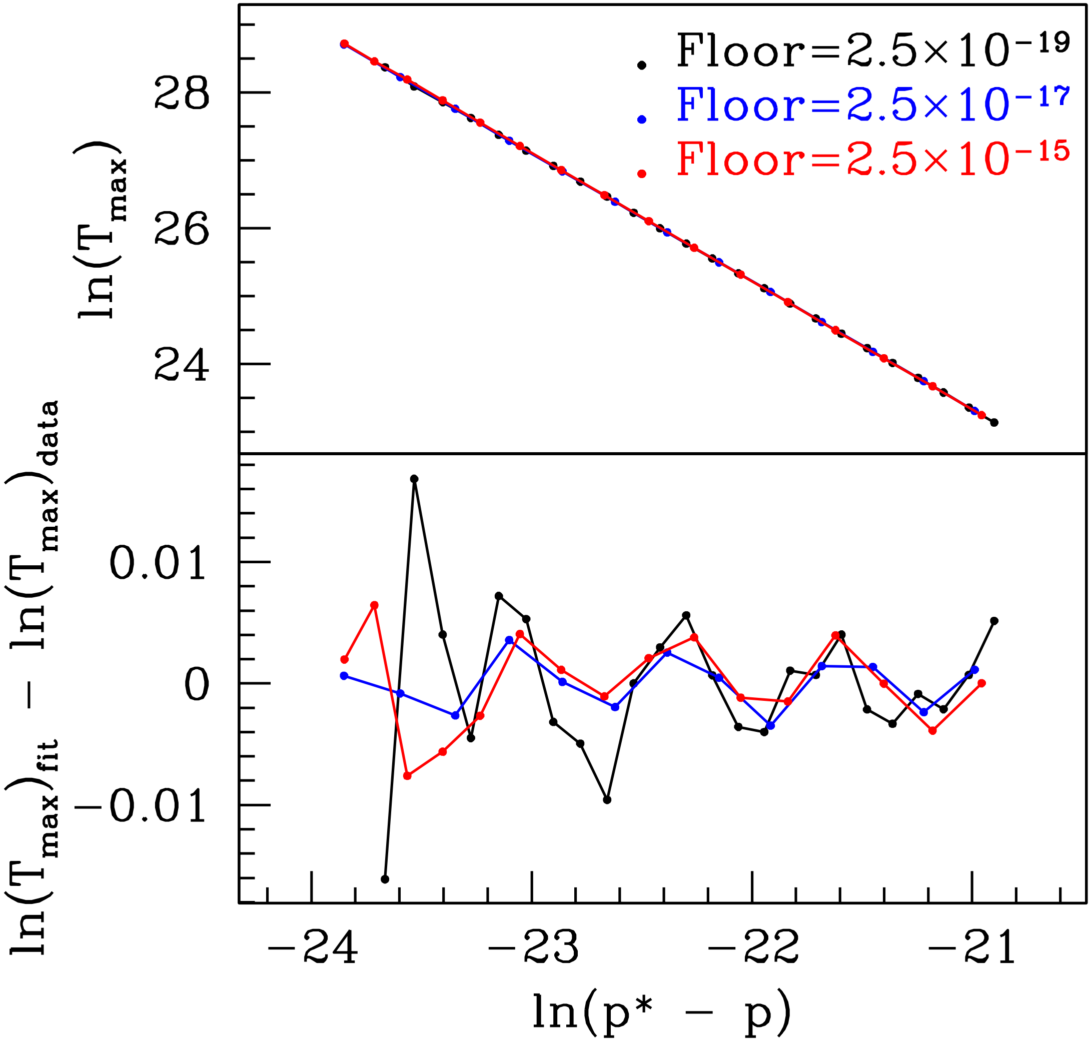
CSS Solutions of Ideal-gas and Ultra-rel.



- Comparison of dimensionless quantities:
  - $\omega \equiv 4\pi r^2 a^2 \rho$
  - $a = \sqrt{g_{rr}}$
  - $v = \frac{a u^r}{\alpha u^t}$ = Eulerian Velocity

    (u^μ = Fluid's 4-velocity)
- Star parameters at $t = 0$:
  - $\rho_\circ\,(r = 0) = 0.05$
  - $P = \rho_\circ^2\ ,\ \epsilon = P/\rho_\circ$

Scaling of $T_{\max}$: Dependence on Fluid's Floor



- $T_{\max} \equiv \text{Global Max.}(T^a{}_a)$

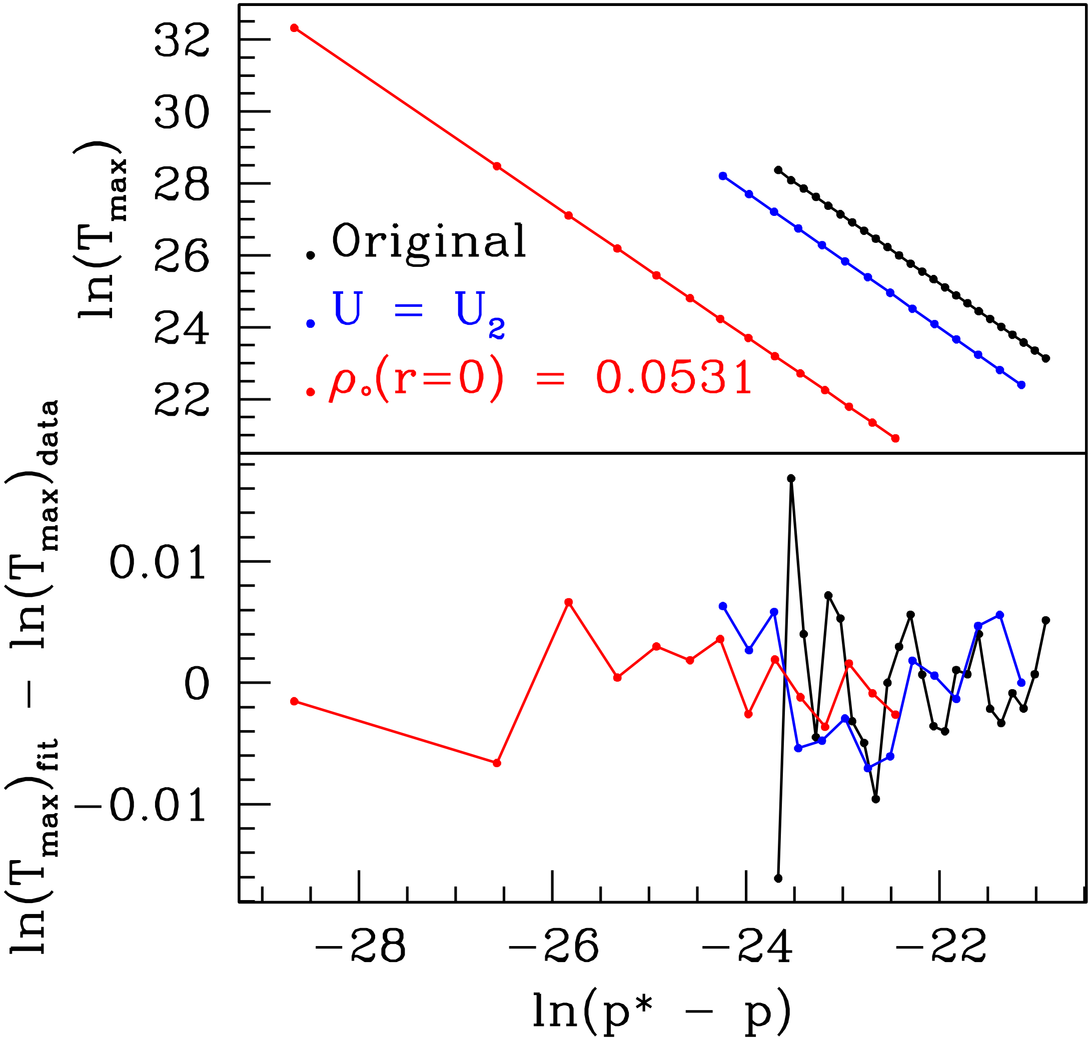
γ	p^*
0.94272	0.46875367383
0.94358	0.46875350285
0.9469707	0.4687516089

- Floor used to prevent

$$v \geq 1 \quad , \quad P, \rho_\circ < 0$$

- No significant effect;

Scaling of $T_{\max}$: Different "Families"



γ	p^*
0.94272	0.46875367383
0.94234392	0.42990315097
0.918693	0.4482047429836

- Suggests scaling is fairly independent of:
  - Functional form of perturbation;
  - Initial star configuration;

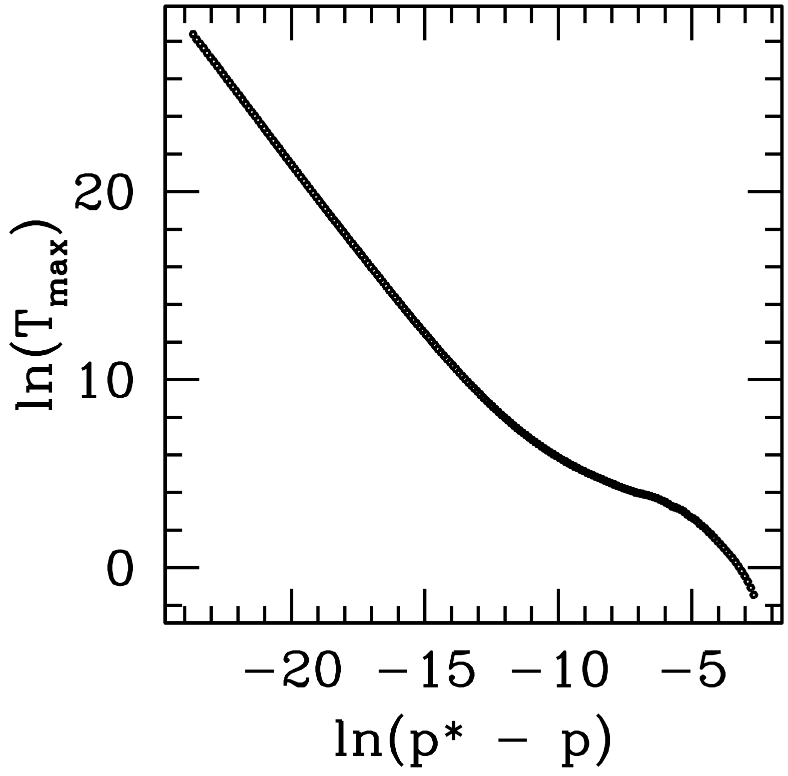
Scaling Parameters

Test Type	ρ_c	Floor	Δr	U	γ	p^*
-	0.05	2.5×10^{-19}	$4h$	U_1	0.94272	0.46875367383
Floor	0.05	2.5×10^{-17}	$4h$	U_1	0.94358	0.46875350285
Floor	0.05	2.5×10^{-15}	$4h$	U_1	0.9469707	0.4687516089
Family	0.05	2.5×10^{-19}	$4h$	U_2	0.94234392	0.42990315097
Family	0.0531	2.5×10^{-19}	$4h$	U_1	0.918693	0.4482047429836

- Our average : $\gamma \; = \; 0.94 \pm 0.01$
- Brady et al. (2002) (averaged over diff. methods): $\gamma = 0.95 \pm 0.02$

Conclusion

- (Ideal-gas Type-II Sol'n.) $\simeq$ (Ultra-rel. Type-II Sol'n.) for $\Gamma = 2$
- $\gamma_{\rm ideal} \simeq \gamma_{\rm ultra-rel.}$
- Novak (2001) did not sufficiently tune toward p^*



Supporting Agencies

- NSERC = National Sciences and Engineering Research Council of Canada
- CIAR = Canadian Institute for Advance Research
- CFI = Canada Foundation for Innovation
- BCKDF = British Columbia Knowledge Development Fund

Back to Title page