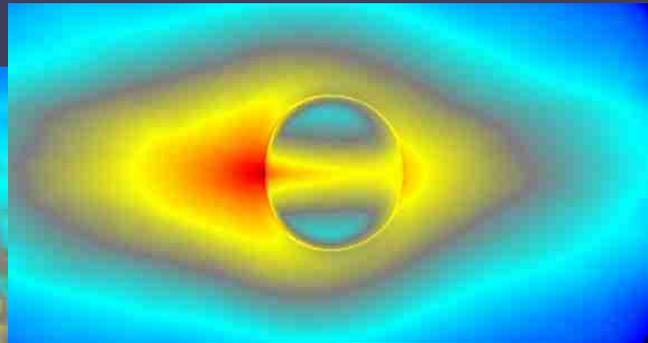
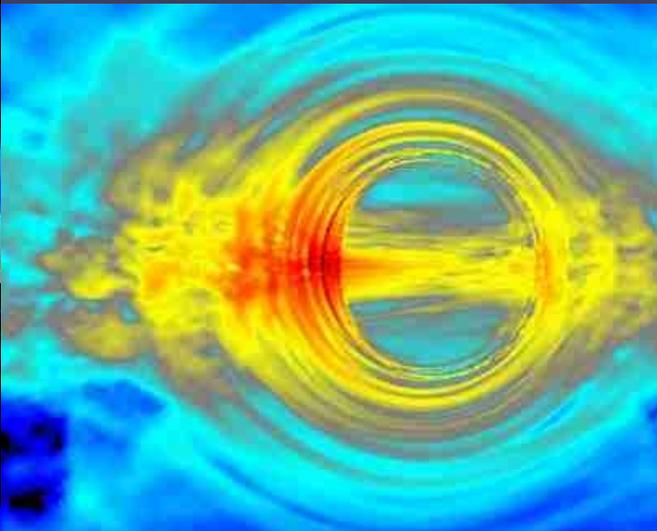
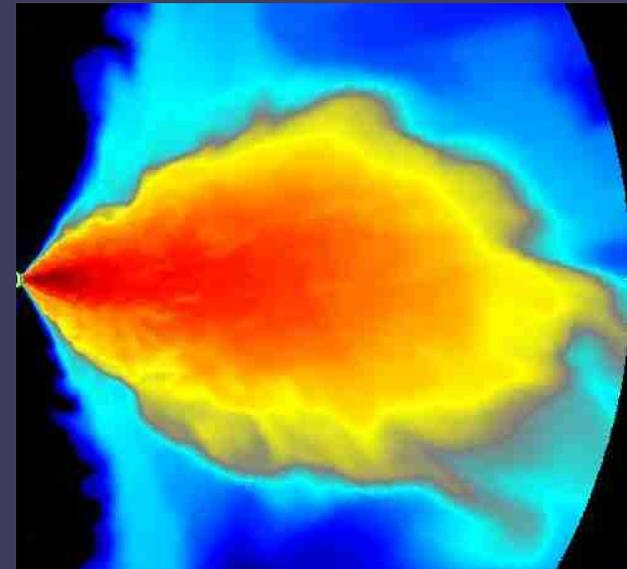


# Calculating the Radiative Efficiency of Thin Disks with 3D GRMHD Simulations

Scott C. Noble, Julian H. Krolik  
John F. Hawley (UVA)

CAS Seminar  
JHU  
November 25<sup>th</sup>, 2008

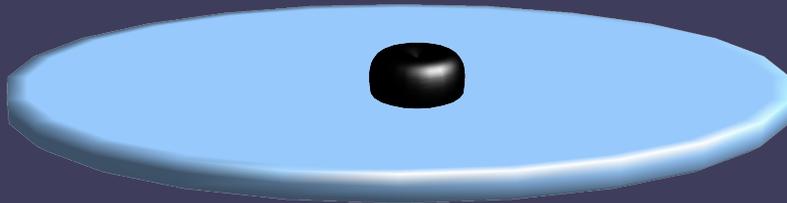


# Astrophysical Disks

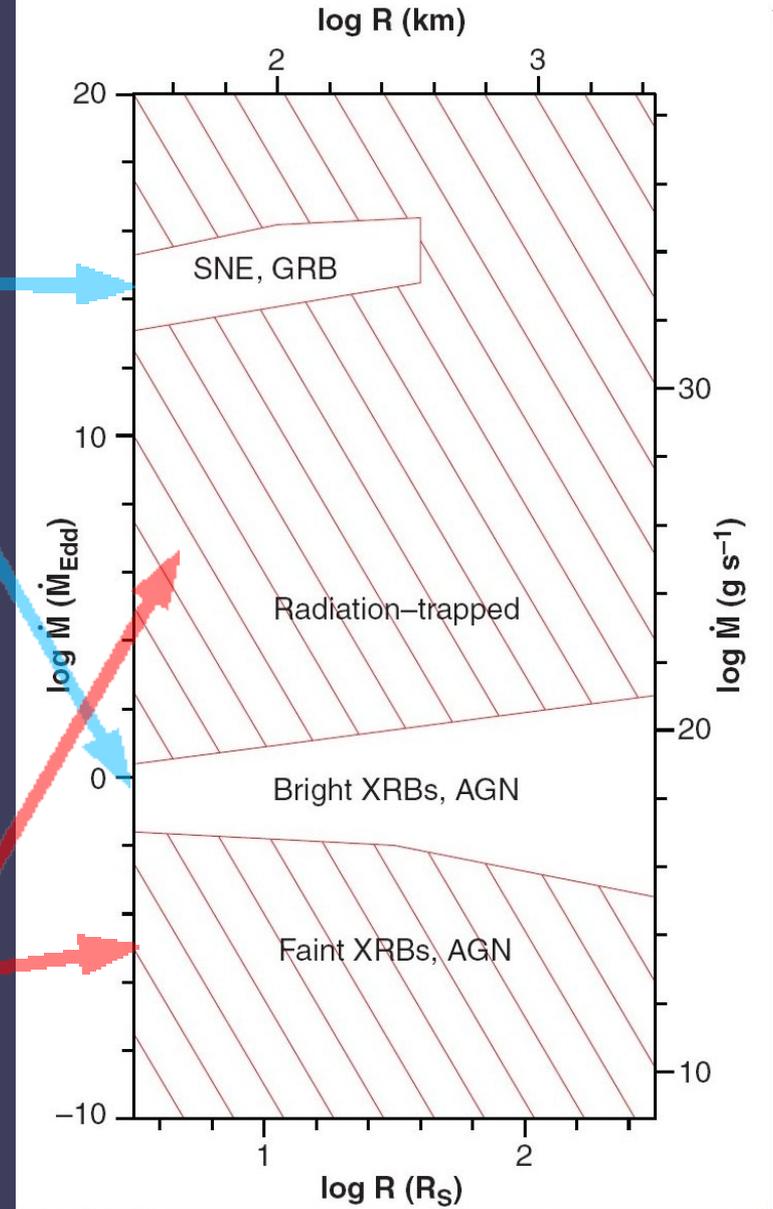
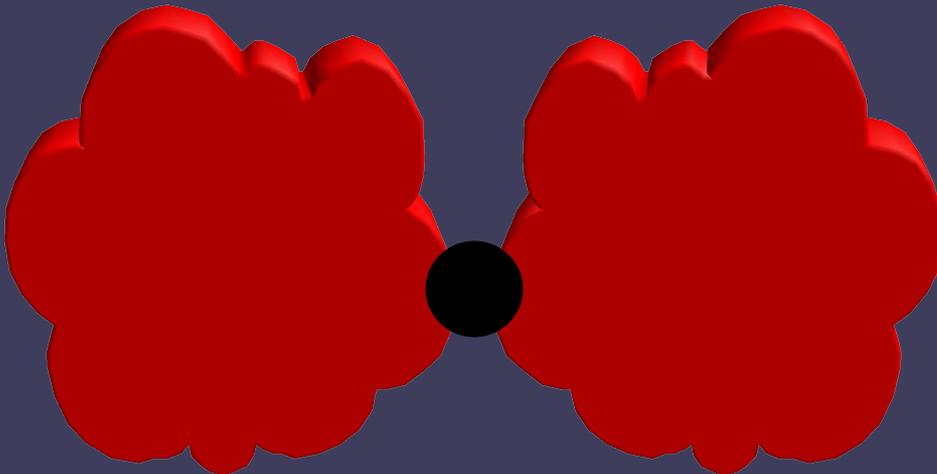
Disk Type	Gravity Model
Galaxies, Stellar Disks	Newtonian
X-ray binaries, AGN	Stationary metric
Collapsars, SN fall-back disks	Full GR

# Radiative Efficiency of Disks

- Radiatively Efficient (thin disks)



- Radiatively Inefficient (thick disks)



Narayan & Quataert (2005)

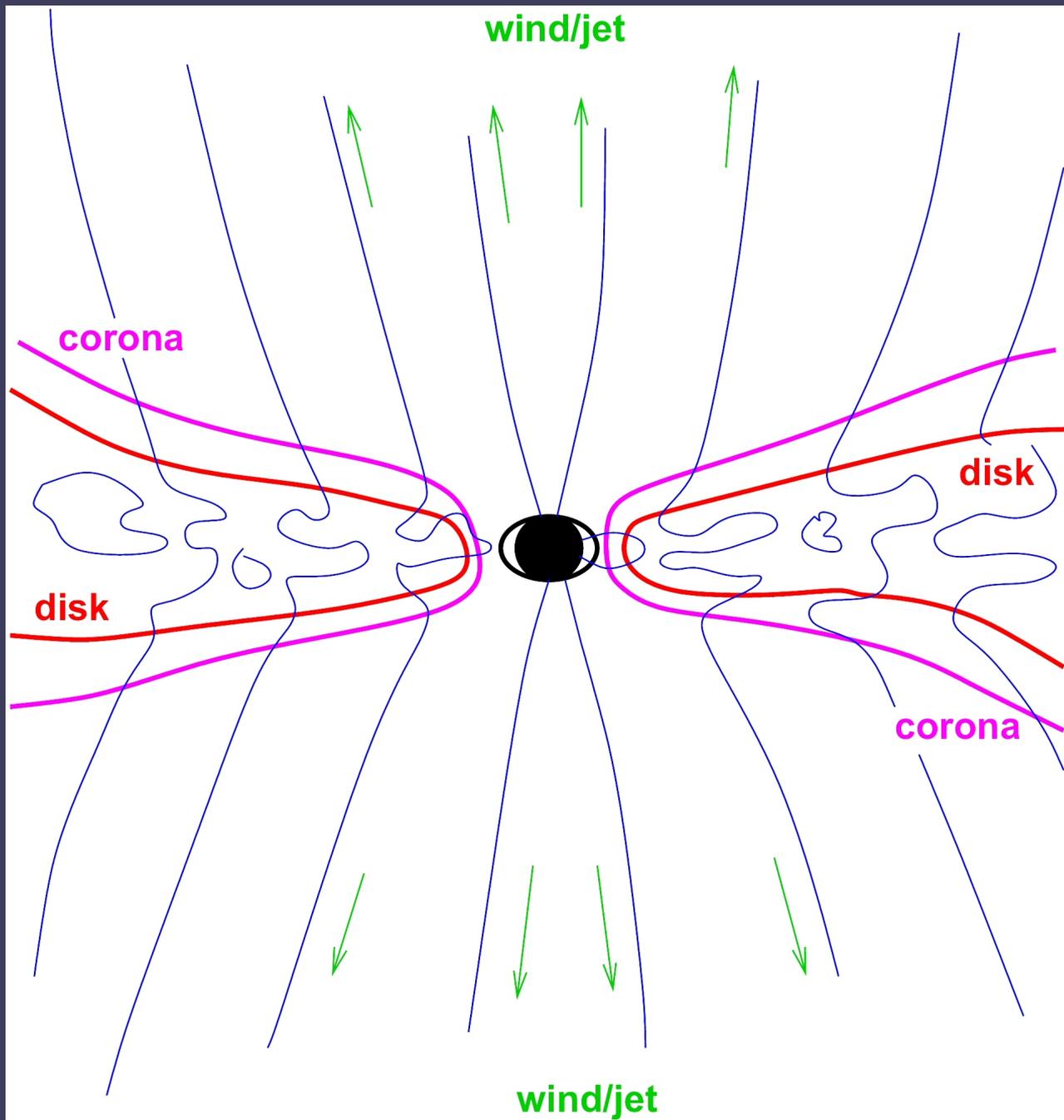


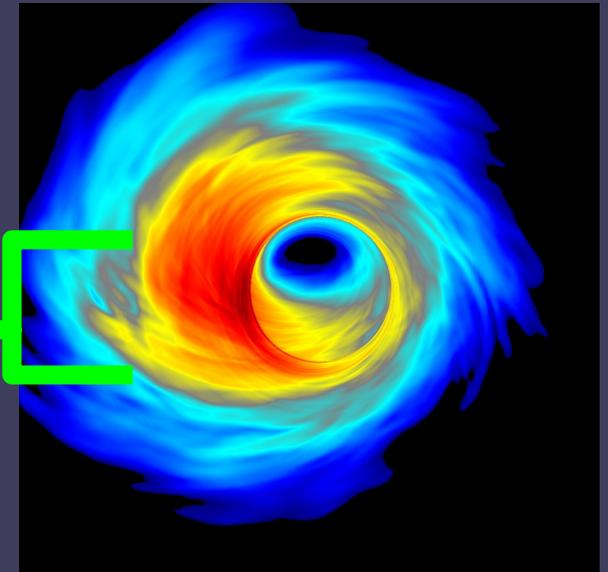
Illustration by  
C. Gammie

# Electromagnetic BH Measurements

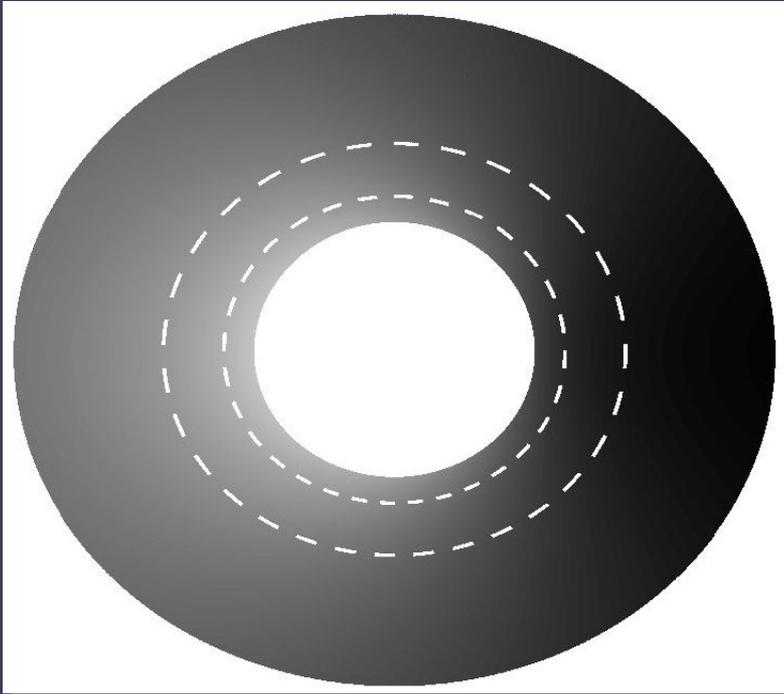
- Variability:
  - e.g. QPOs, short-time scale var.
- Spectral Fitting: e.g. Thermal emission

$$L = A R_{in}^2 T_{max}^4 \quad R_{in} = R_{in}(M, a)$$

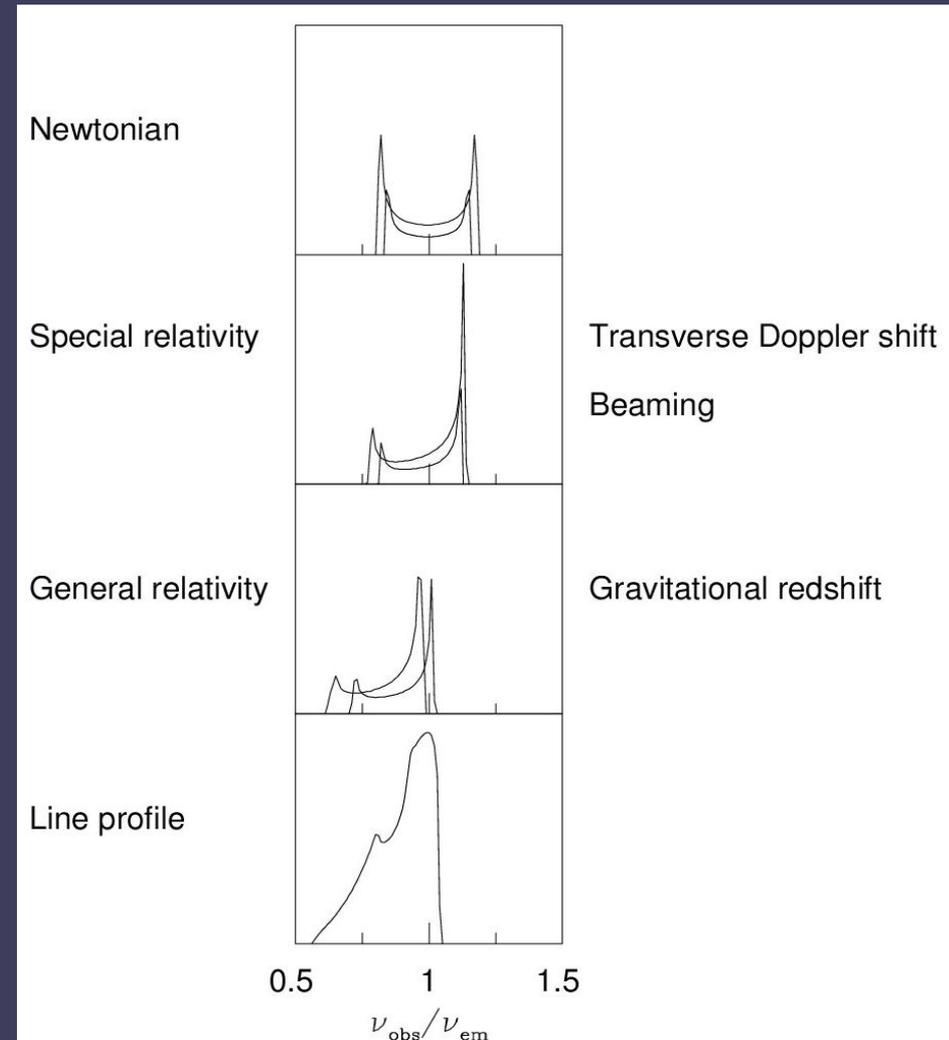
- Directly Resolving Event Horizon: e.g., Sgr A\*
  - Silhouette size =  $D(M, a)$



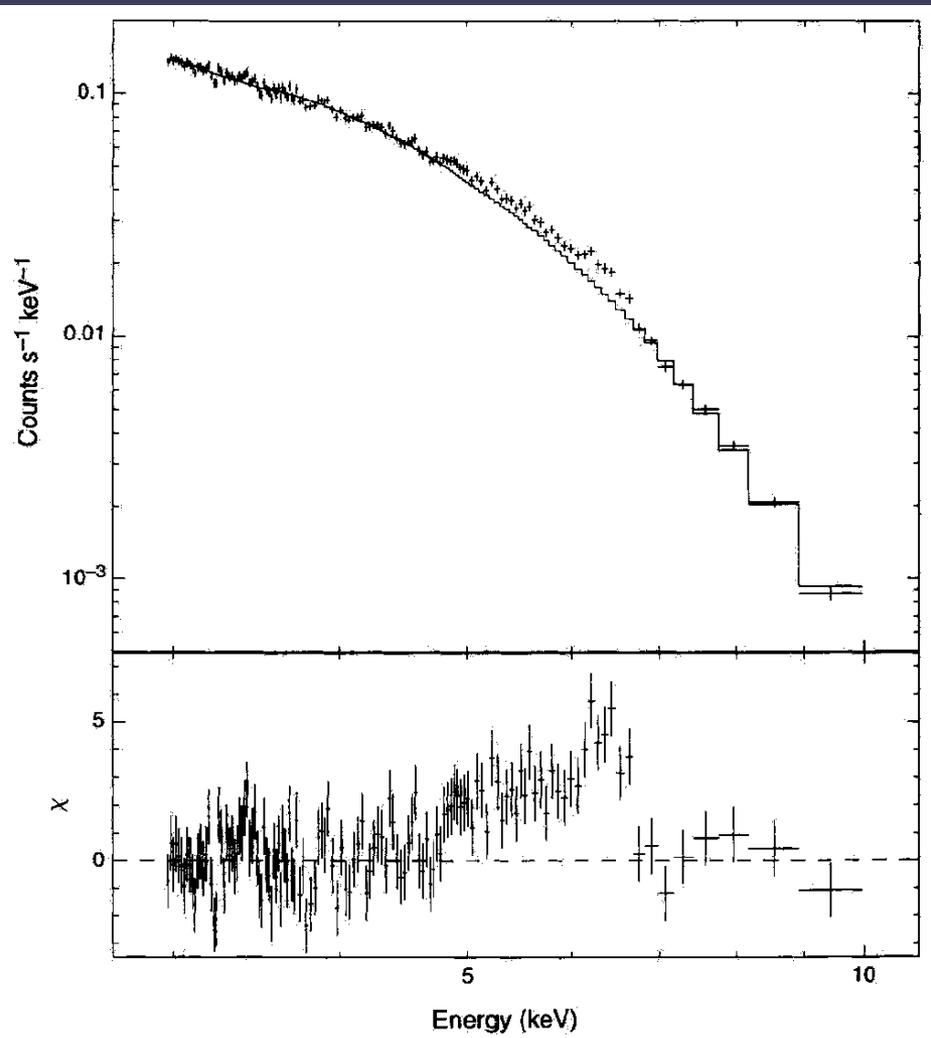
# Relativistic Iron-Lines



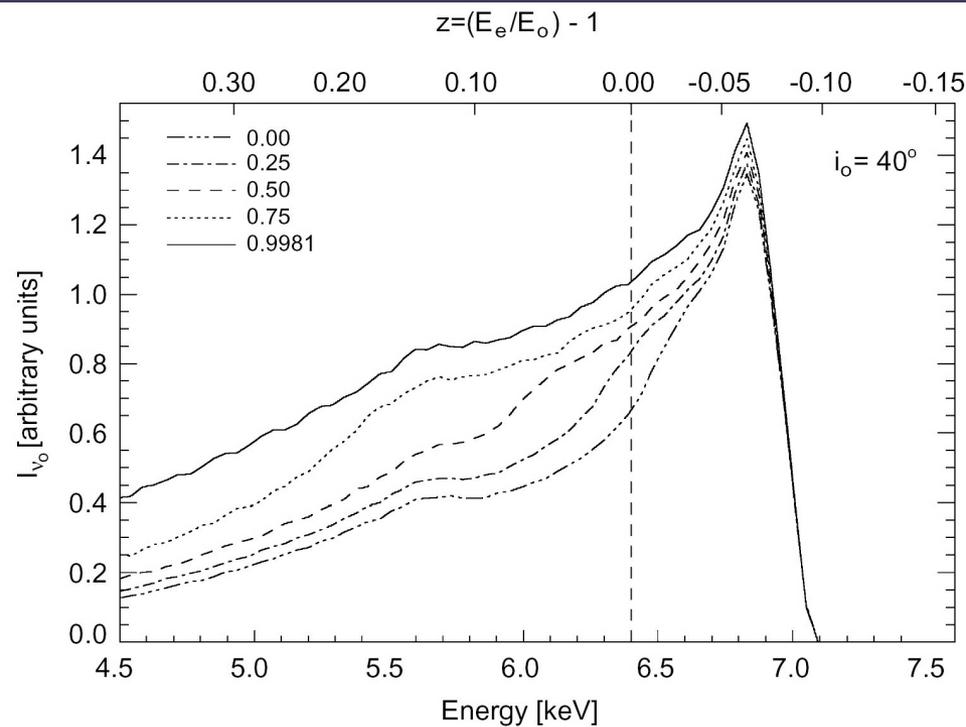
Fabian et al. (2000)



# Relativistic Iron-Lines

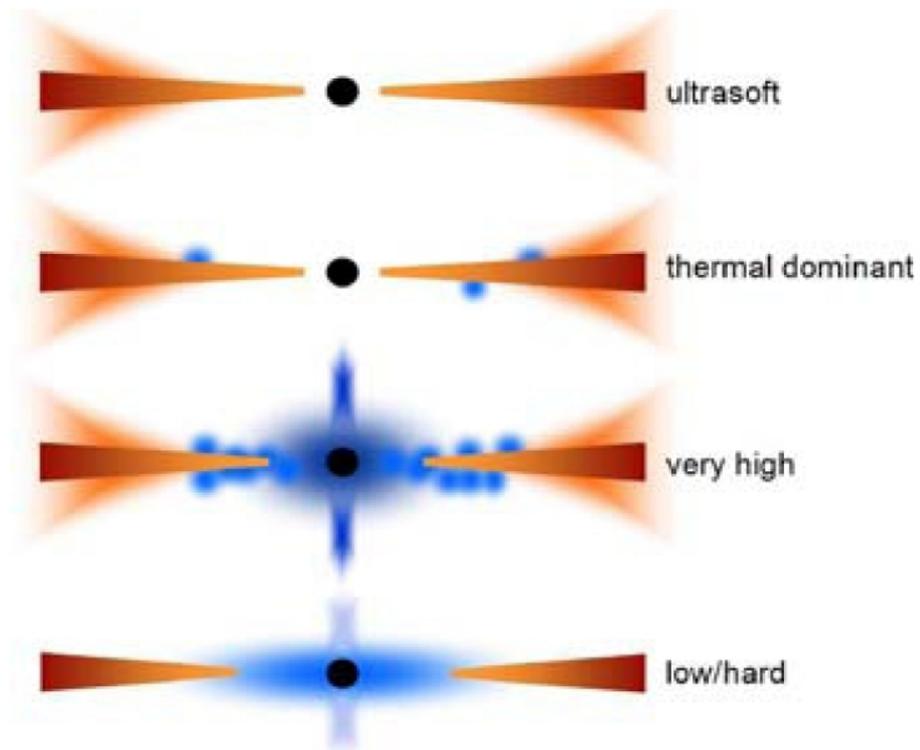
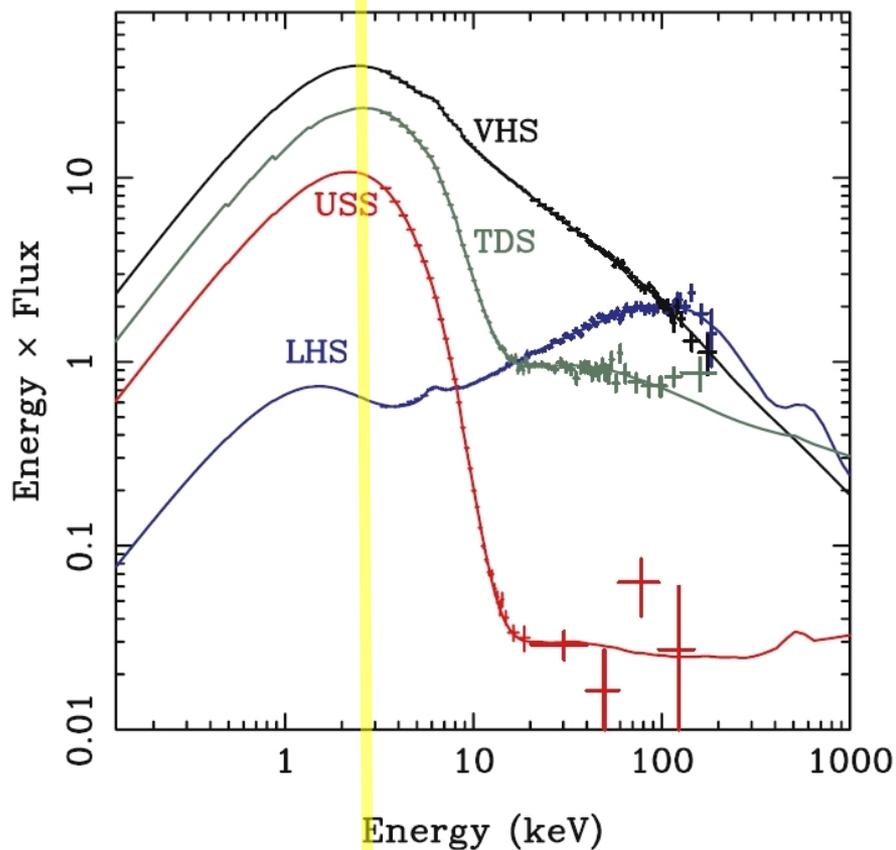


Tanaka et al. (1995)  
MCG 6-30-15



Reynolds & Nowak (2003)

# Accretion States



Done, Gierlinski & Kubota (2007)

$$L = A R_{in}^2 T_{max}^4$$

$$R_{in} = R_{in}(M, a) \sim R_{isco}$$

# Spectral Fits for BH Spin

TABLE 1

BLACK HOLE SPIN ESTIMATES USING THE MEAN OBSERVED VALUES OF  $M$ ,  $D$ , AND  $i$

Candidate	Observation Date	Satellite	Detector	$a_*$ (D05)	$a_*$ (ST95)
GRO J1655–40 .....	1995 Aug 15	<i>ASCA</i>	GIS2	~0.85	~0.8
			GIS3	~0.80	~0.75
	1997 Feb 25–28	<i>ASCA</i>	GIS2	~0.75 <sup>a</sup>	~0.70
			GIS3	~0.75 <sup>a</sup>	~0.7
	1997 Feb 26	<i>RXTE</i>	PCA	~0.75 <sup>a</sup>	~0.65
1997 (several)	<i>RXTE</i>	PCA	0.65–0.75 <sup>a</sup>	0.55–0.65	
4U 1543–47 .....	2002 (several)	<i>RXTE</i>	PCA	0.75–0.85 <sup>a</sup>	0.55–0.65

<sup>a</sup> Values adopted in this Letter.

Shafee et al. (2006)

OBJECT	POWER LAW	
	Mean	Standard Deviation
GRS 1915+105 <sup>a</sup>	0.998	0.001
GRS 1915+105 <sup>b</sup>	0.998	0.001

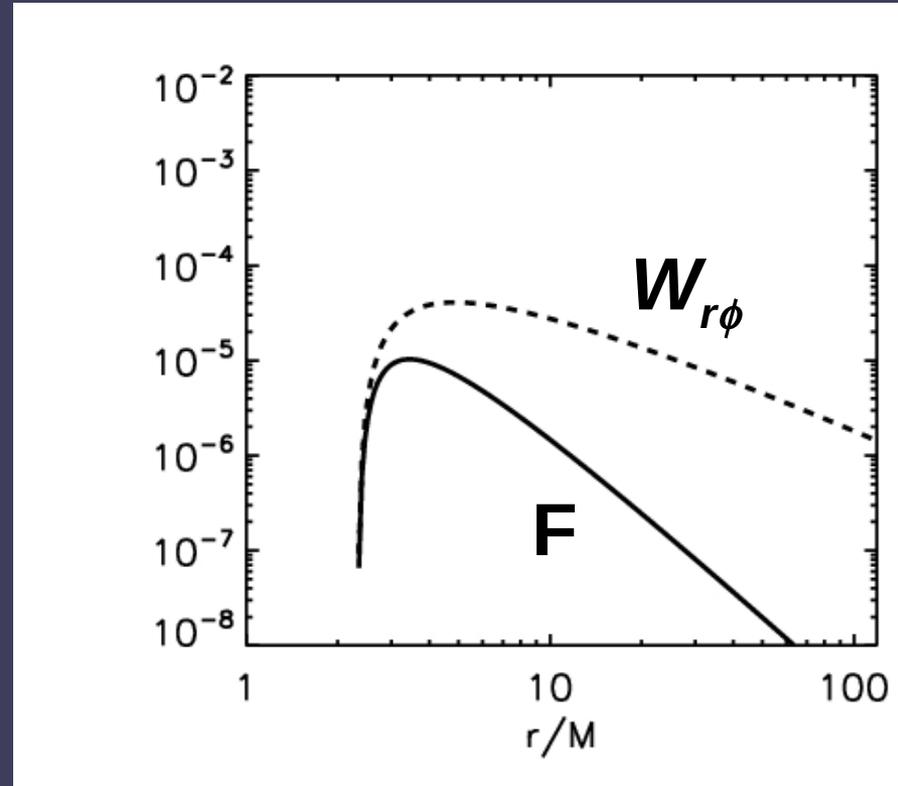
McClintock et al. (2006)

# Steady-State Models: Novikov & Thorne (1973)

## Assumptions:

- 1) Stationary gravity
- 2) Equatorial Keplerian Flow
  - Thin, cold disks
- 3) Time-independent
- 4) Work done by stress locally dissipated into heat
- 5) Conservation of  $M, E, L$
- 6) Zero Stress at ISCO
  - Eliminated d.o.f.
  - Condition thought to be suspect from very start

(Thorne 1974, Page & Thorne 1974)

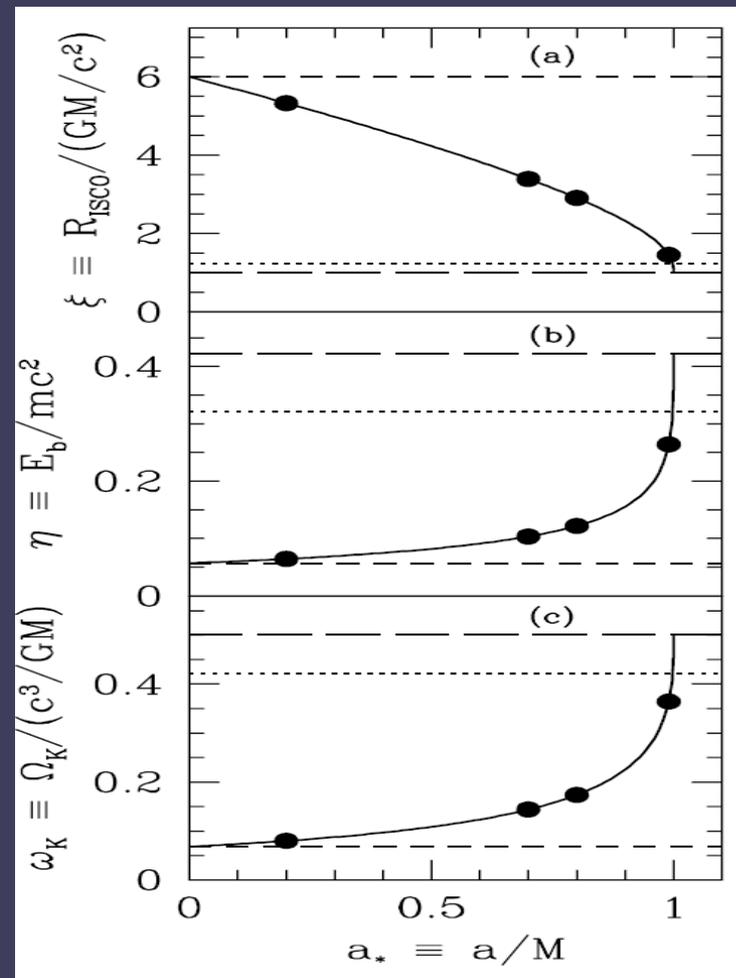


$$\begin{aligned}\eta &= 1 - \dot{E} / \dot{M} \\ &= 1 - \epsilon_{ISCO}\end{aligned}$$

# Steady-State Models: Novikov & Thorne (1973)

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    - Condition thought to be suspect from very start
- (Thorne 1974, Page & Thorne 1974)



$$\eta = 1 - \dot{E} / \dot{M}$$
$$= 1 - \epsilon_{\text{ISCO}}$$

# Steady-State Models: $\alpha$ Disks

- Shakura & Sunyaev (1973):

$$T_{\phi}^r = -\alpha P$$

$$P = \rho c_s^2 \quad t_{\phi}^r = -\alpha c_s^2$$

- No stress at sonic point:

$$\rightarrow R_{\text{in}} = R_s$$

e.g.:

Muchotrzeb & Paczynski (1982)

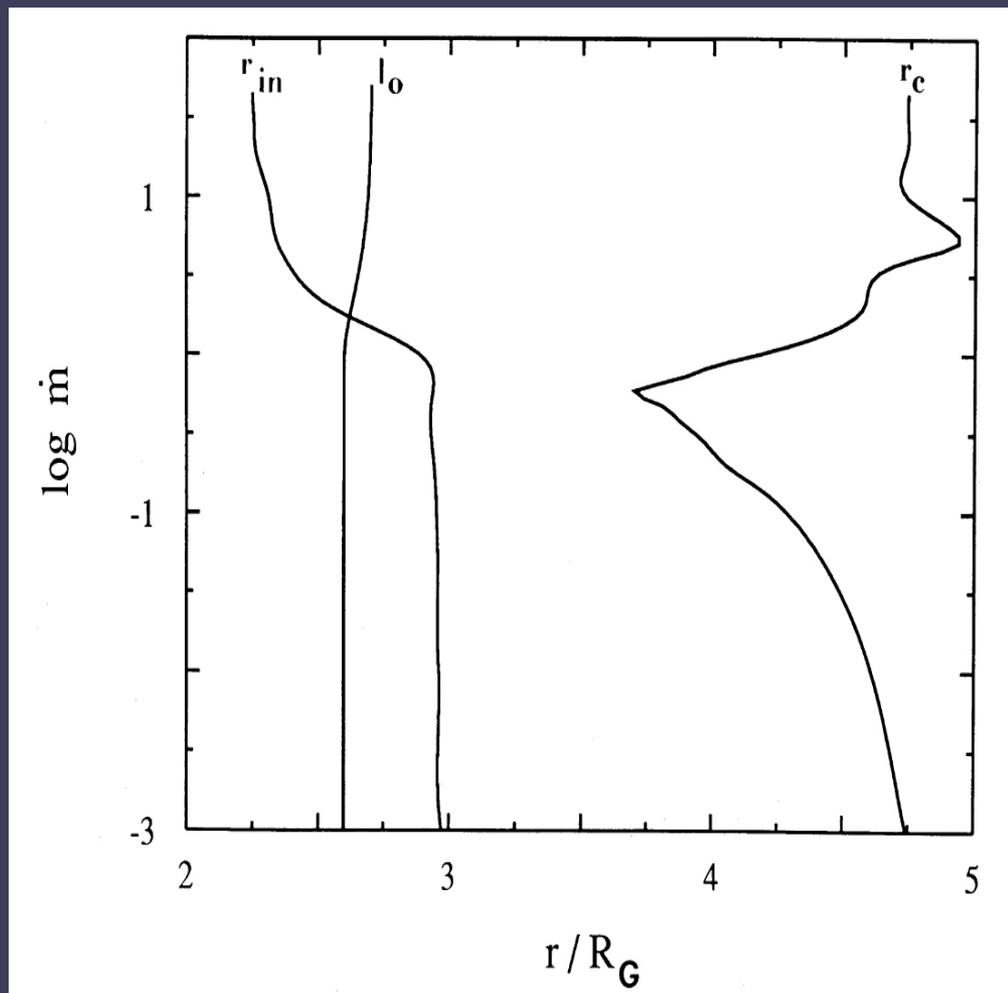
Abramowicz, et al. (1988)

Afshordi & Paczynski (2003)

(Schwarzschild BHs)

- Variable  $\alpha$

e.g., Shafee, Narayan, McClintock (2008)



Abramowicz, et al. (1988)

$$\eta \sim 1 - \epsilon_{\text{ISCO}}$$

# Steady-State Models: $\alpha$ Disks

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e.g.:

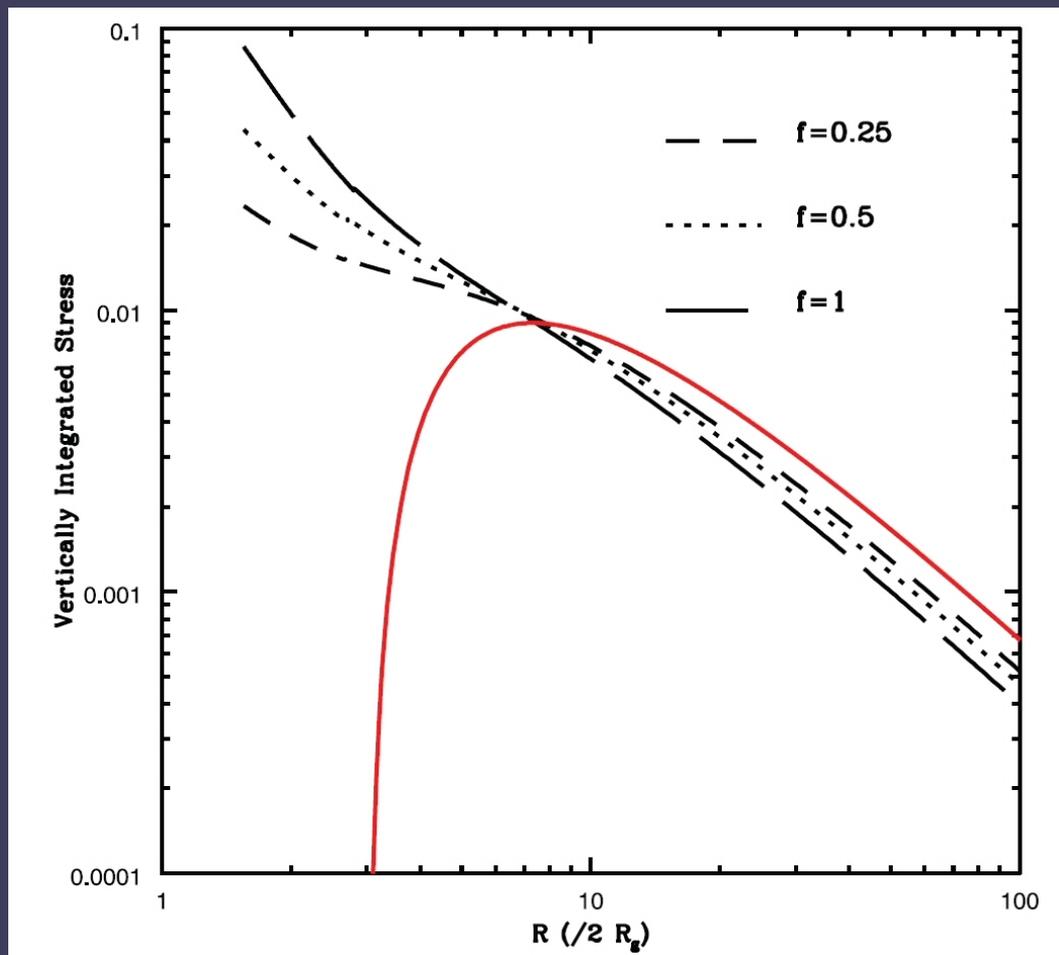
Muchotrzeb & Paczynski (1982)

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(Schwarzschild BHs)

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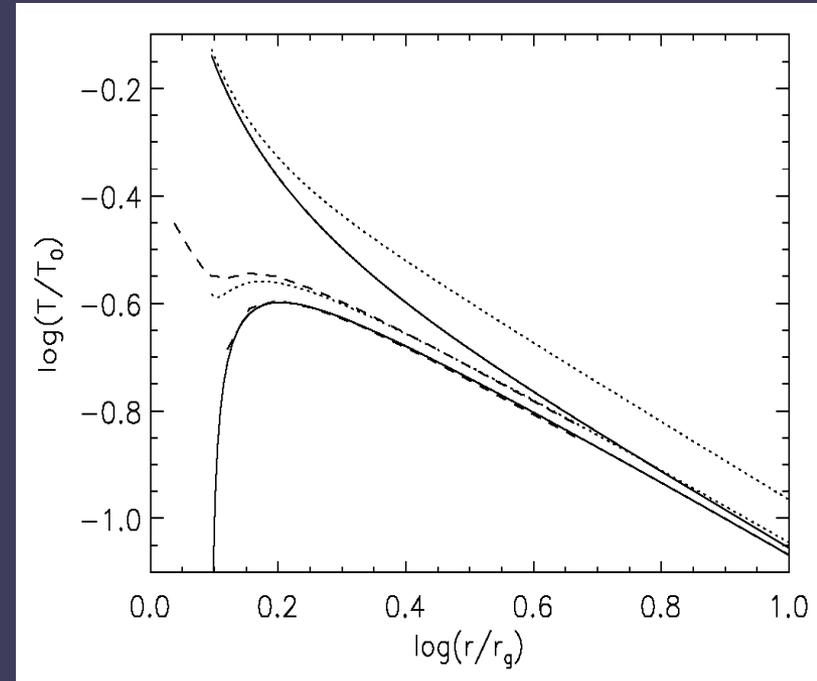
Shafee, Narayan, McClintock (2008)

$$\eta \sim 1 - \epsilon_{\text{ISCO}}$$

# Steady-State Models: Finite Torque Disks

- Krolik (1999)
  - B-field dynamically significant for  $r < r_{\text{ISCO}}$
- Gammie's Inflow model (1999)
  - Matched interior model to thin disk  $\rightarrow \eta > 1$  possible
- Agol & Krolik (2000)
  - Parameterize ISCO B.C. with  $\eta$
  - $\eta$  reduced by increased probability of photon capture

$\rightarrow$  Need dynamical models!!!

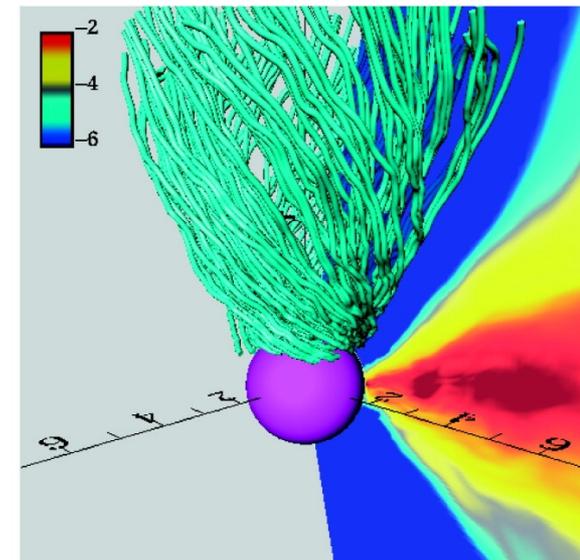
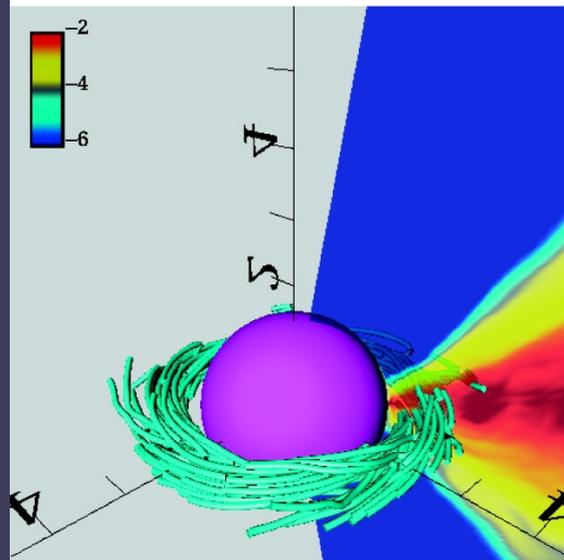
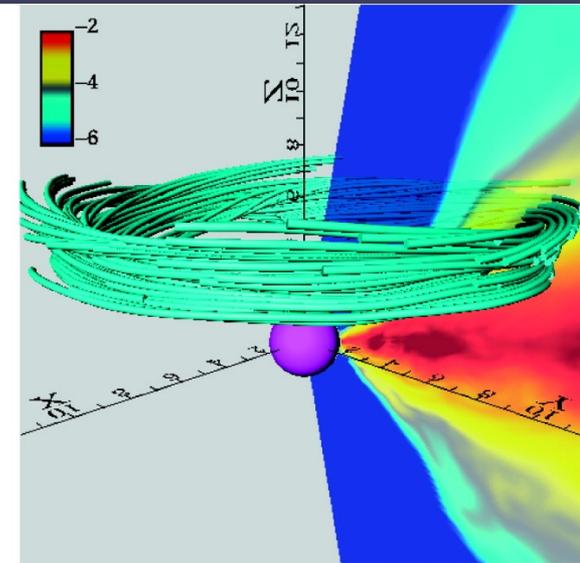
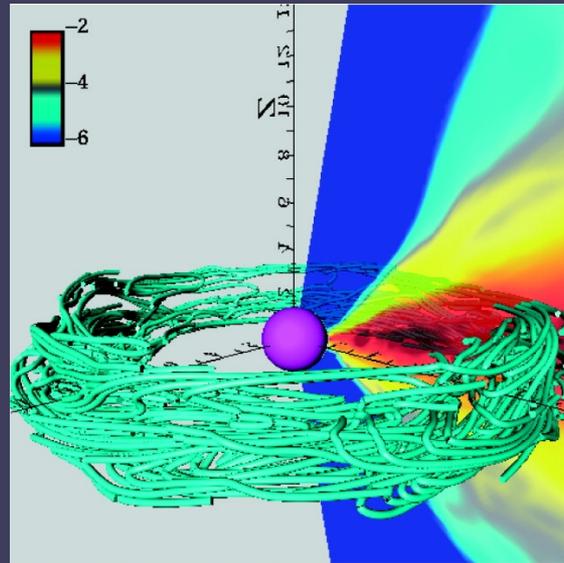


# Dynamical Global Disk Models

- De Villiers, Hawley, Hirose, Krolik (2003-2006)

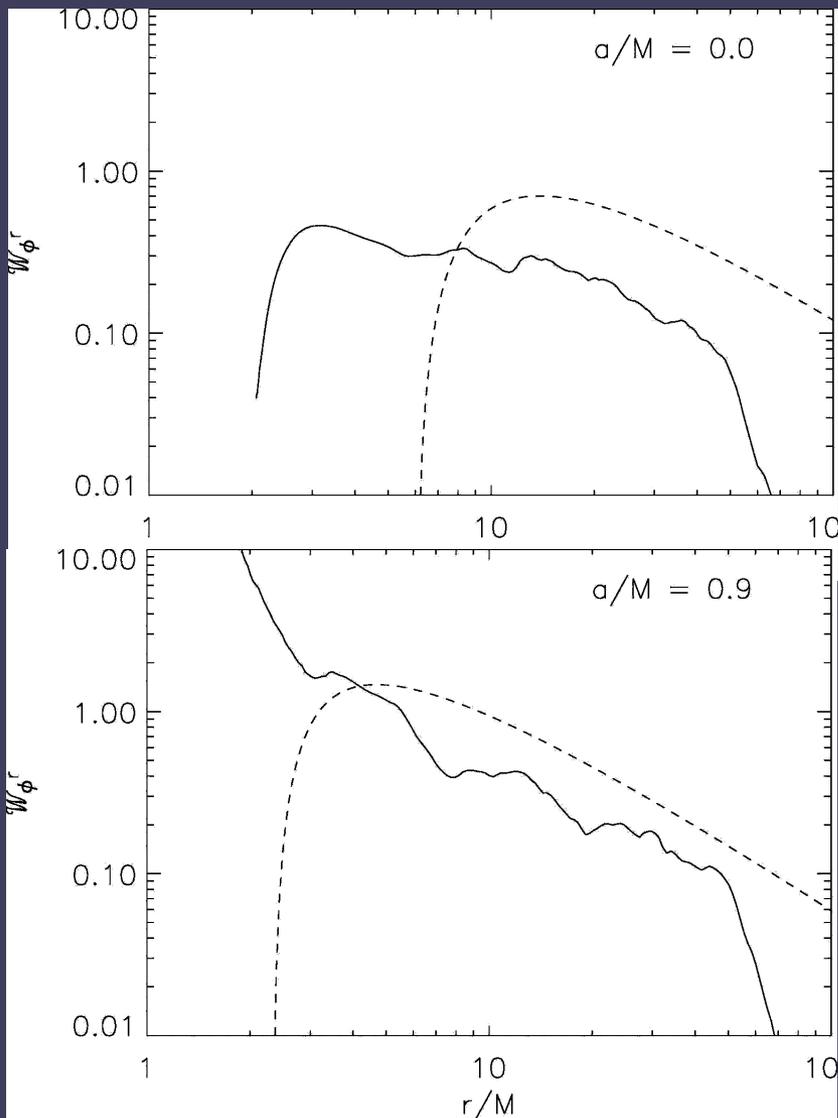
- MRI develops from weak initial field.

- Significant field within ISCO up to the horizon.

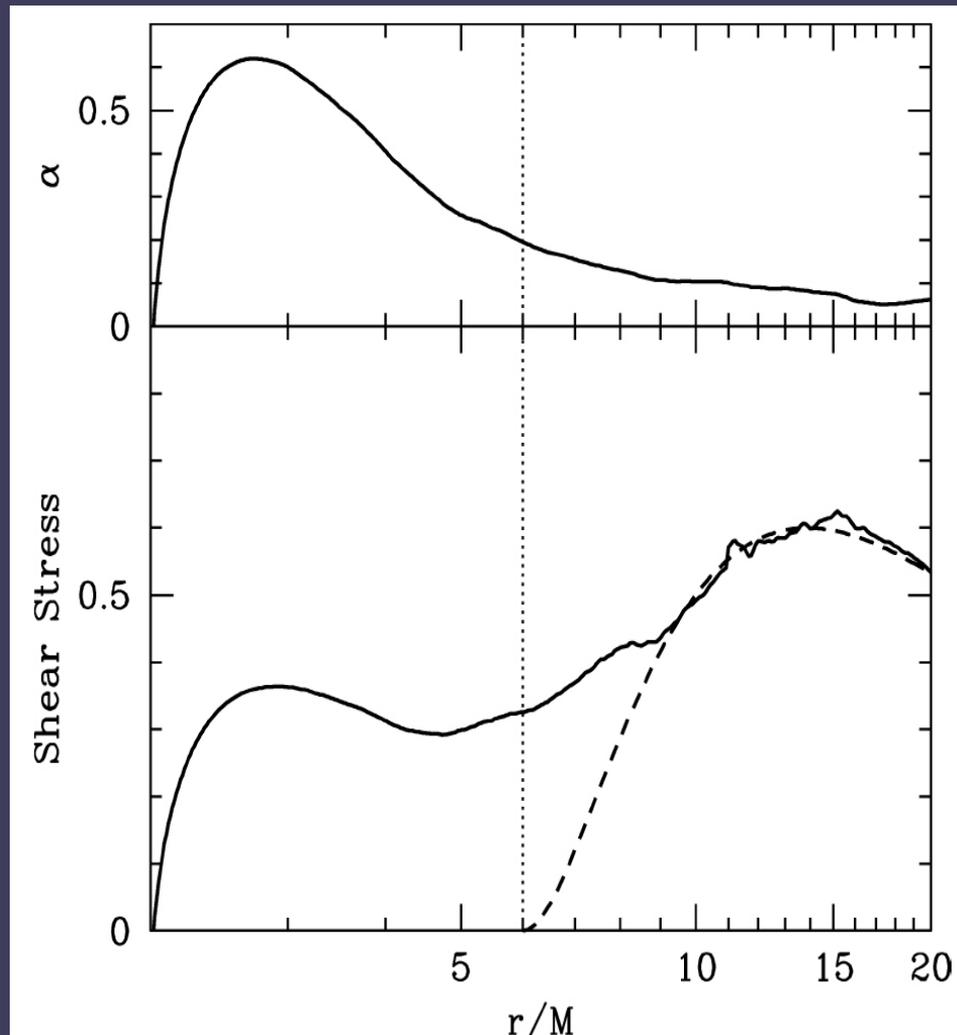


Hirose, Krolik, De Villiers, Hawley (2004)

# Dynamical Global Disk Models

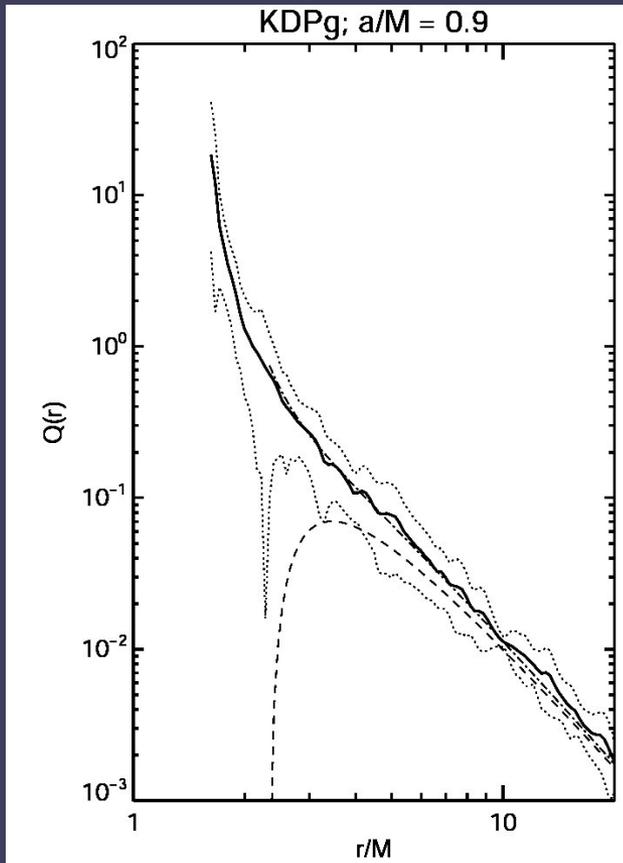


Krolik, Hawley, Hirose (2005)  
 $H/R \sim 0.1 - 0.15$



Shafee et al. (2008)  
 $H/R \sim 0.05$

# Inner Radiation Edge



- Beckwith, Hawley & Krolik (2008)
- Models dissipation stress as EM stress
- Large dissipation near horizon compensated partially by capture losses and gravitational redshift.
- Used (non-conserv.) int. energy code (dVH) assuming adiabatic flow
  - Fails to completely capture heat from shocks and reconnection events
  - Need a conservative code with explicit cooling to directly measure dissipation.

$$S^{\mu\nu} u_{\nu;\mu} = Q_{;\theta}^{\theta}$$

$$S^{\mu\nu} = T_{EM}^{\mu\nu}$$

# Our Method: Simulations

- **HARM:**

Gammie, McKinney, Toth (2003)

$$\nabla_{\nu} {}^*F^{\mu\nu} = 0$$

- Axisymmetric (2D)

- Total energy conserving  
(dissipation  $\rightarrow$  heat)

$$\nabla_{\mu} (\rho u^{\mu}) = 0$$

- Modern Shock Capturing techniques  
(greater accuracy)

$$\nabla_{\mu} T^{\mu}_{\nu} = 0$$

- Improvements:

- 3D

- More accurate (parabolic interp. In reconstruction and constraint transport schemes)

- Assume flow is isentropic when  $P_{\text{gas}} \ll P_{\text{mag}}$

# Our Method: Simulations

- Improvements:

- 3D
- More accurate (higher effective resolution)
- Stable low density flows

$$\nabla_{\nu} {}^*F^{\mu\nu} = 0$$

- Cooling function:

- Control energy loss rate
- Parameterized by H/R
- $t_{\text{cool}} \sim t_{\text{orb}}$
- Only cool when  $T > T_{\text{target}}$
- Passive radiation
- Radiative flux is stored for self-consistent post-simulation radiative transfer calculation

$$\nabla_{\mu} (\rho u^{\mu}) = 0$$

$$\nabla_{\mu} T^{\mu}_{\nu} = -\mathcal{F}_{\mu}$$

$$H/R \sim 0.08$$

$$a_{\text{BH}} = 0.9M$$

# Cooling Function

- Optically-thin radiation:

$$T^{\mu}_{\nu;\mu} = -F_{\nu}$$

- Isotropic emission:

$$F_{\nu} = f_c u_{\nu}$$

- Cool only when fluid's temperature too high:

$$f_c = s \Omega u (\Delta - 1 + |\Delta - 1|)^q$$
$$\Delta = \frac{u}{\rho T}$$
$$T(r) = \left( \frac{H}{R} r \Omega \right)^2$$

- $\Omega(r < r_{isco})$  is that of a geodesic with constant  $E$  &  $L$  from ISCO

# GRMHD Disk Simulations

$$N_r \times N_\theta \times N_\phi$$

=

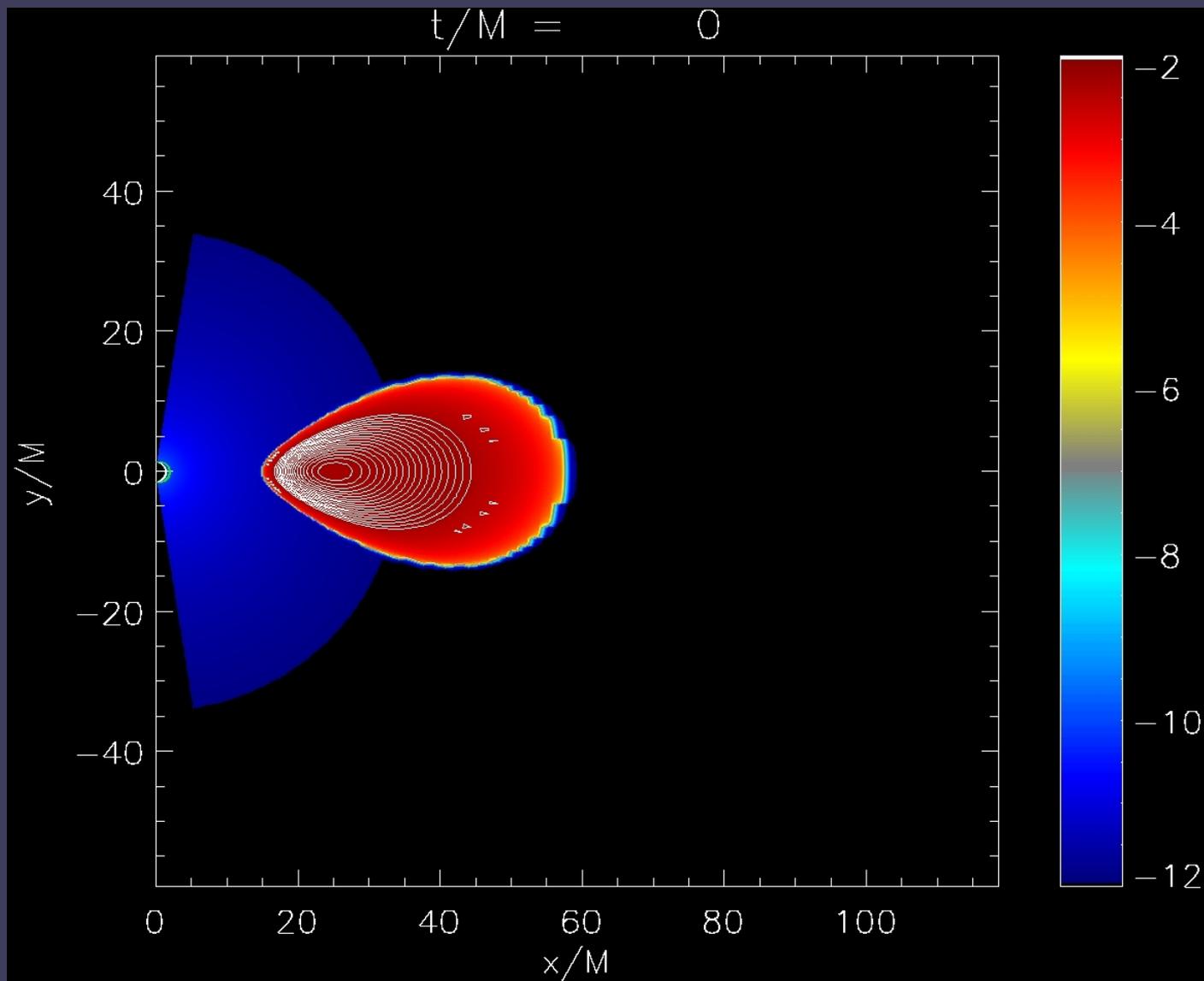
$$192 \times 192 \times 64$$

$$r \in [r_{hor}, 120M]$$

$$\theta \in \pi [0.05, 0.95]$$

$$\phi \in [0, \frac{\pi}{2}]$$

$$a = 0.9M$$



# GRMHD Disk Simulations

$$N_r \times N_\theta \times N_\phi$$

=

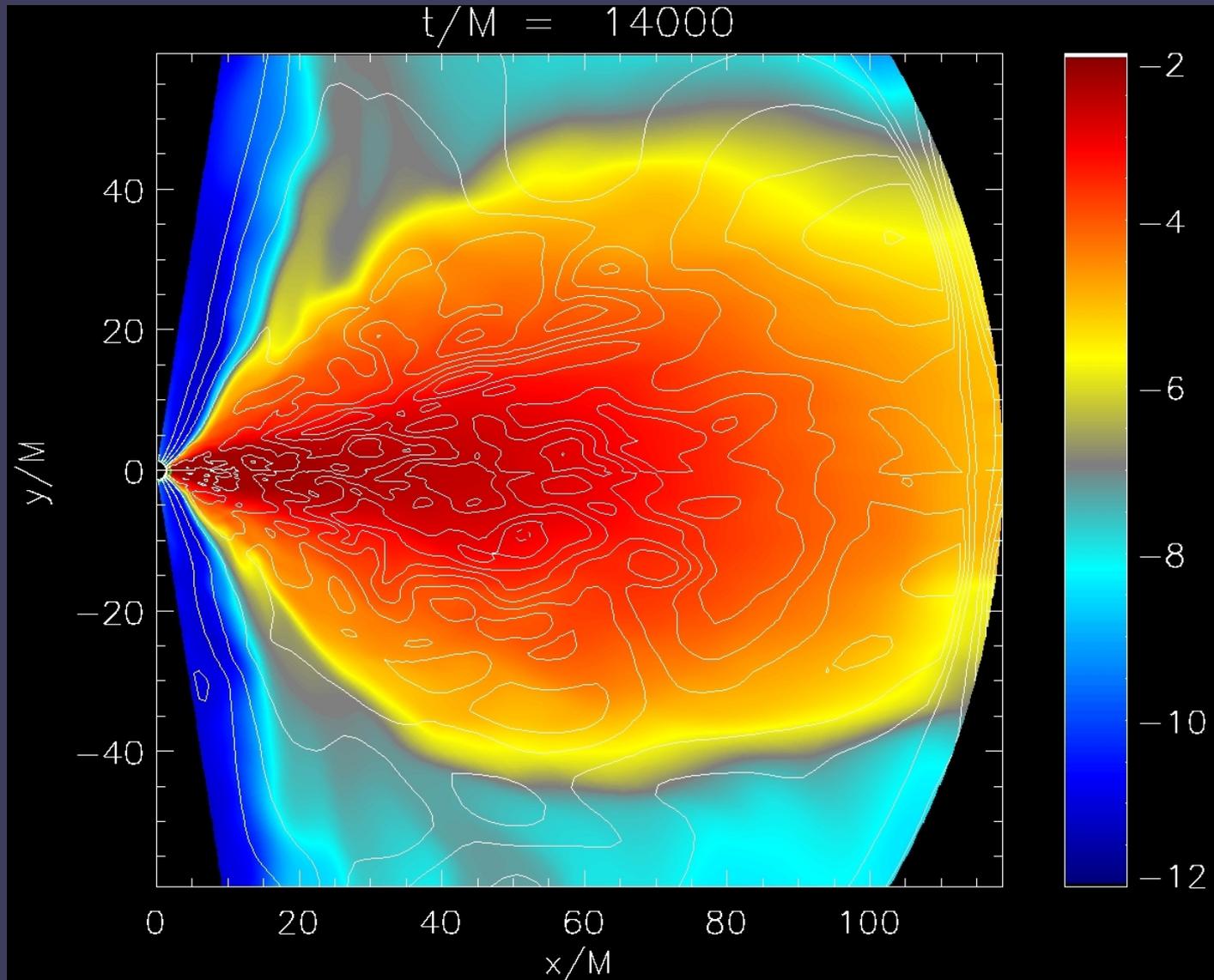
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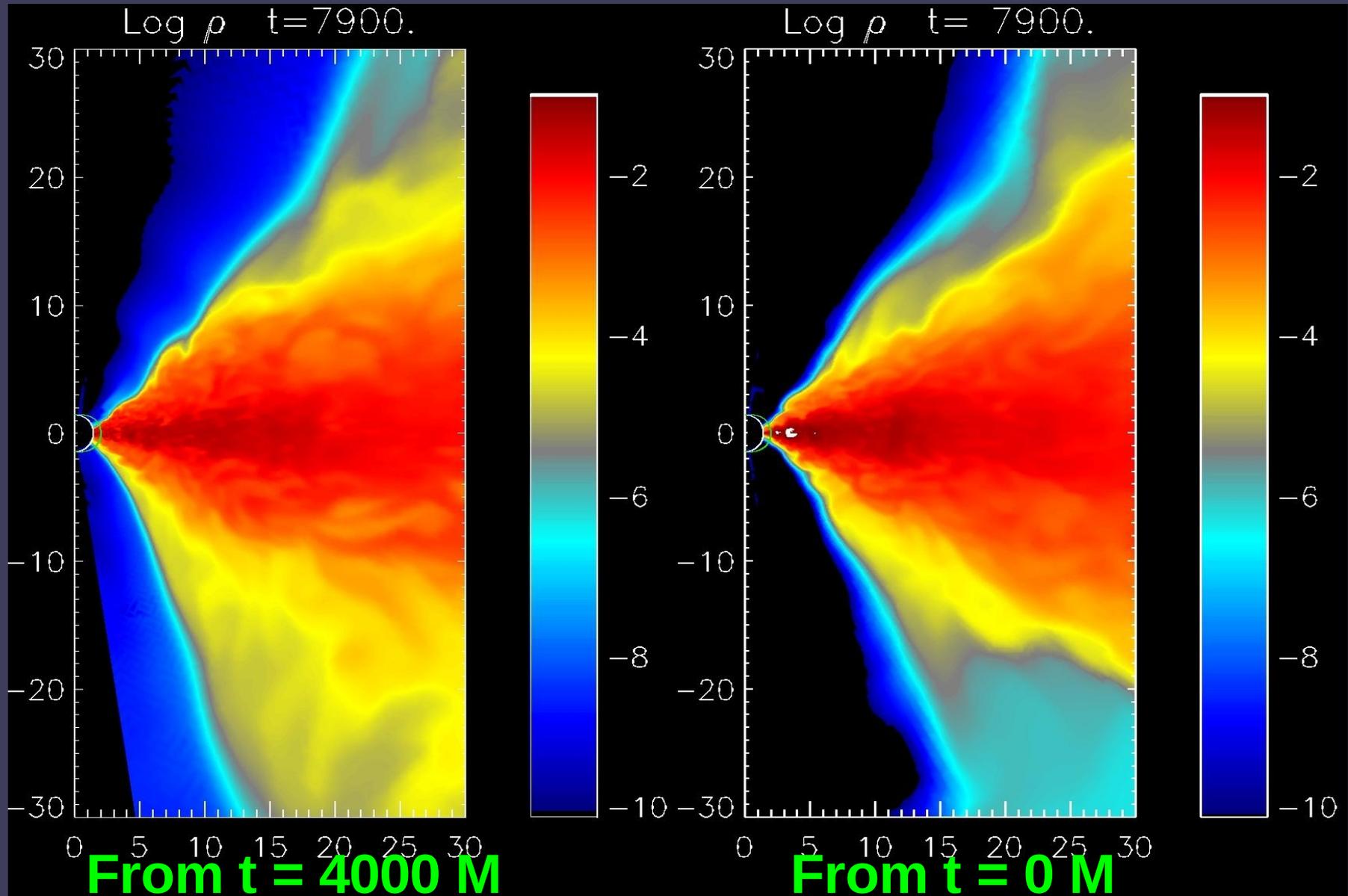
$$\phi \in [0, \frac{\pi}{2}]$$

$$a = 0.9M$$



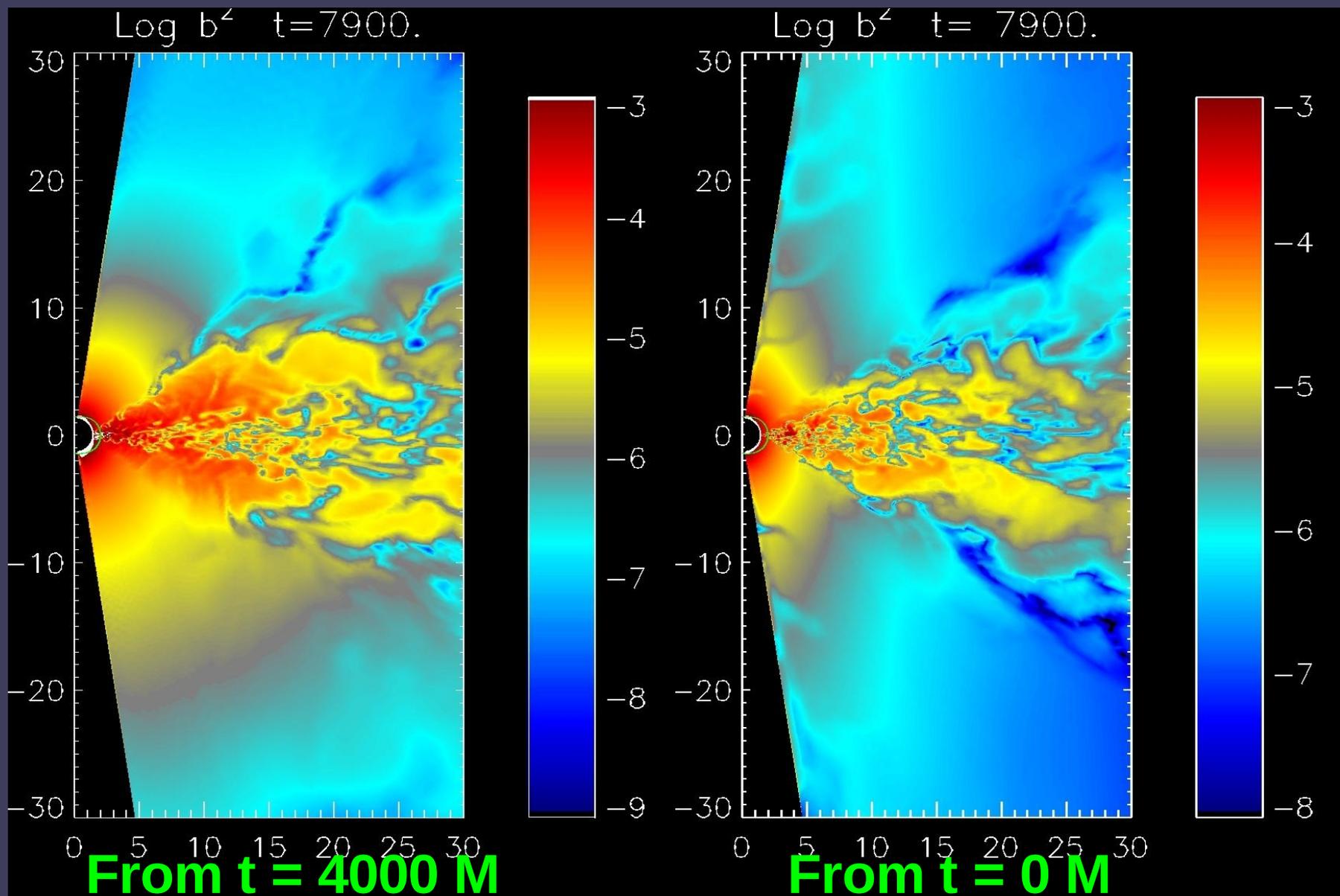
# Cooled #1 vs. Cooled #2

$\log(\rho)$

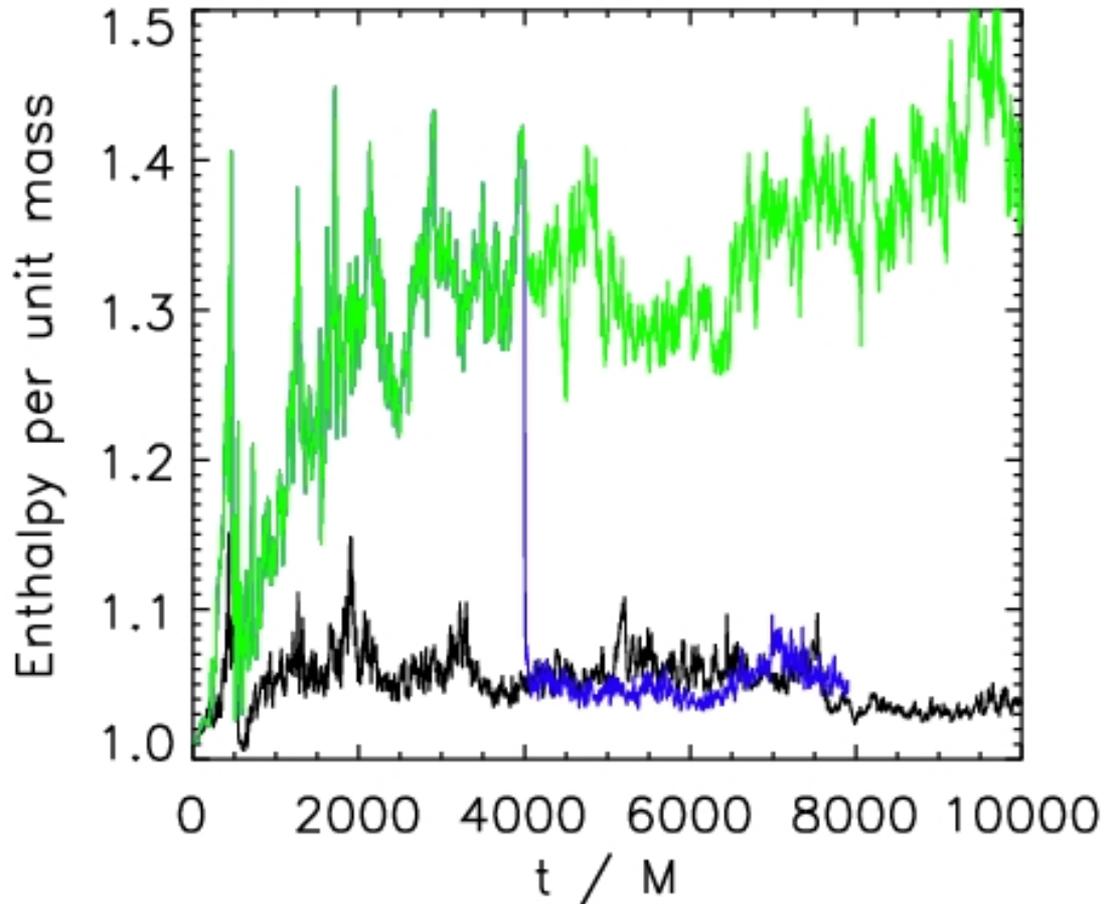


# Cooled #1 vs. Cooled #2

$\log(P_{mag})$



# Cooling Efficacy

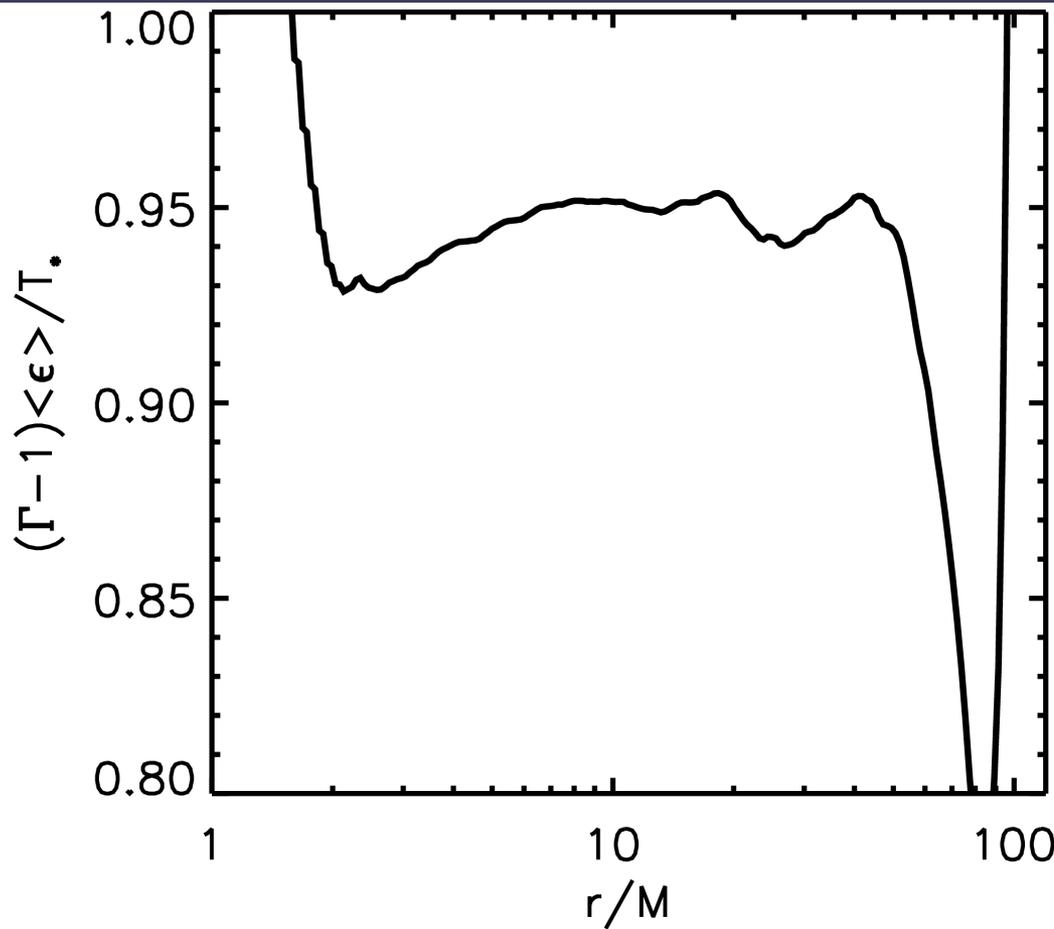


Cooled from  $t=0M$

Cooled from  $t=4000M$

Uncooled

# Target Temperature

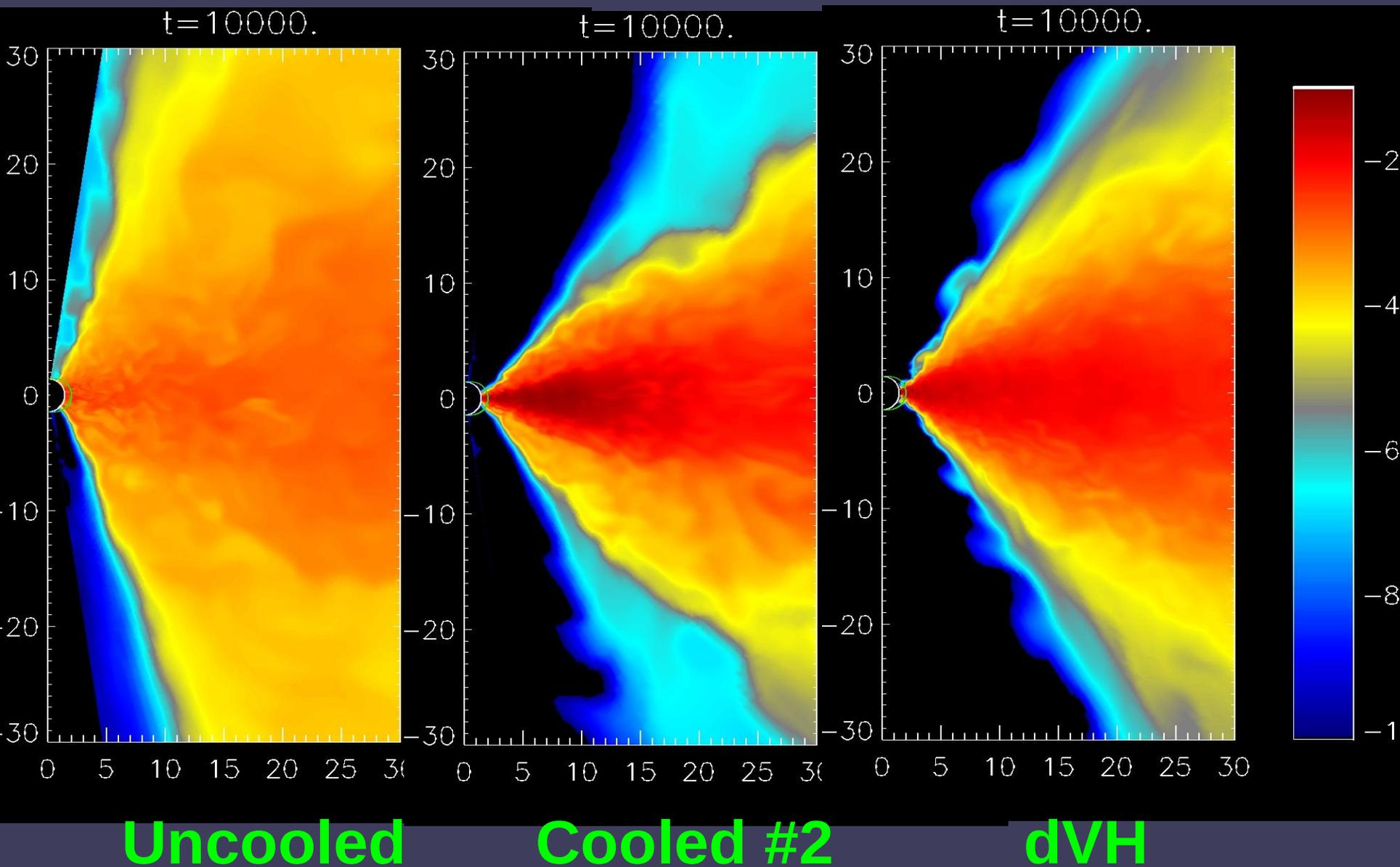


Reaching to within 5% of  
Target Temperature

Cooling Rate  $>\sim$  Diss. Rate

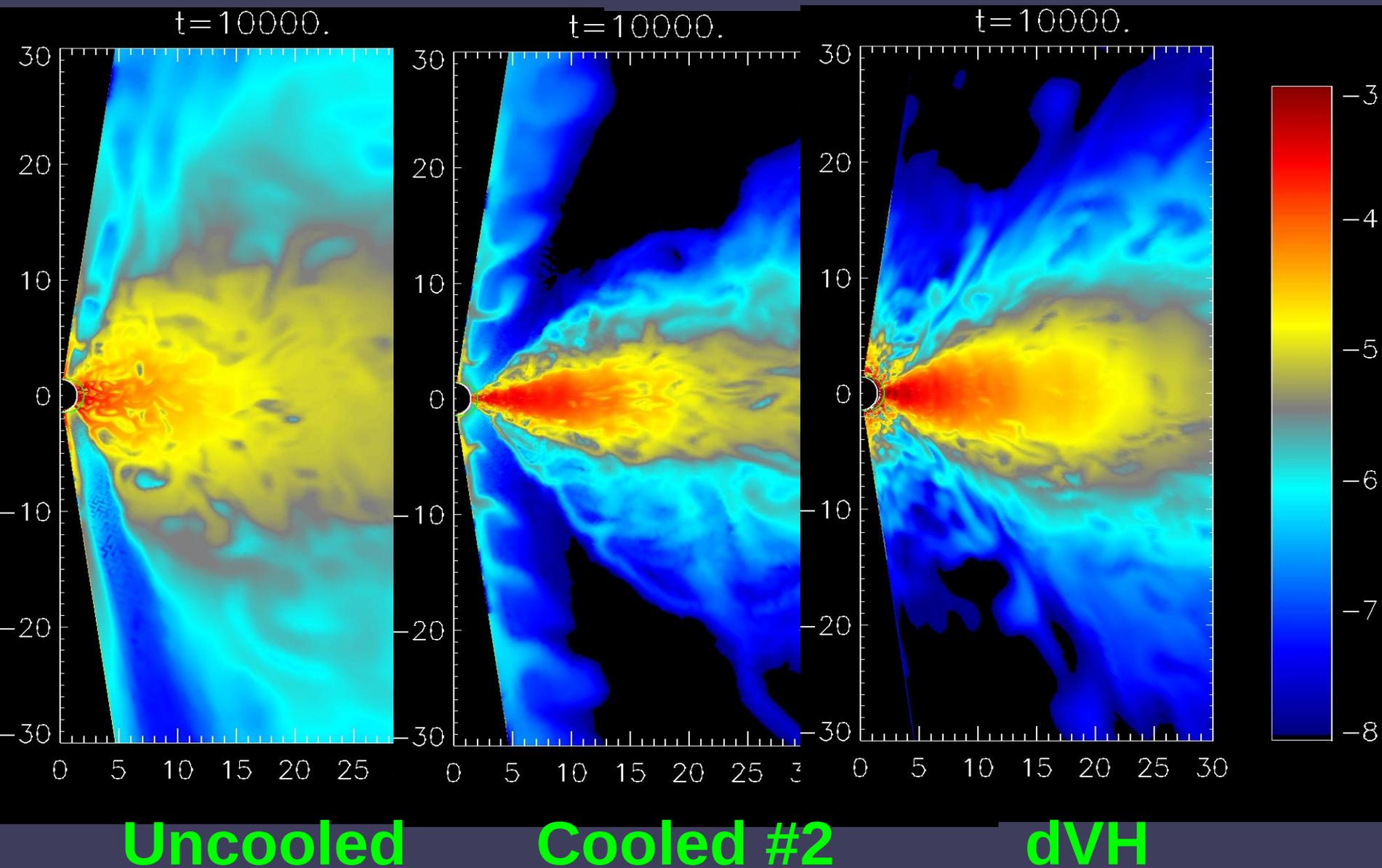
# HARM3D vs. dVH

$\log(\rho)$



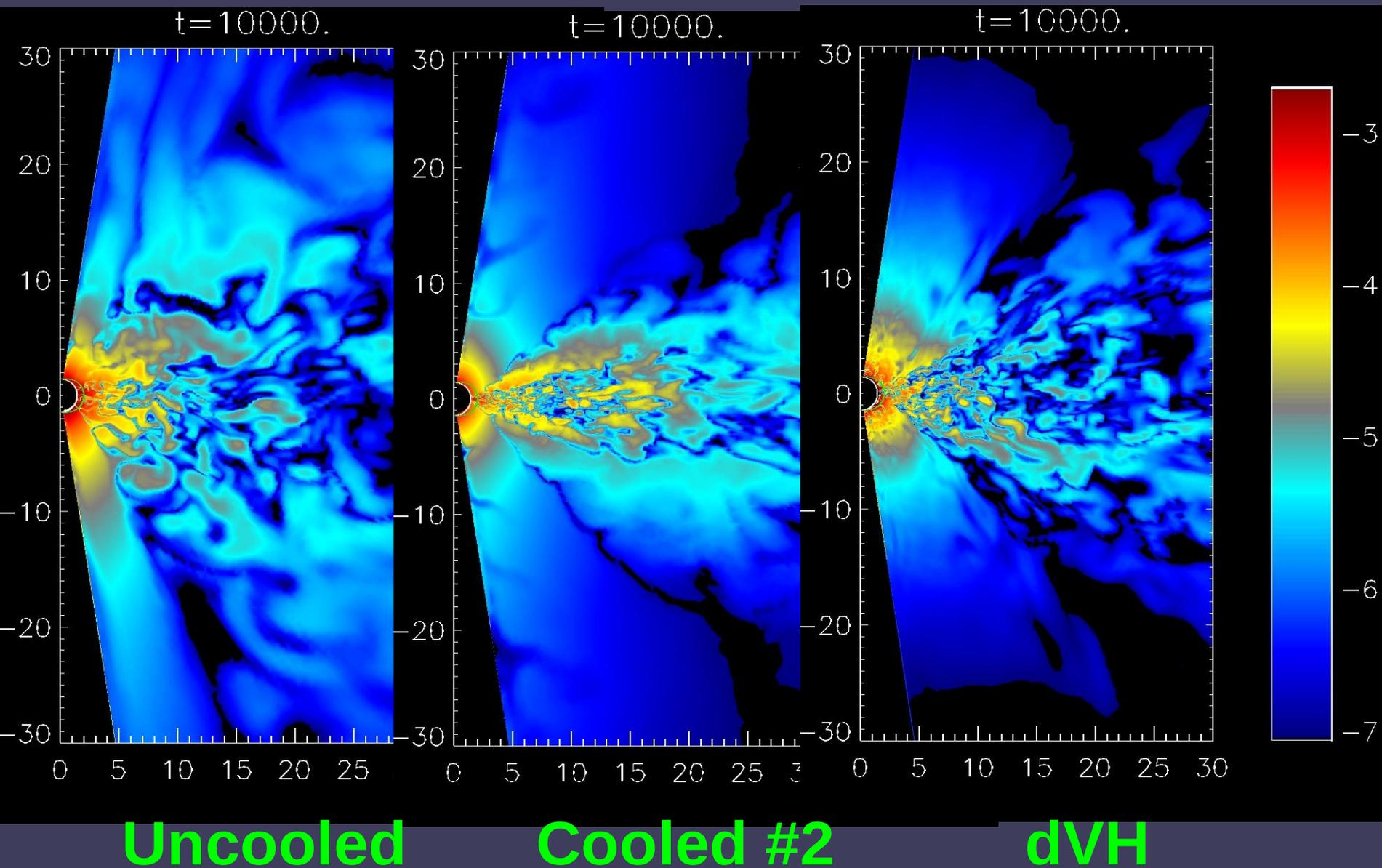
# HARM3D vs. dVH

$\log(P)$



# HARM3D vs. dVH

$\log(P_{mag})$



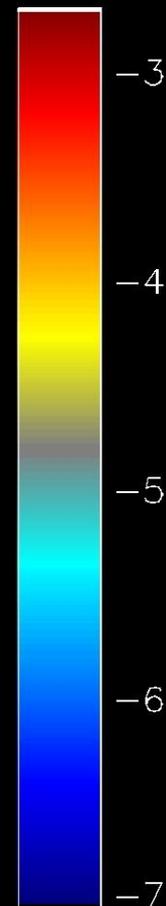
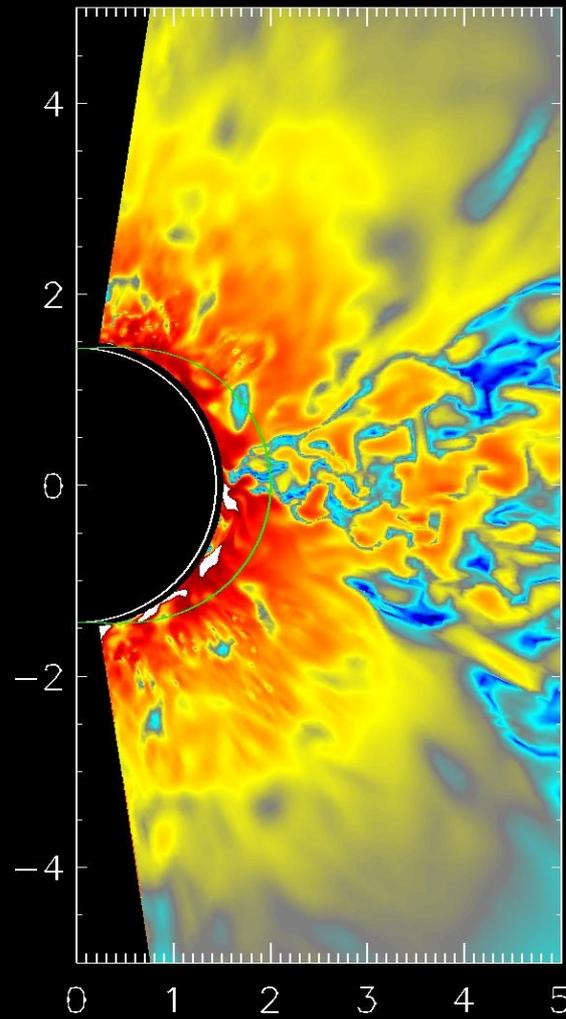
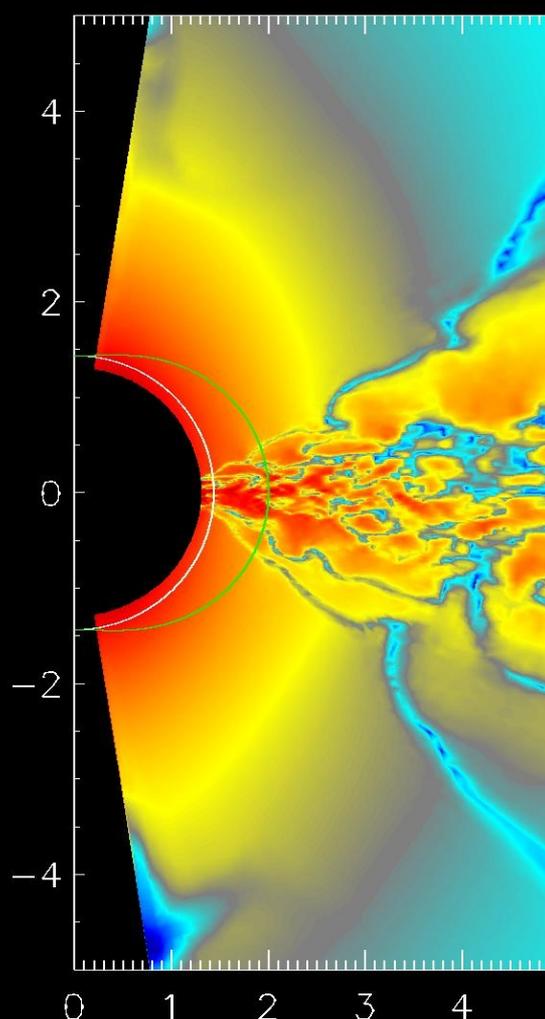
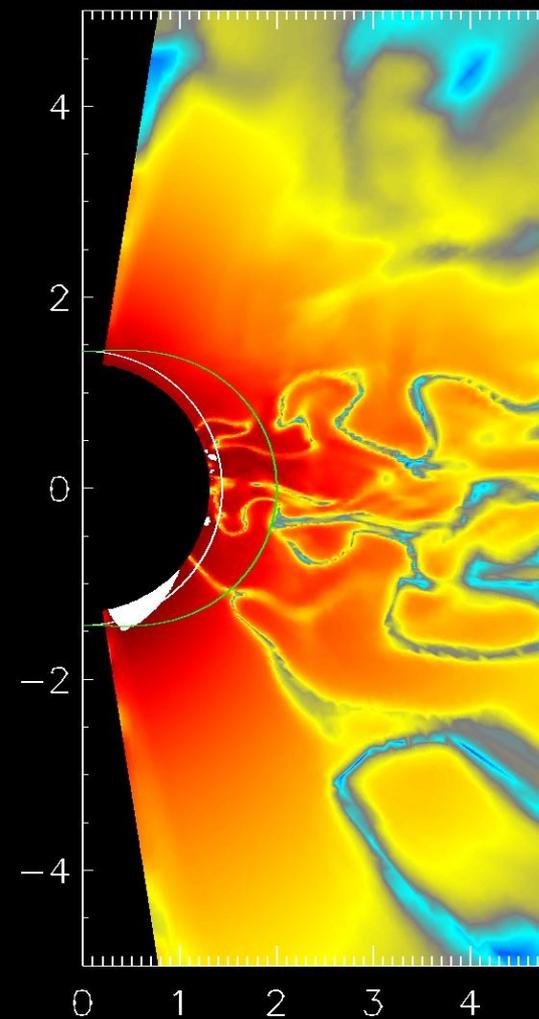
# HARM3D vs. dVH

$\log(P_{mag})$

t=10000.

t=10000.

t=10000.

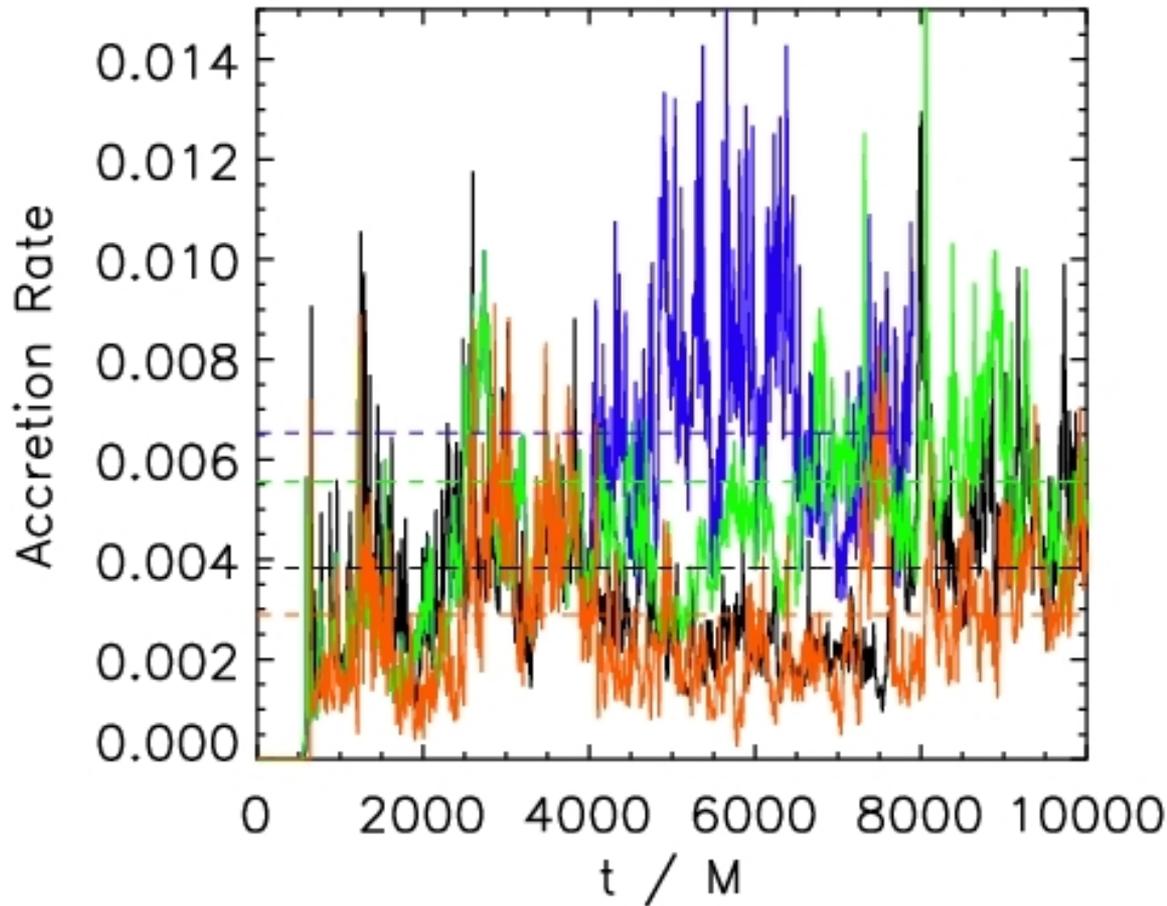


Uncooled

Cooled #2

dVH

# HARM3D vs. dVH

 $\dot{M}$ 

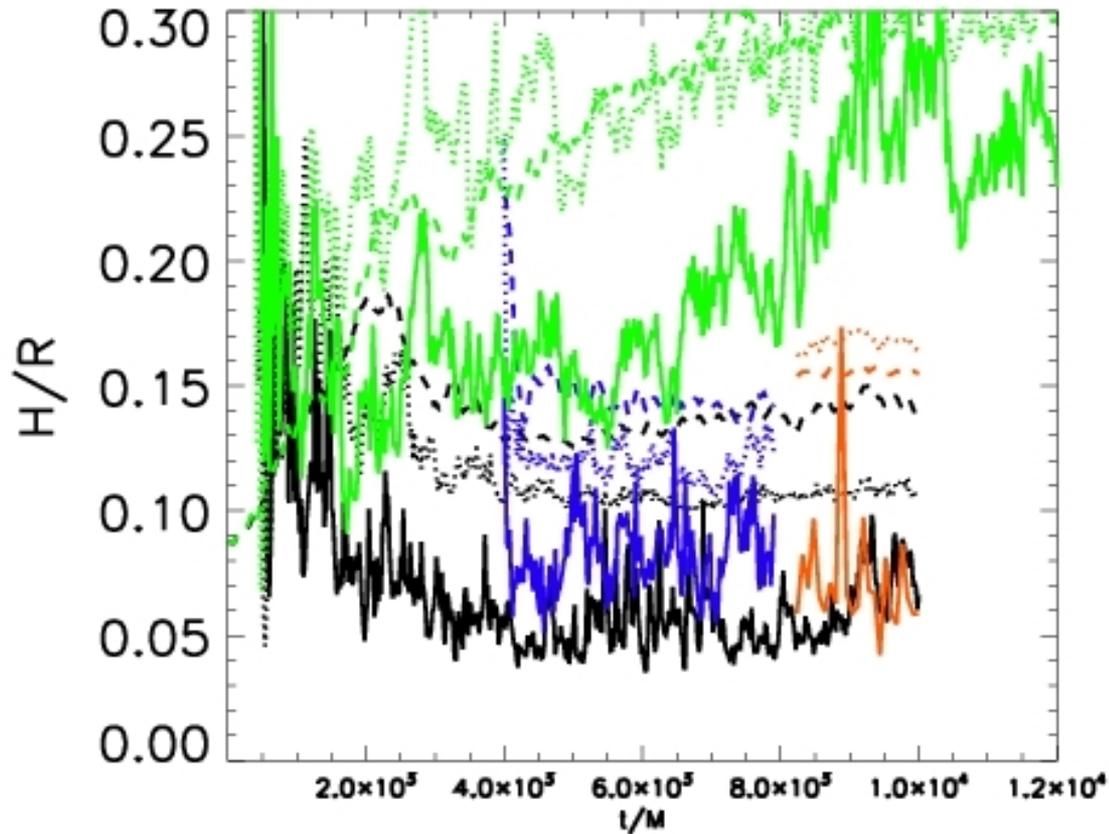
Cooled from  $t=0M$

Cooled from  $t=4000M$

Uncooled

dVH

# HARM3D vs. dVH



Cooled from  $t=0M$

Cooled from  $t=4000M$

Uncooled

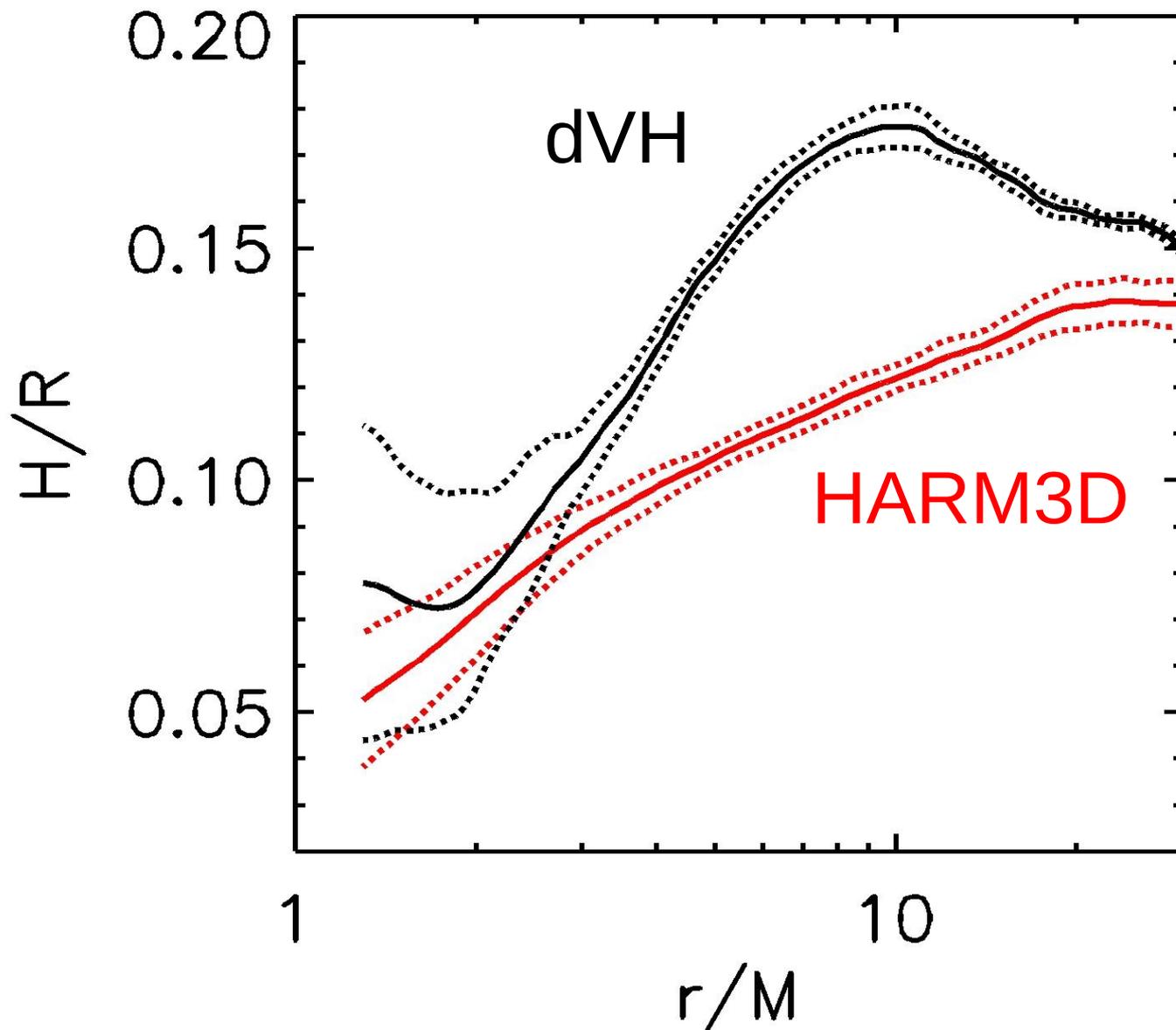
dVH

Solid :  $r = 1.6$

Dotted :  $r = 5$

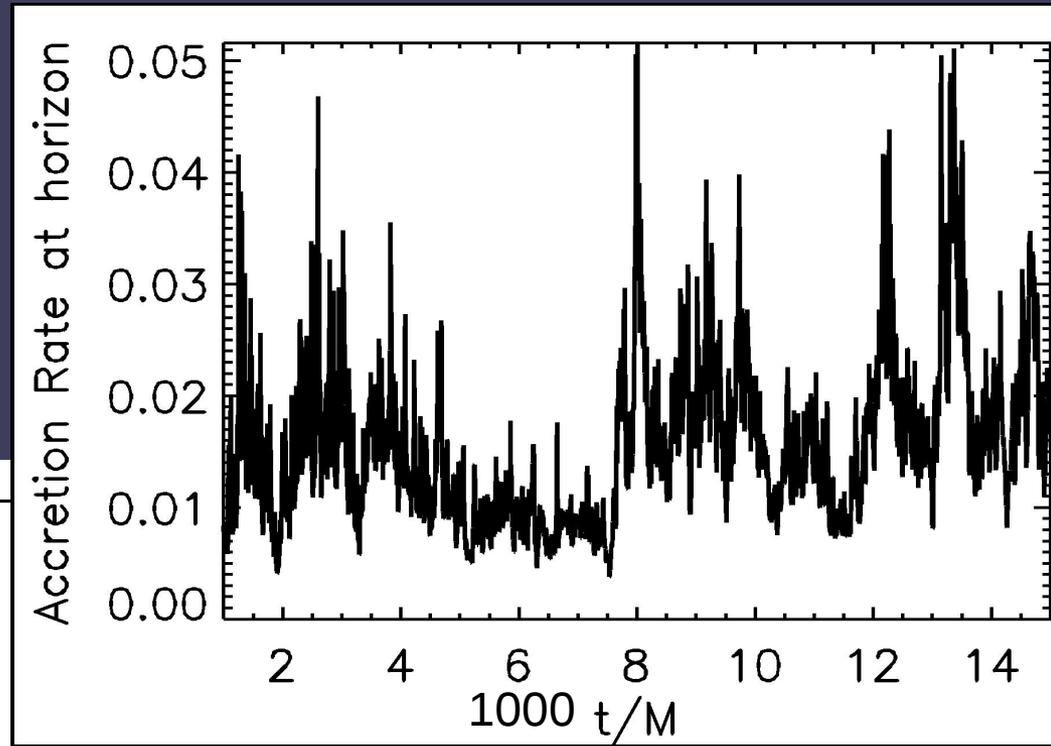
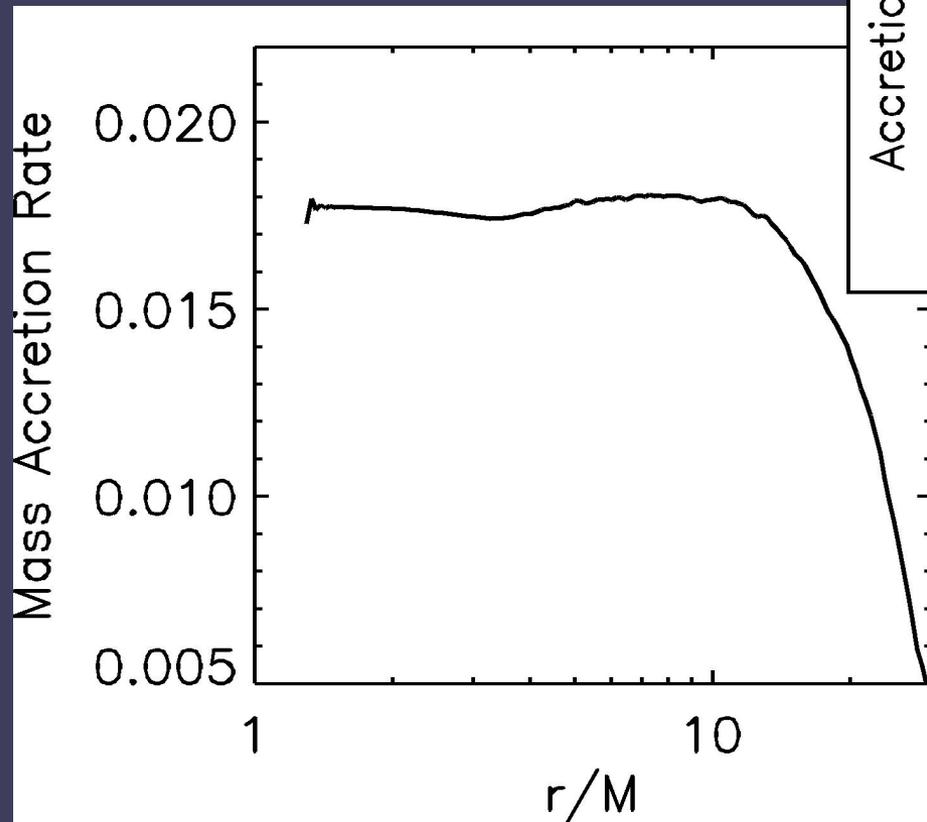
Dashed :  $r = 20$

# Disk Thickness



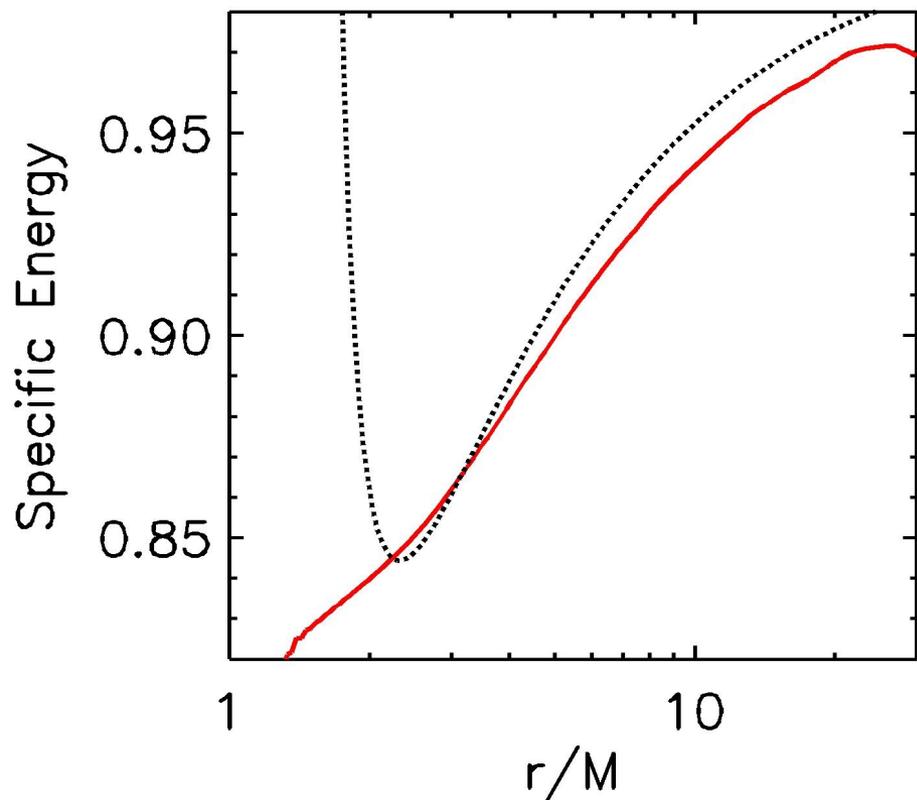
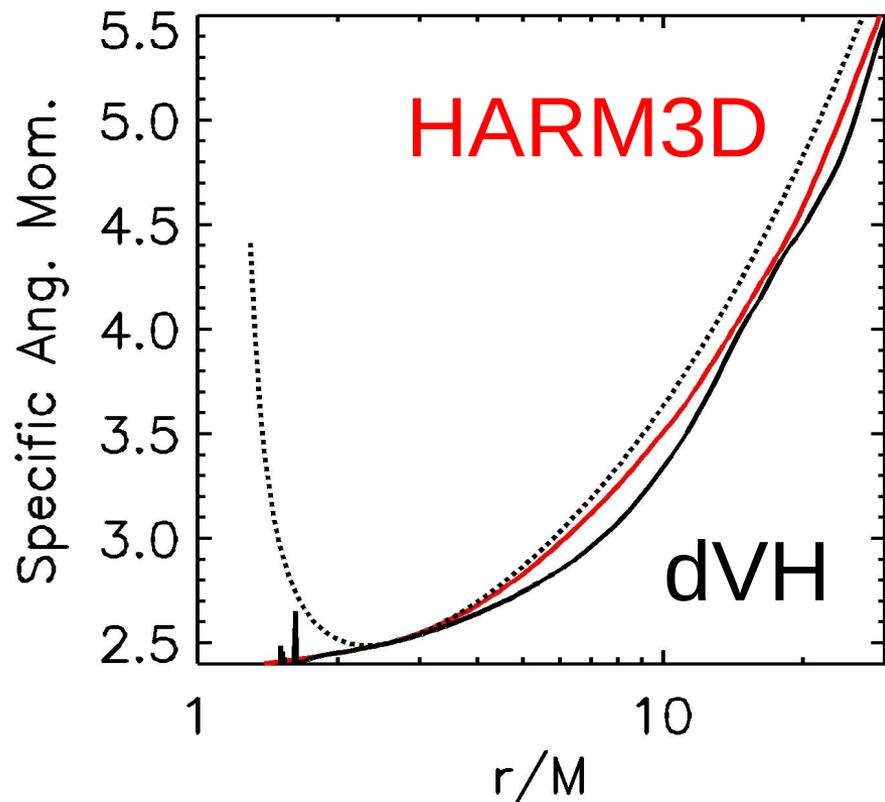
# Accretion Rate

Steady State Period = 7000 – 15000M

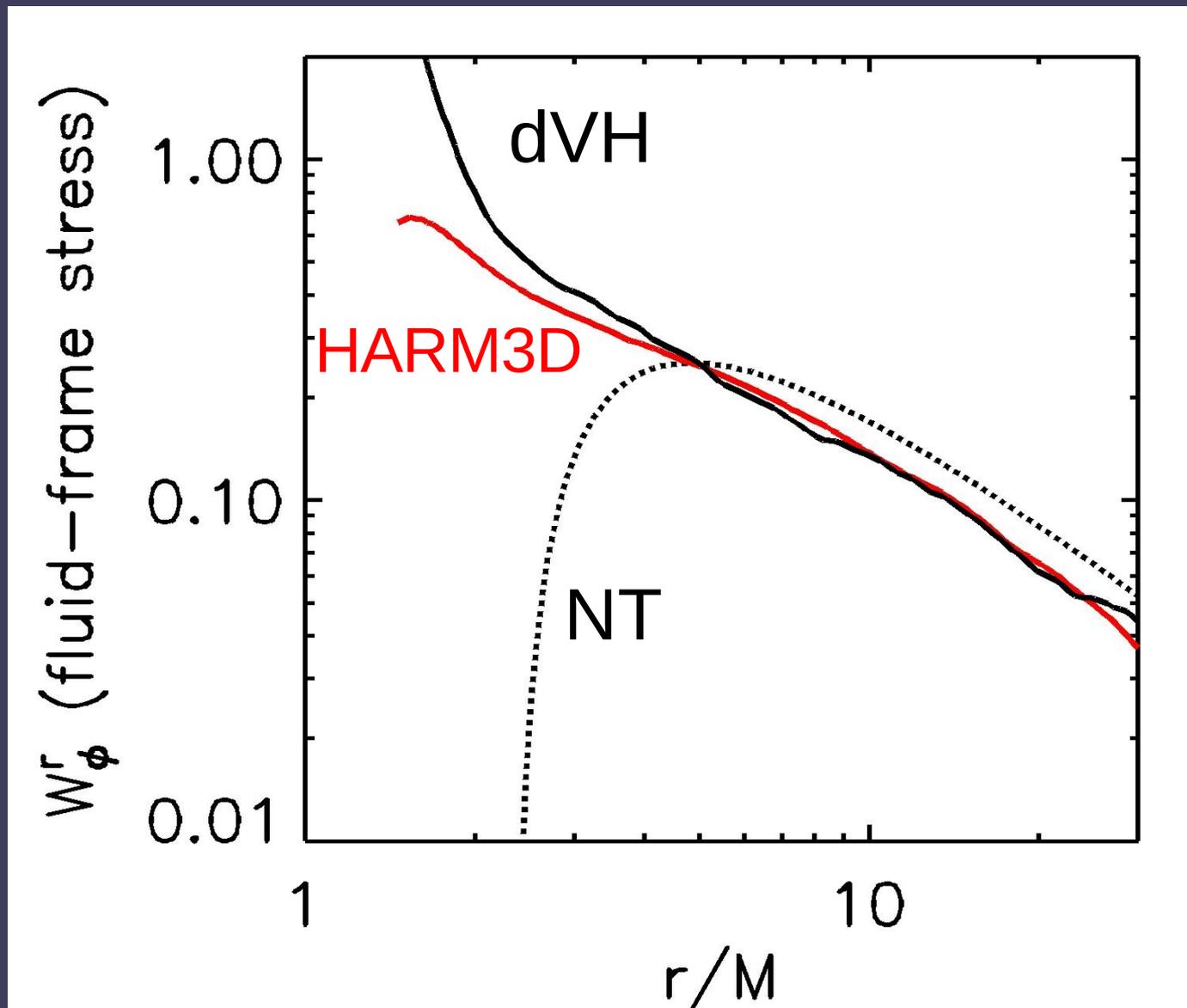


Steady State Region = Horizon – 12M

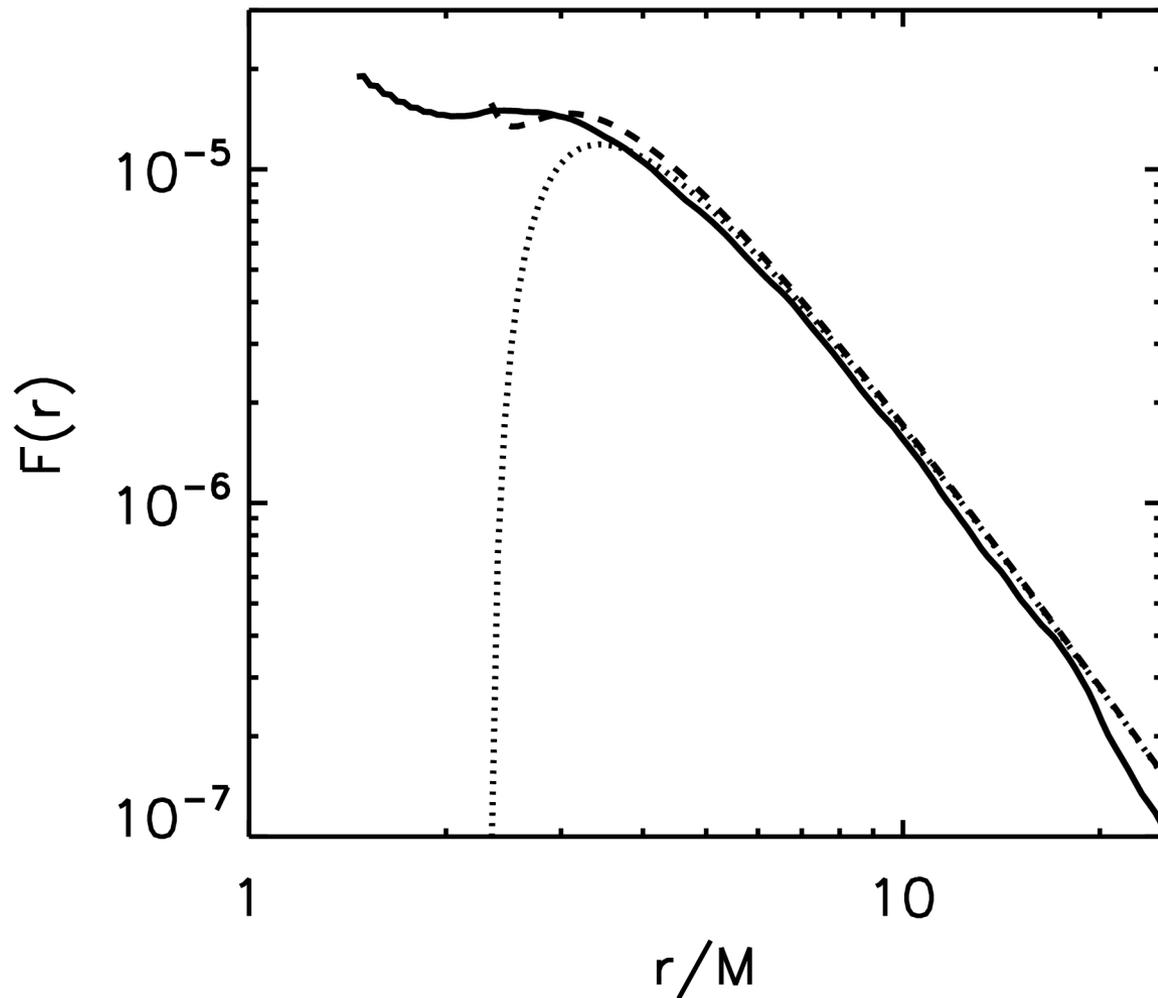
# Departure from Keplerian Motion



# Magnetic Stress



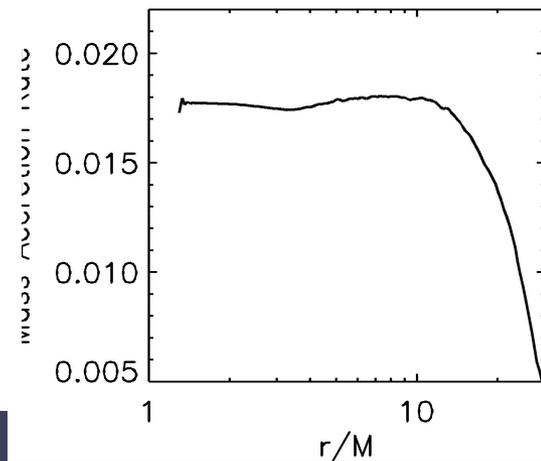
# Fluid Frame Flux



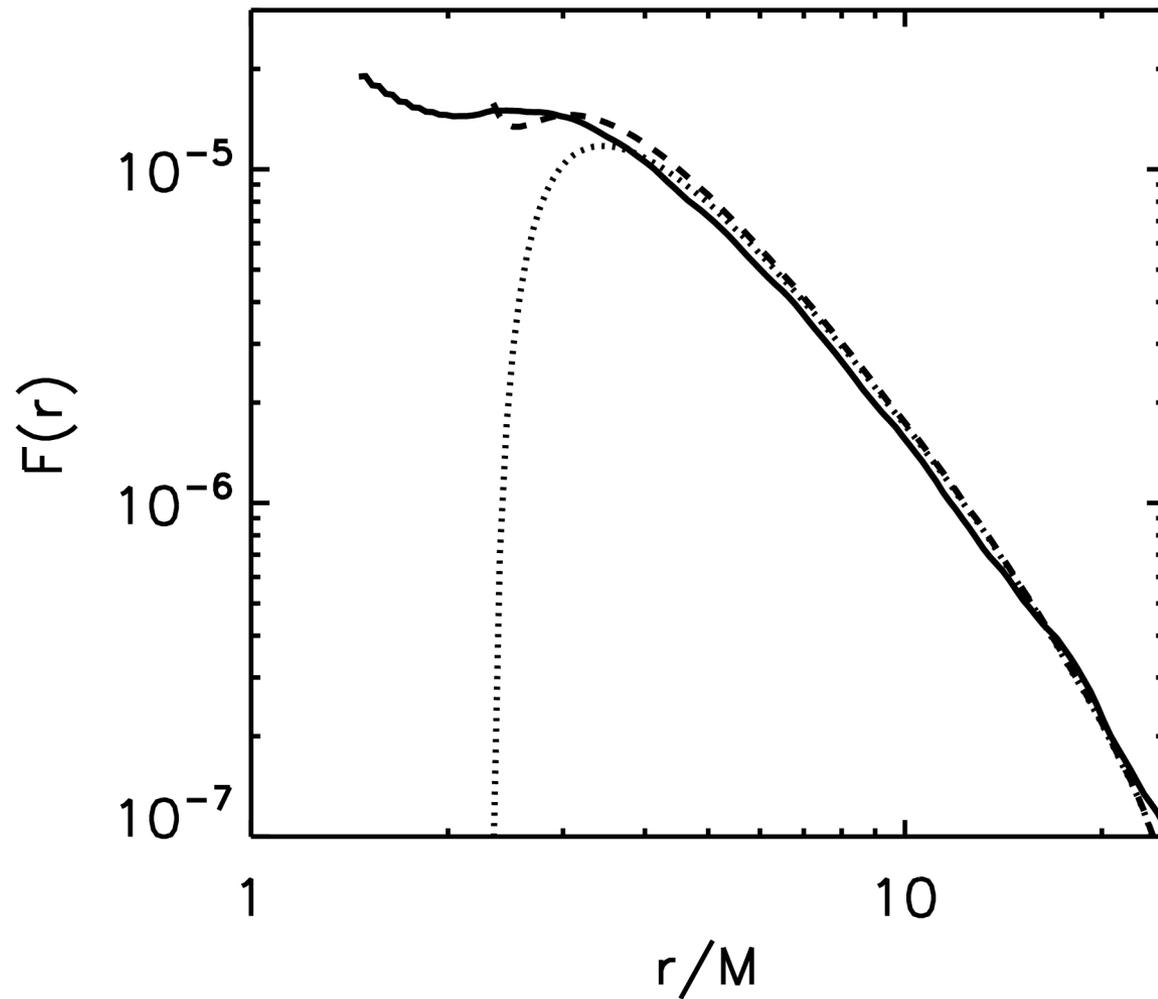
Agol & Krolik (2000)  
model

$$\Delta \eta = 0.01$$

$$\Delta \eta / \eta = 7\%$$



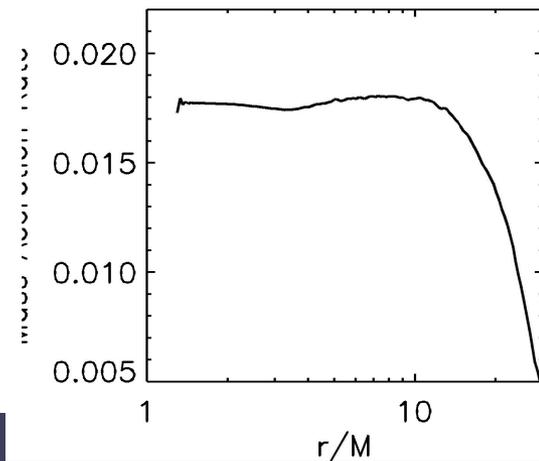
# Fluid Frame Flux



Agol & Krolik (2000)  
model

$$\Delta \eta = 0.01$$

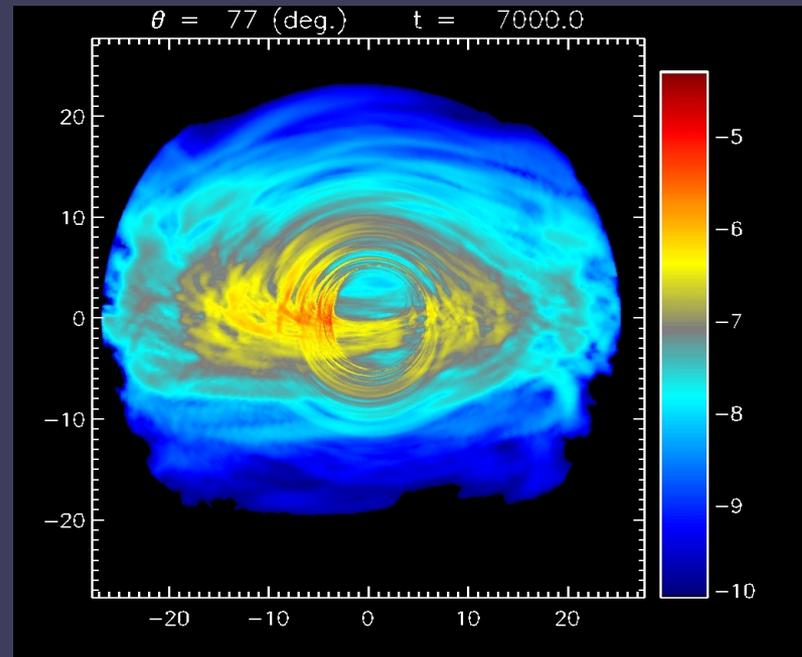
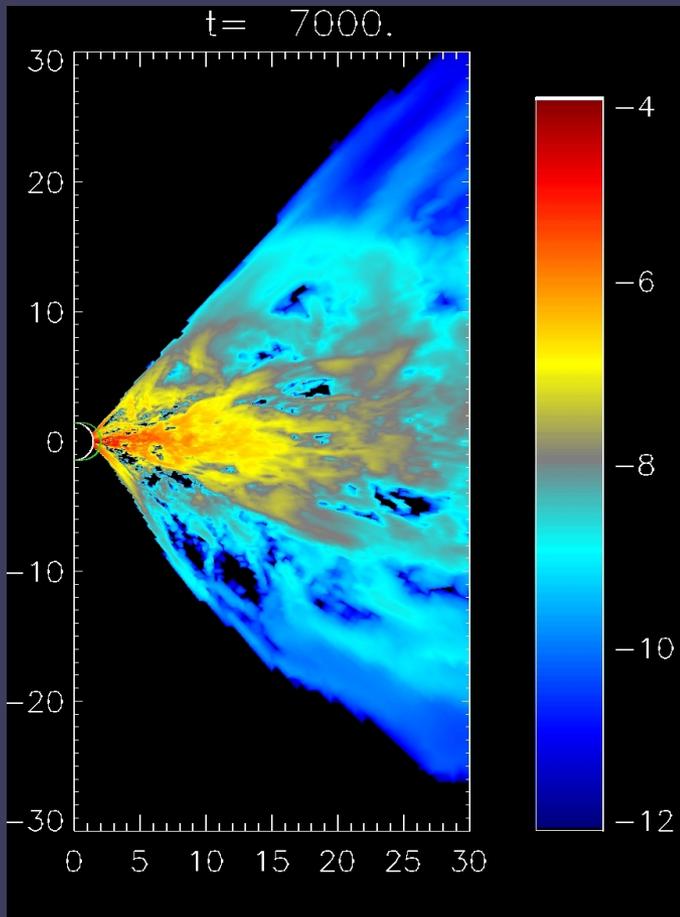
$$\Delta \eta / \eta = 7\%$$



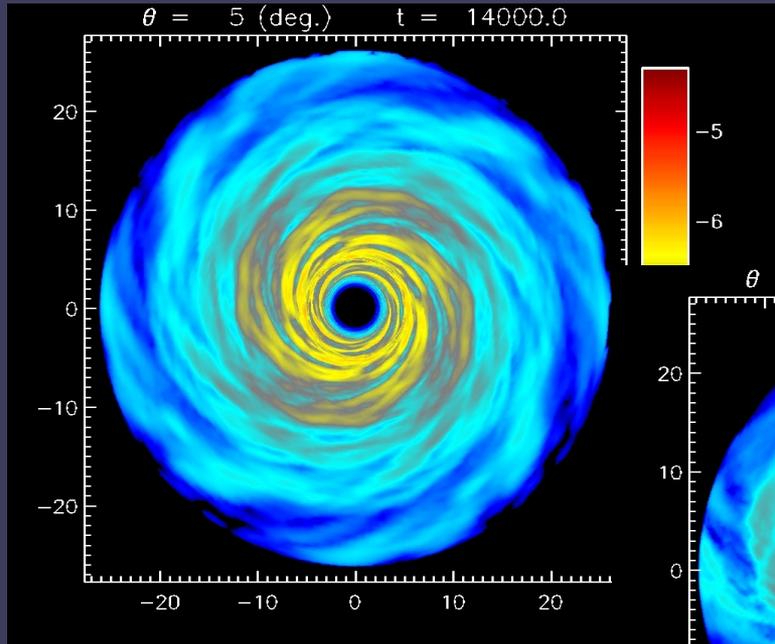
# Our Method: Radiative Transfer

$$j_\nu = \frac{f_c}{4\pi\nu^2}$$

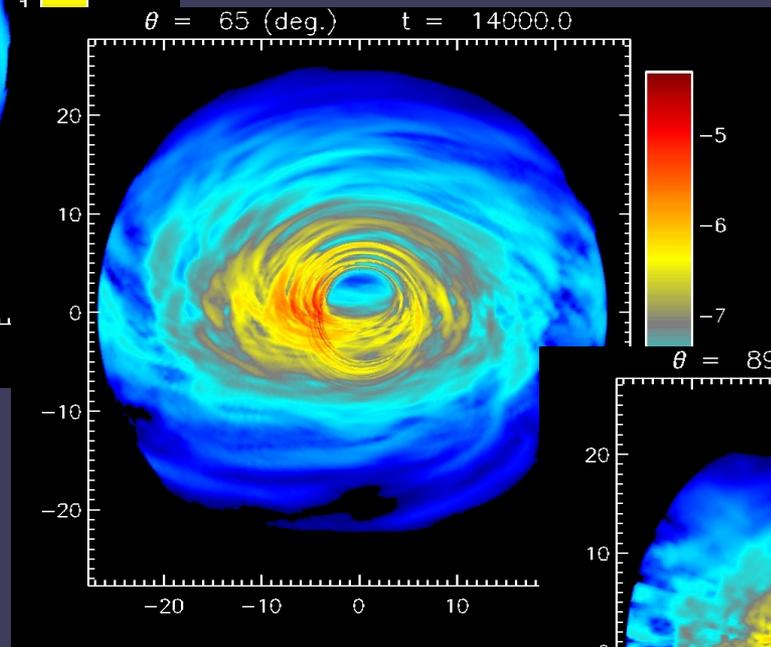
- Full GR radiative transfer
  - GR geodesic integration
  - Doppler shifts
  - Gravitational redshift
  - Relativistic beaming
  - Uses simulation's fluid vel.
  - Inclination angle survey
  - Time domain survey



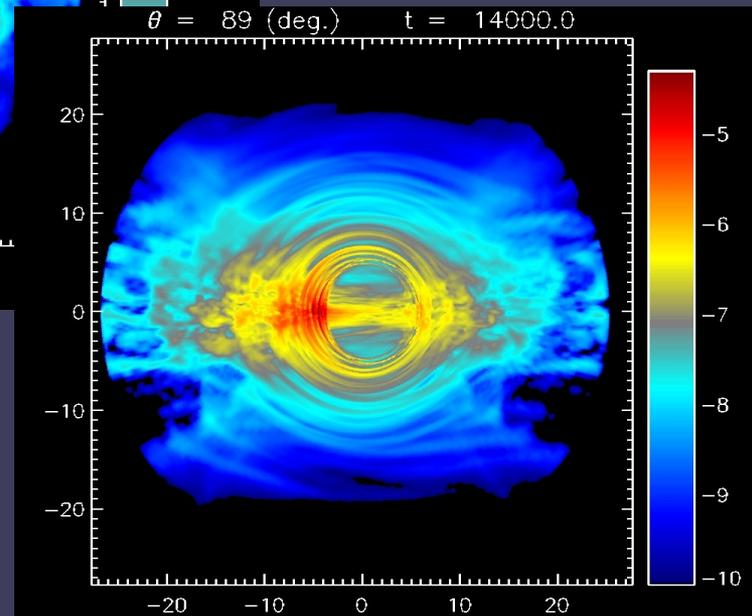
# Observer-Frame Intensity: Inclination



$i=5^\circ$



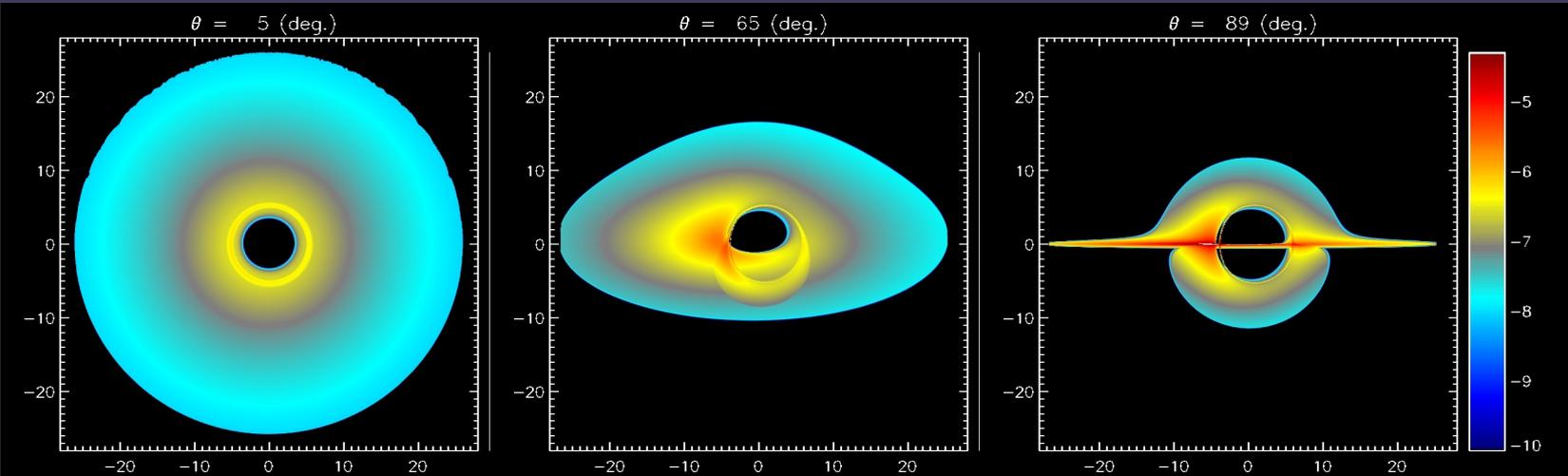
$i=65^\circ$



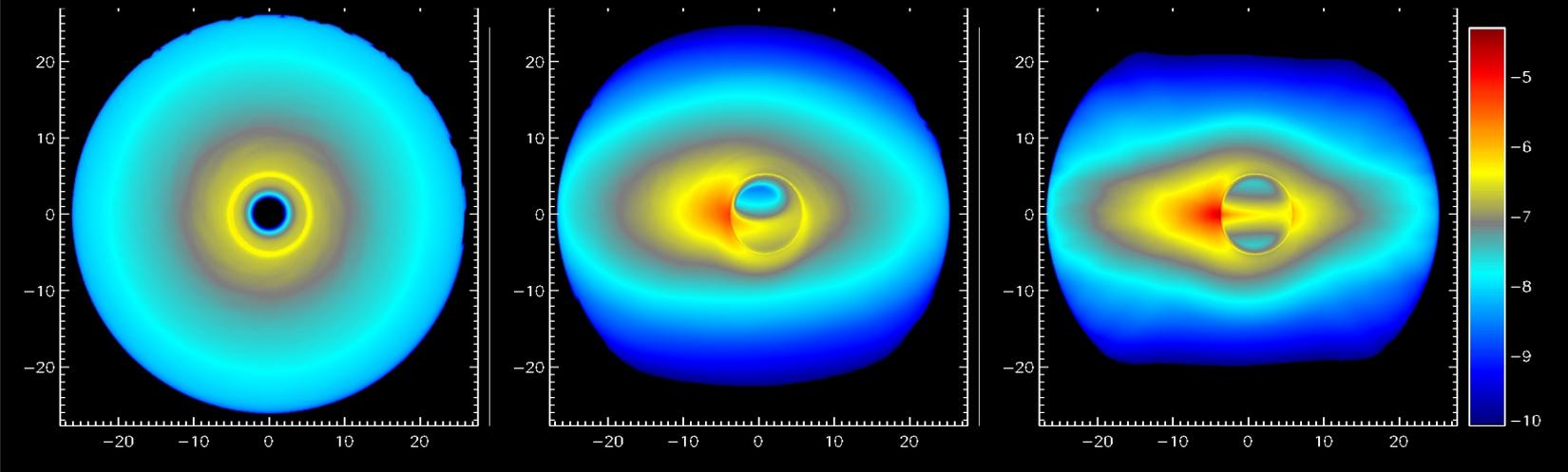
$i=89^\circ$

# Observer-Frame Intensity: Time Average

NT



HARM

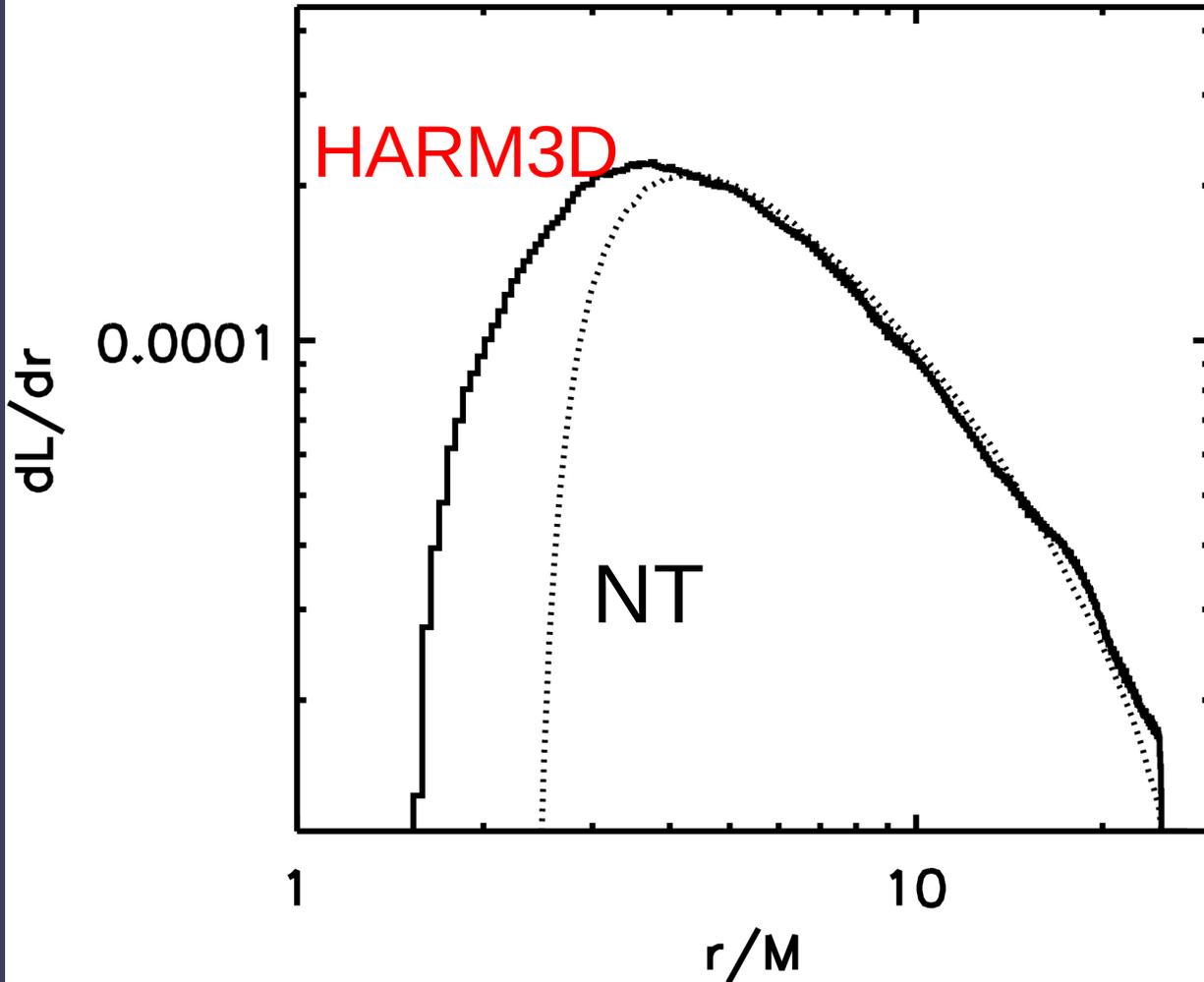


$i=5^\circ$

$i=65^\circ$

$i=89^\circ$

# Observer Frame Luminosity: Angle/Time Average



Assume NT profile  
for  $r > 12M$ .

$$\eta_{H3D} = 0.151$$

$$\eta_{NT} = 0.143$$

$$\Delta\eta/\eta = 6\%$$

$$\Delta R_{in}/R_{in} \sim 80\%$$

$$\Delta T_{max}/T_{max} = 30\%$$

If disk emitted retained heat:  $\Delta\eta/\eta \sim 20\%$

# Summary & Conclusions

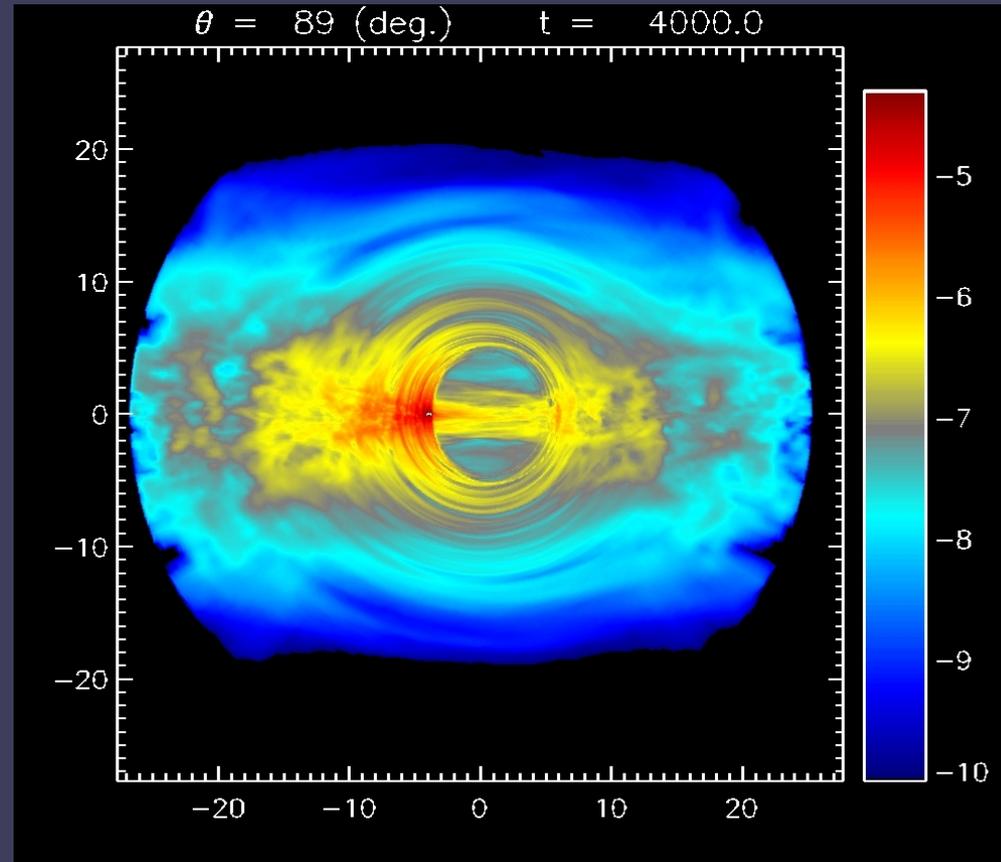
- We now have the tools to self-consistently measure  $dL/dr$  from GRMHD disks
  - 3D Conservative GRMHD simulations
  - GR Radiative Transfer
- Luminosity from within ISCO diminished by
  - Photon capture by the black hole
  - Gravitational redshift
  - $t_{\text{cool}} > t_{\text{inflow}}$
- Possibly greater difference for  $a_{\text{BH}} < 0.9$  when ISCO is further out of the potential well.

# Summary & Conclusions

- Comparison between cooled HARM3d and dVH runs:
  - HARM3d has less reconnection at horizon, more along the cutout boundary
  - HARM3d produces less power in the jet, reducing its relative efficiency to dVH
- dVH has enhanced stress w/o enhanced magnetic field strength
- Accretion rates surprisingly similar
- Sudden cooling can trap magnetic field and enhance accretion

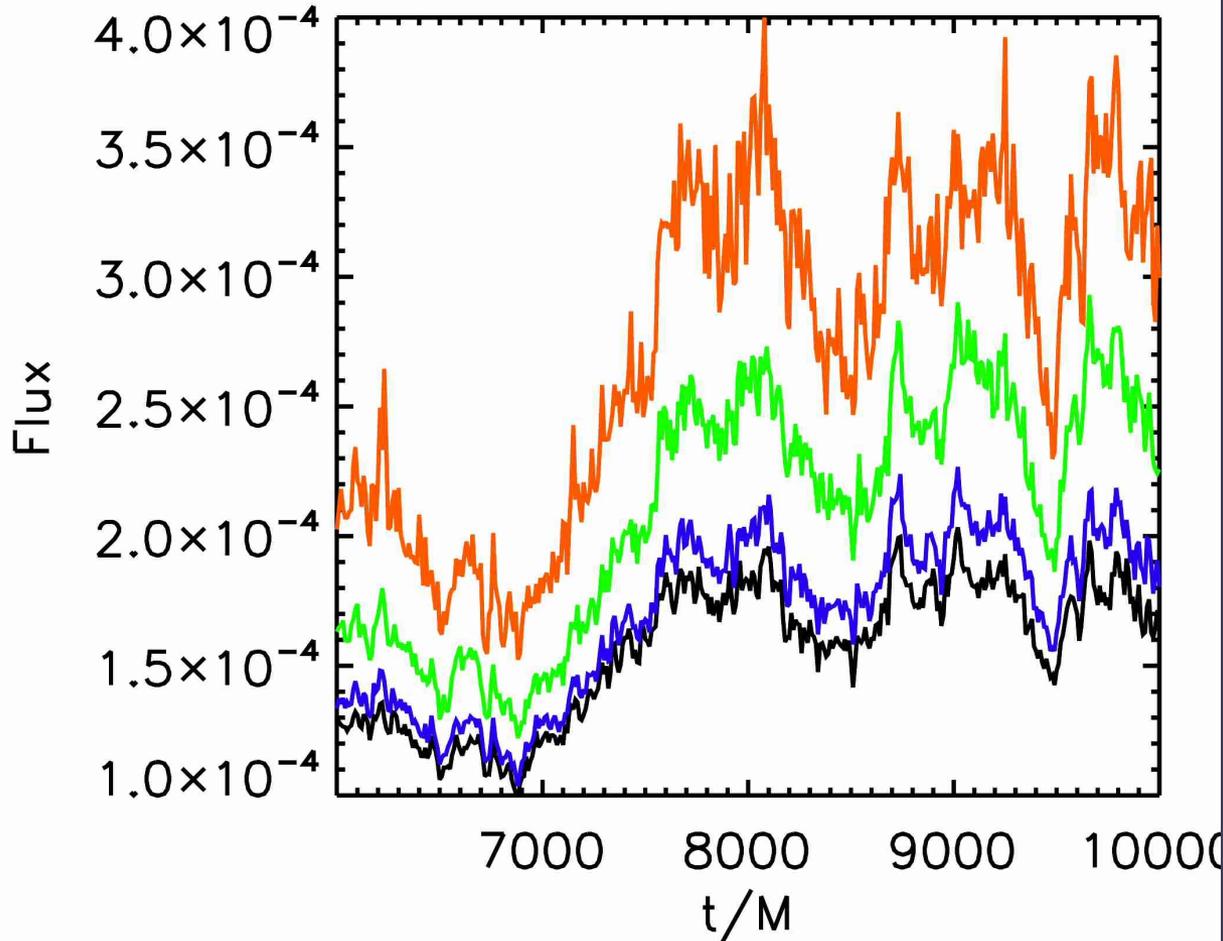
# Future Work

- Explore parameter space:
  - More spins
  - More H/R 's
  - More H(R) 's



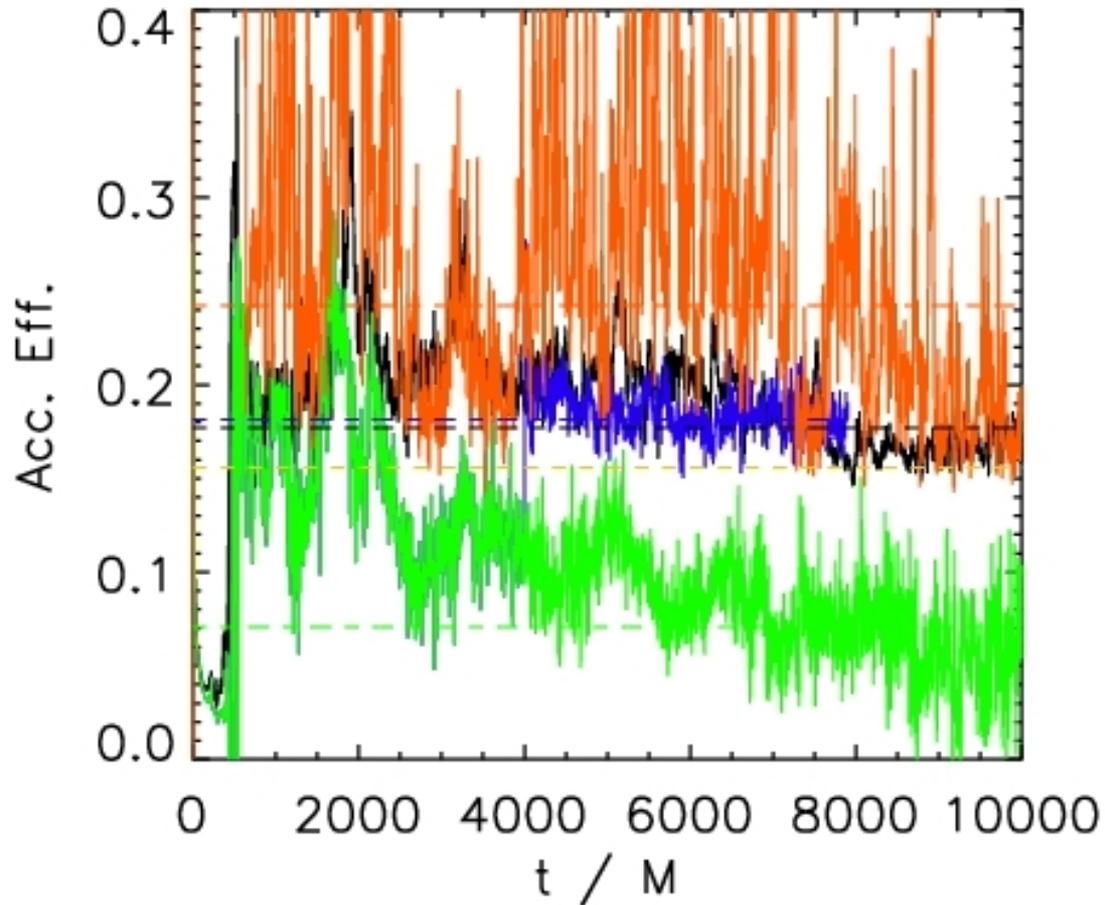
- Time variability analysis
  - Impossible with steady-state models

# Variability of Dissipated Flux



$\theta = 5 \text{ deg.}$   
 $\theta = 35 \text{ deg.}$   
 $\theta = 65 \text{ deg.}$   
 $\theta = 89 \text{ deg.}$

# HARM3D vs. dVH



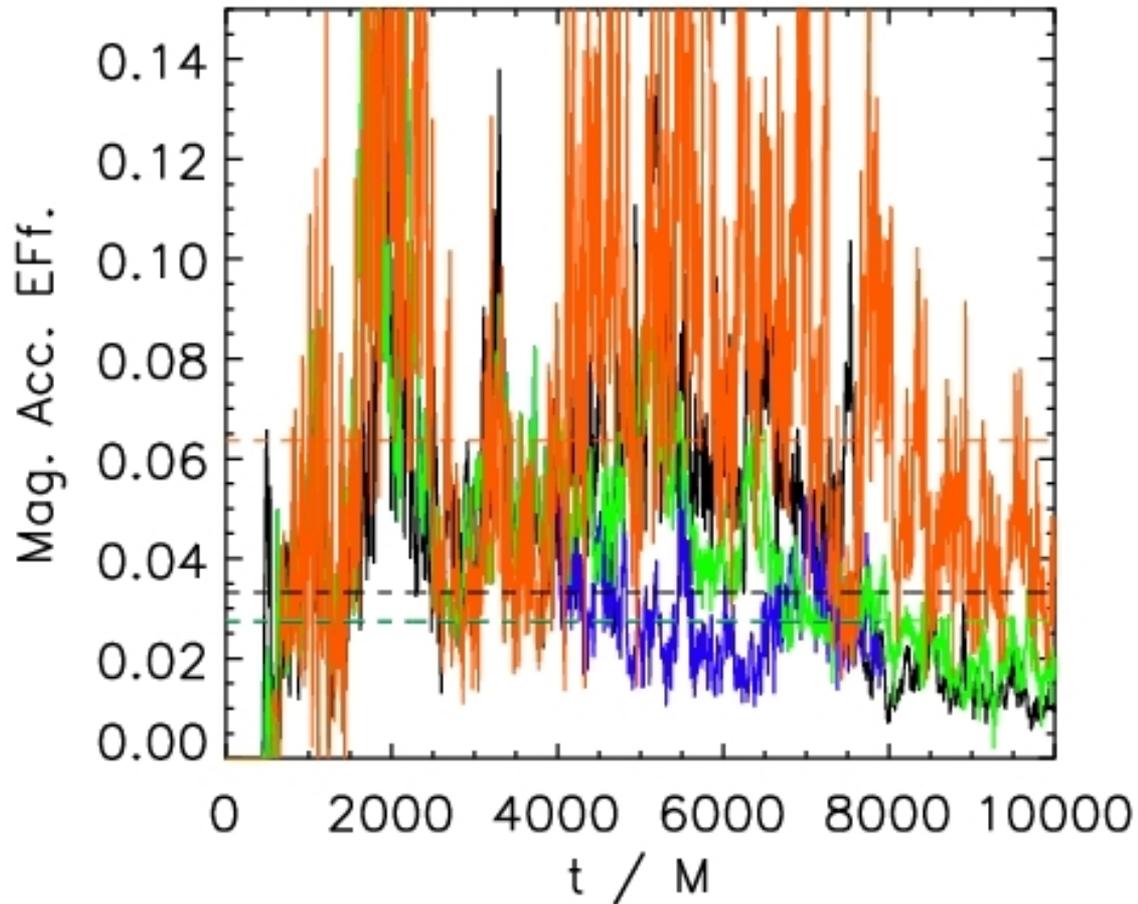
Cooled from  $t=0M$

Cooled from  $t=4000M$

Uncooled

dVH

# HARM3D vs. dVH



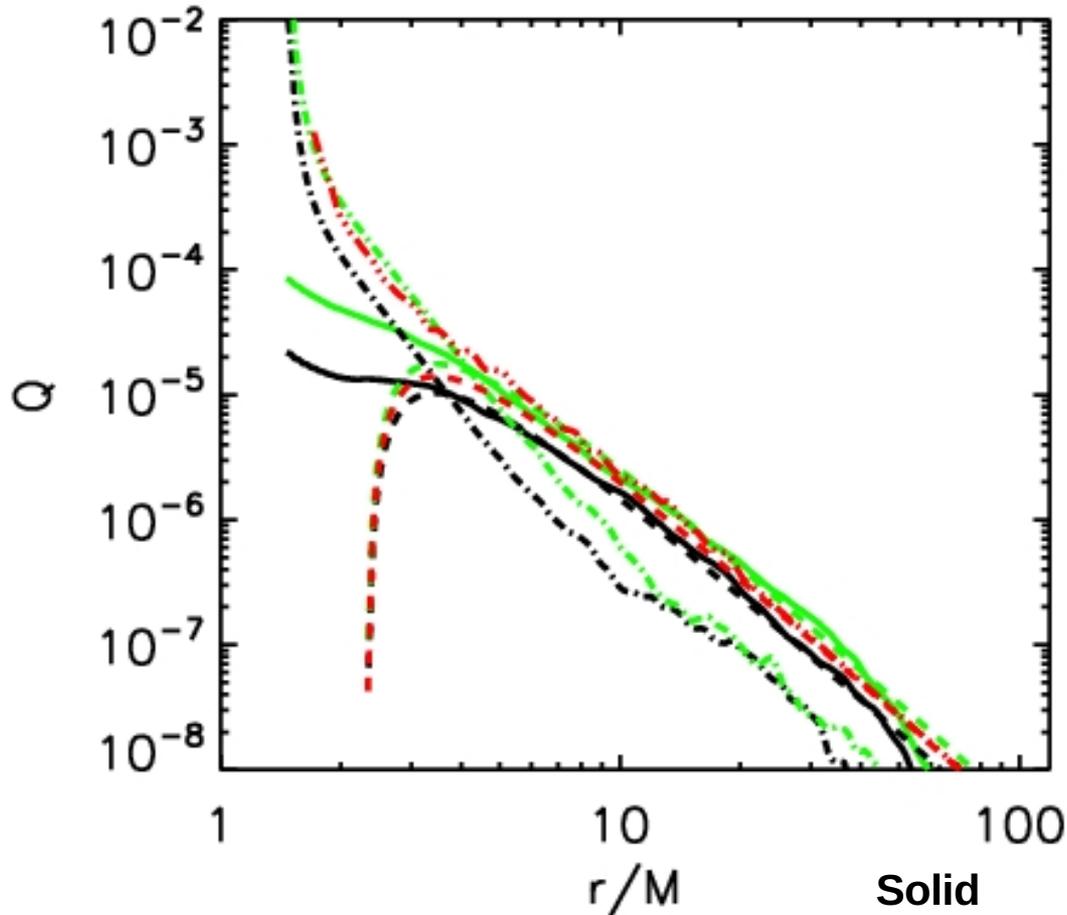
Cooled from  $t=0M$

Cooled from  $t=4000M$

Uncooled

dVH

# HARM3D vs. dVH



Cooled from  $t=0M$

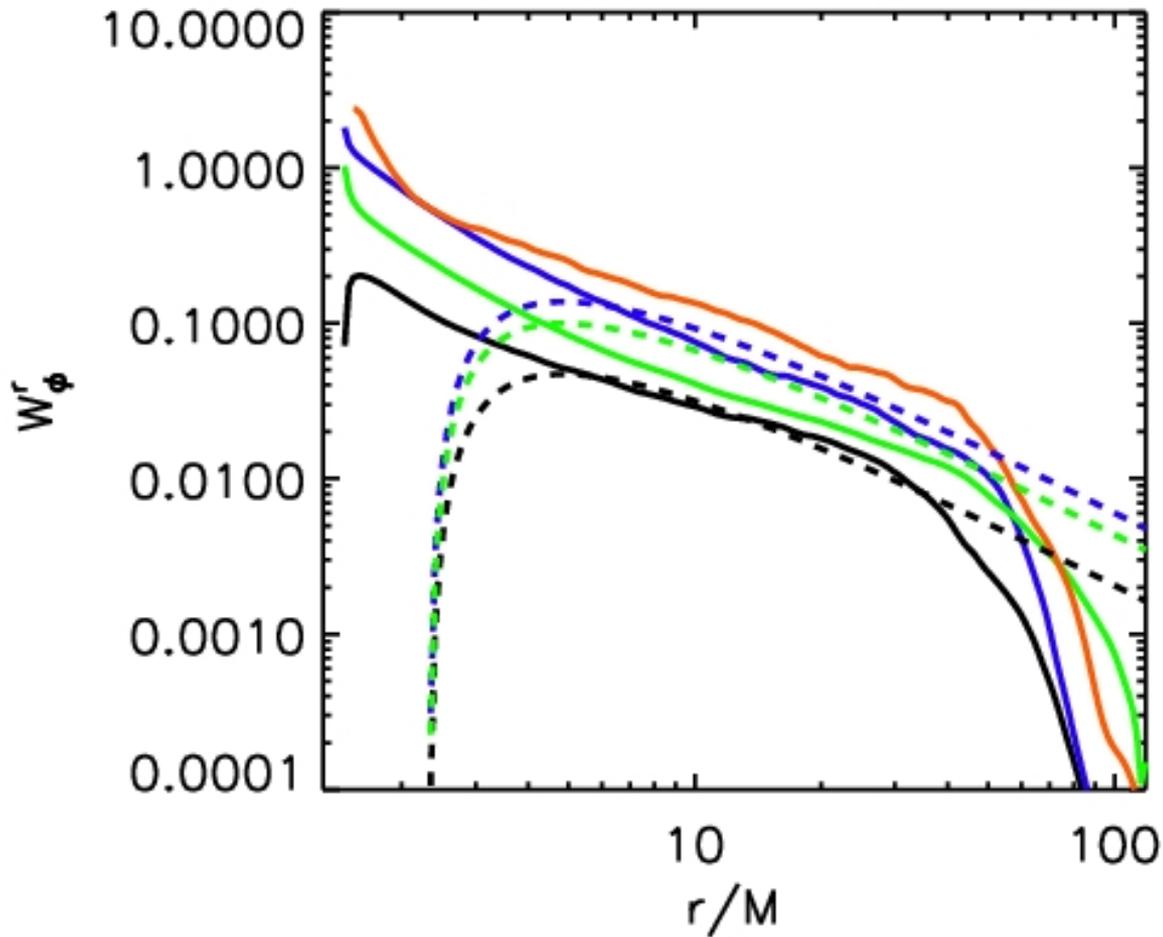
Cooled from  $t=4000M$

dVH

Solid : Local Dissipation  
Dashed : Novikov-Thorne  
Dot-Dashed : Beckwith et al. (2008)

# HARM3D vs. dVH

*Stress*



Cooled from  $t=0M$

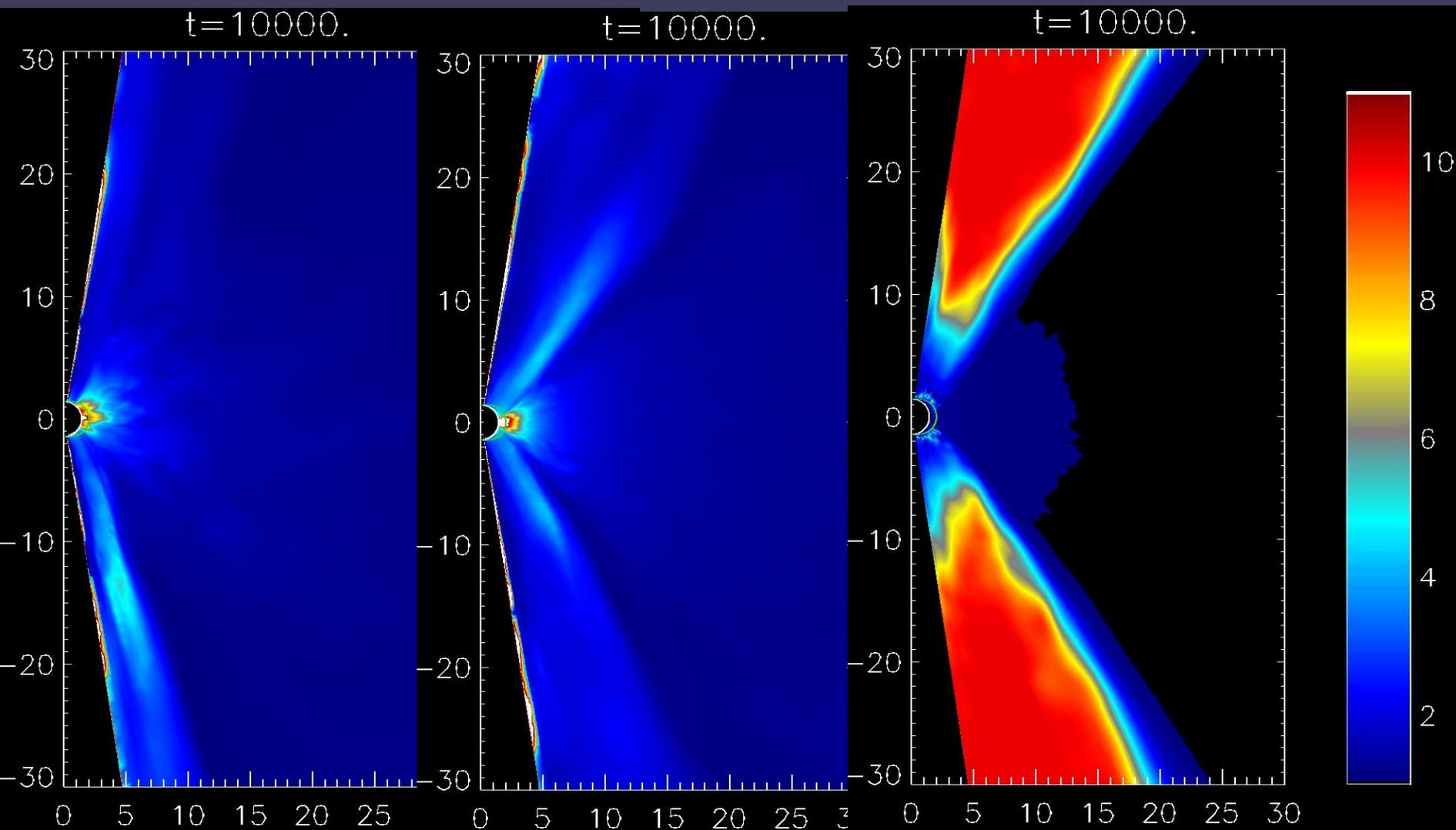
Cooled from  $t=4000M$

Uncooled

dVH

# HARM3D vs. dVH

$$\gamma(\phi - avg)$$



Uncooled

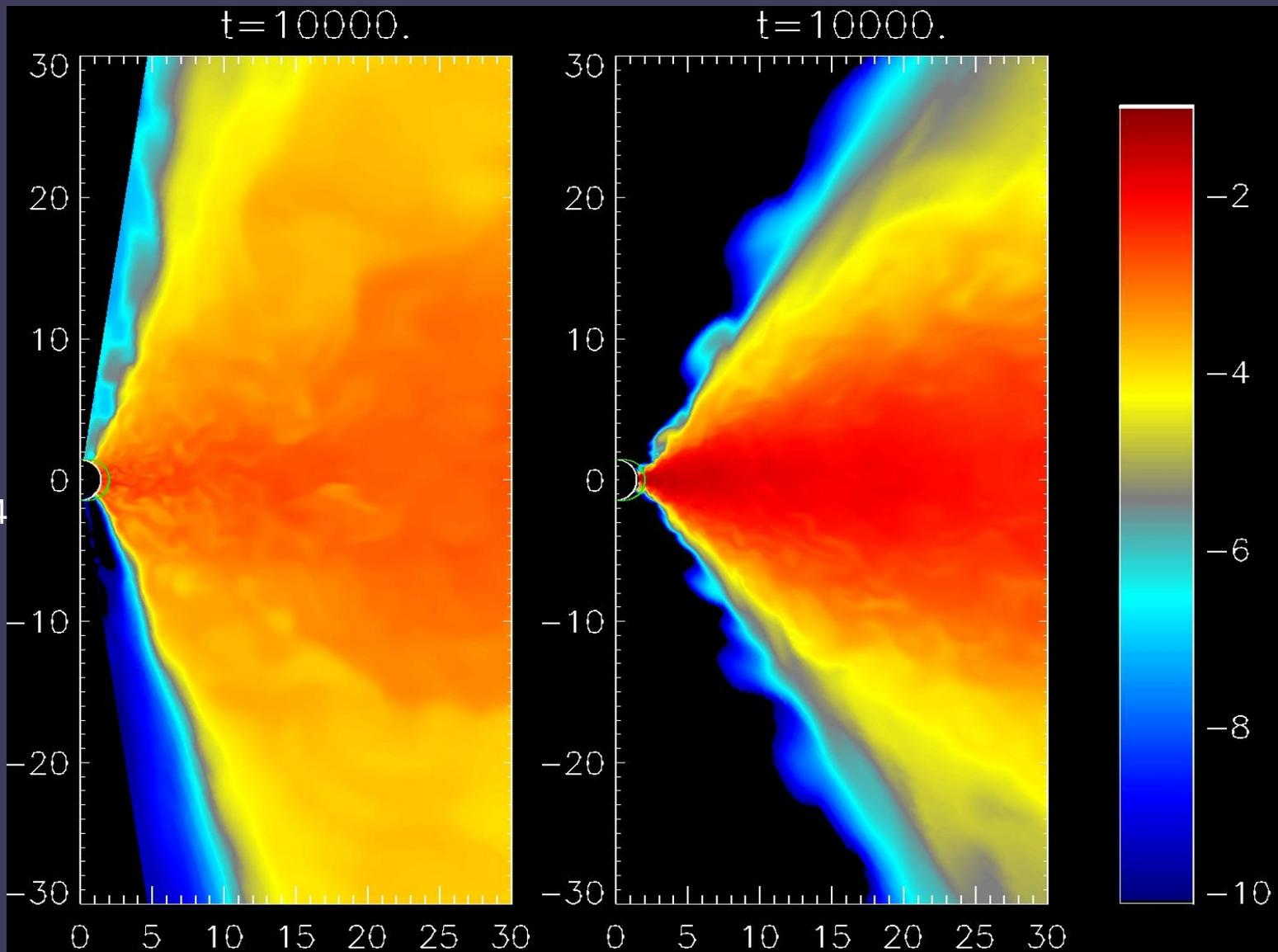
Cooled #2

dVH

# HARM3D vs. dVH

$\log(\rho)$

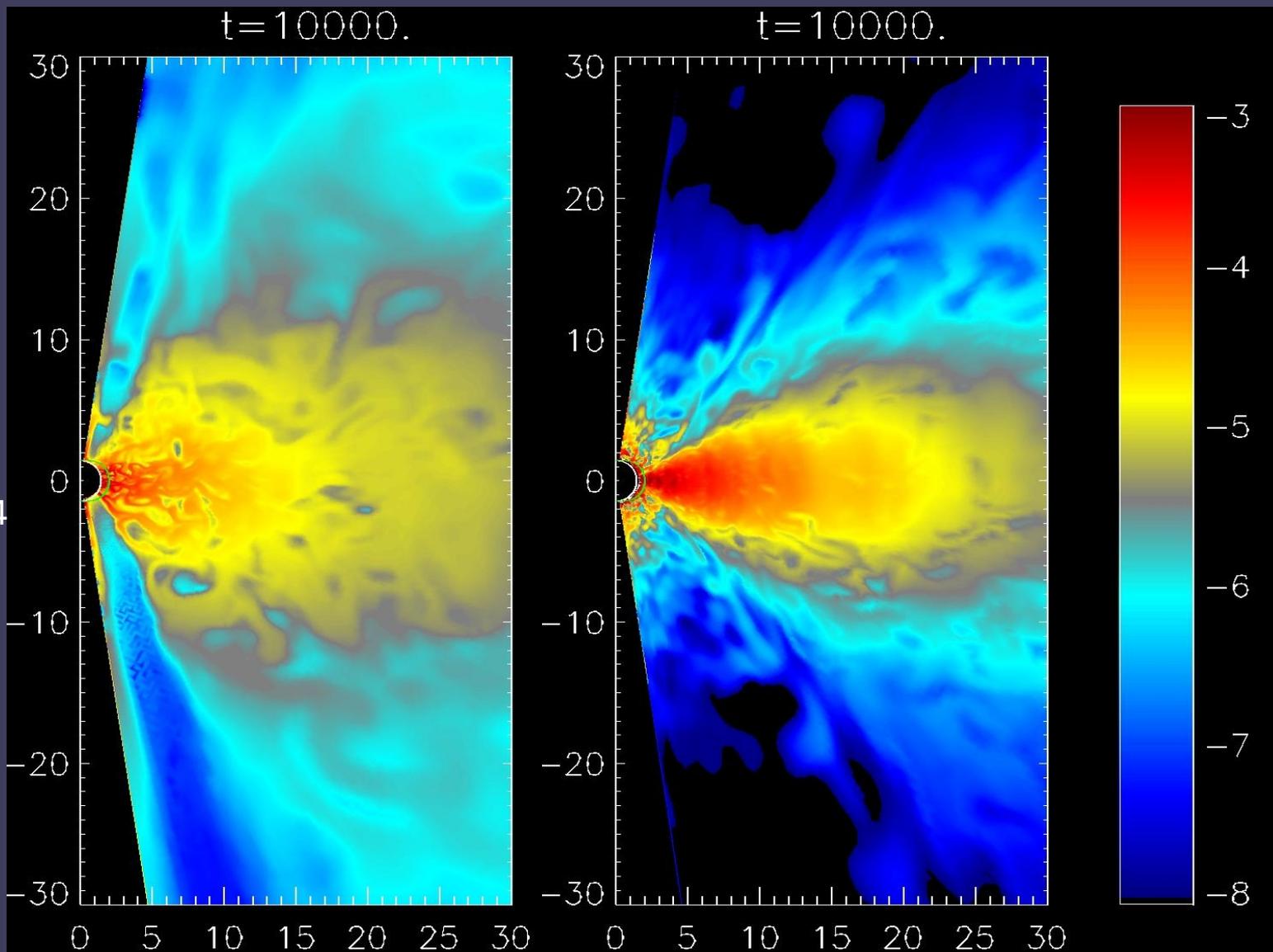
192x192x64  
a = 0.9 M



# HARM3D vs. dVH

$\log(P)$

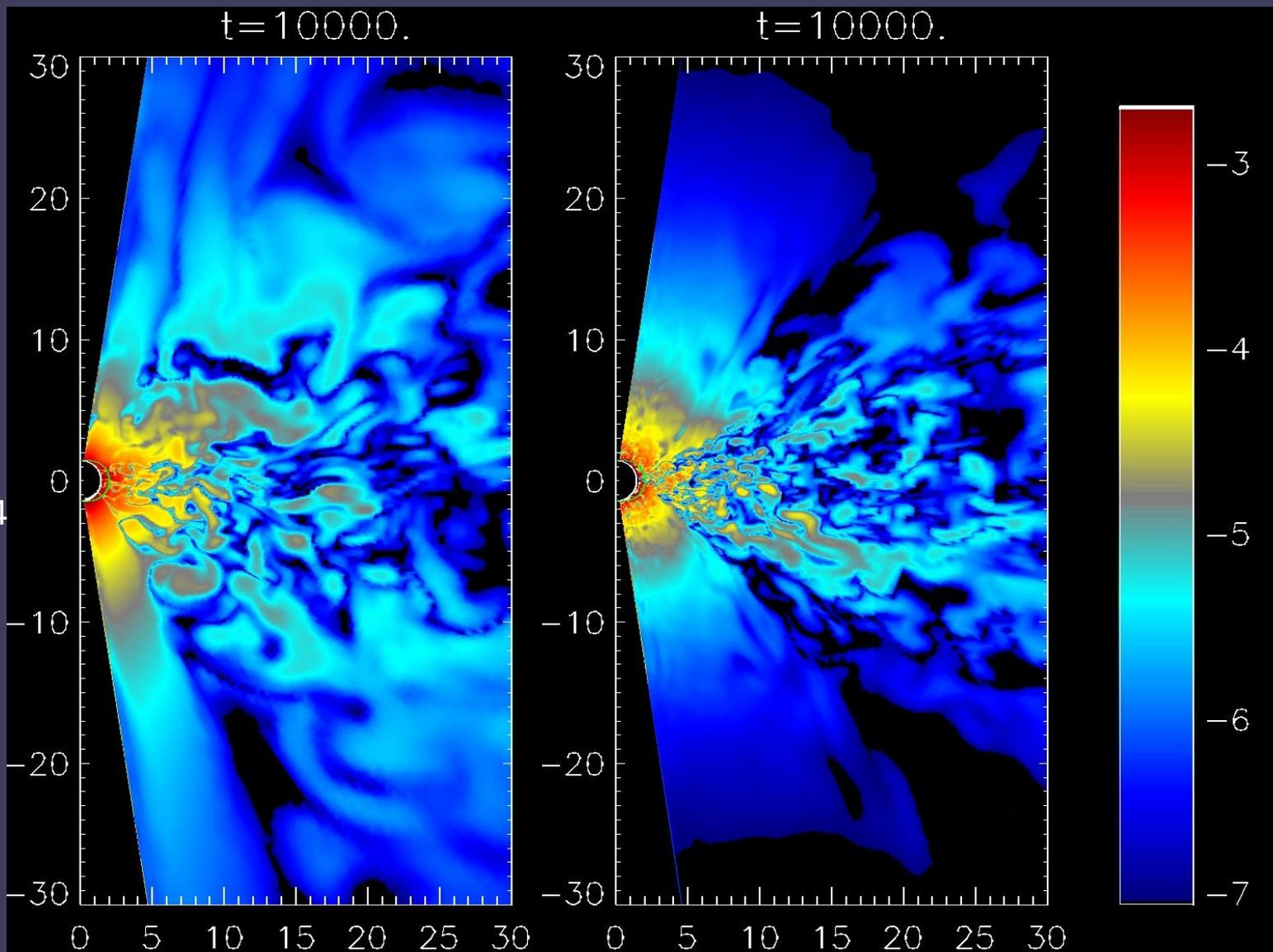
192x192x64  
a = 0.9 M



# HARM3D vs. dVH

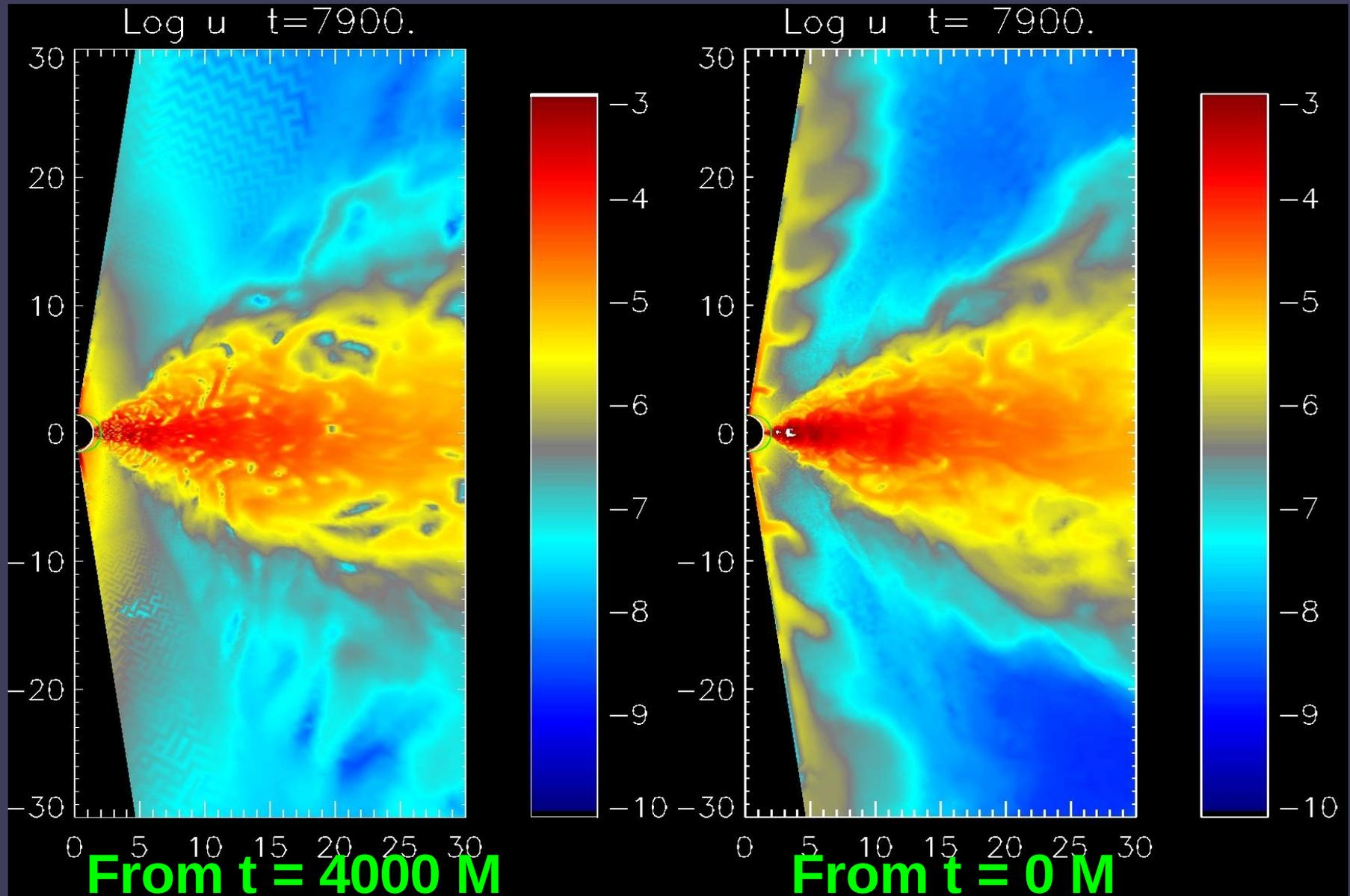
$\log(P_{mag})$

192x192x64  
a = 0.9 M



# Cooled #1 vs. Cooled #2

$\log(P)$

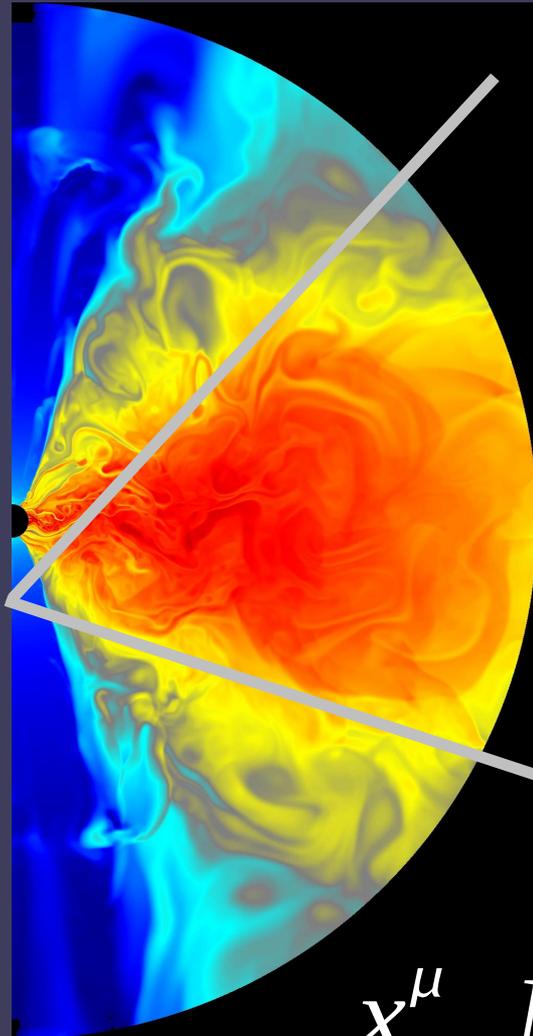


# Radiation Transfer in GR: Step #1

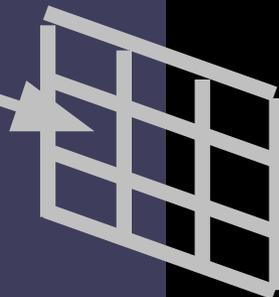
- Post-processing calculation
- Assume geodesic motion (no scattering):
- Rays start from Camera;
- Aimed at Camera, integrated to source
- Integrated back in time;
- A geodesic per image pixel ;
- Camera can be aimed anywhere at any angle;

$$\frac{\partial x^\mu}{\partial \lambda} = N^\mu$$

$$\frac{\partial N_\mu}{\partial \lambda} = \Gamma^\nu_{\mu\eta} N_\nu N^\eta$$

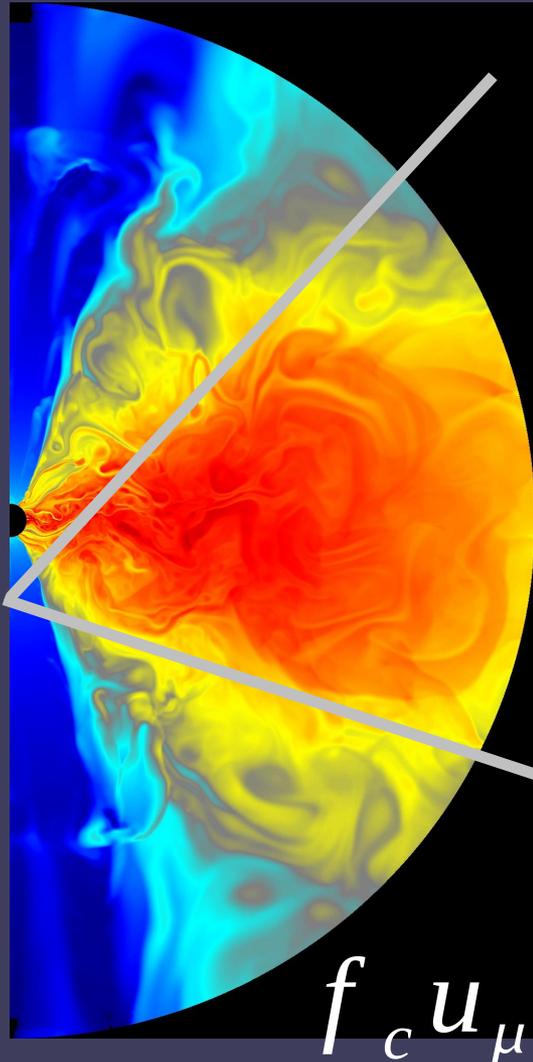


$x^\mu, N^\mu$

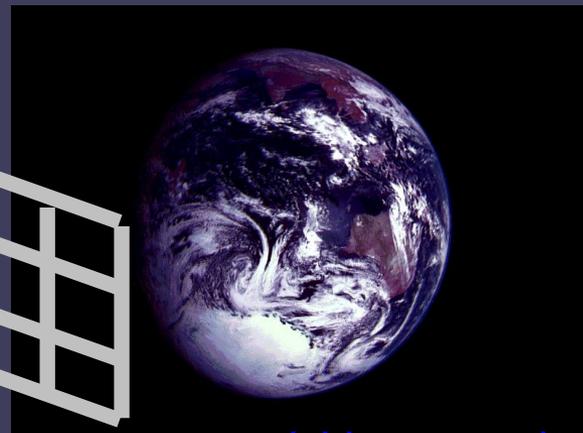


(objects not shown to scale)

# Radiation Transfer in GR: Step #2

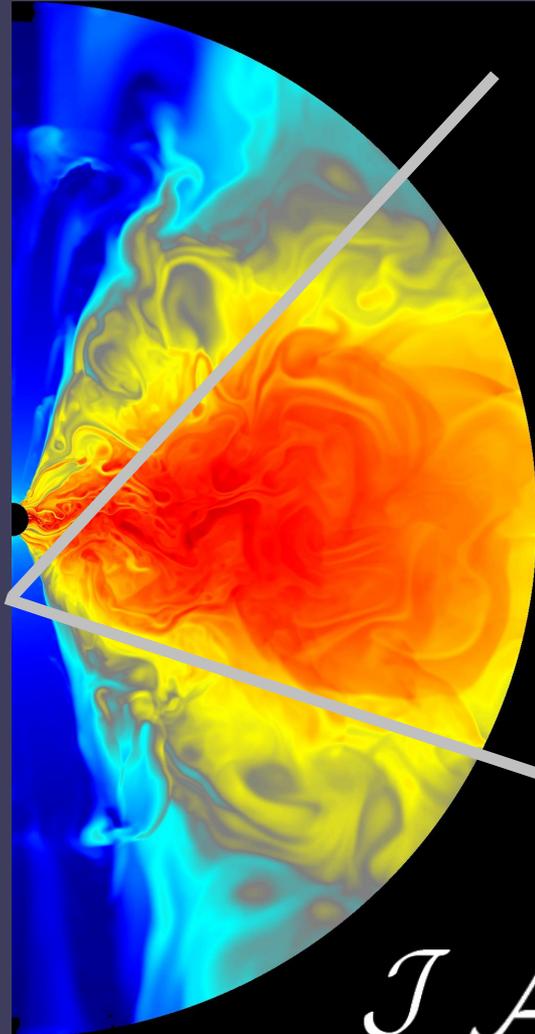


- Interpolate simulation data along rays
- Spatially interpolate single timeslice per image
  - Assume  $t_{\text{dyn}} \gg t_{\text{crossing}}$



(objects not shown to scale)

# Radiation Transfer in GR: Step #3



- Calculate frame-independent quantities:

$$\mathcal{J} = \frac{j_\nu}{\nu^2}$$

$$\mathcal{A} = \nu \alpha_\nu$$

$$\mathcal{I} = I_\nu / \nu^3$$

- Integrate frame-independent RT equation along geodesics:

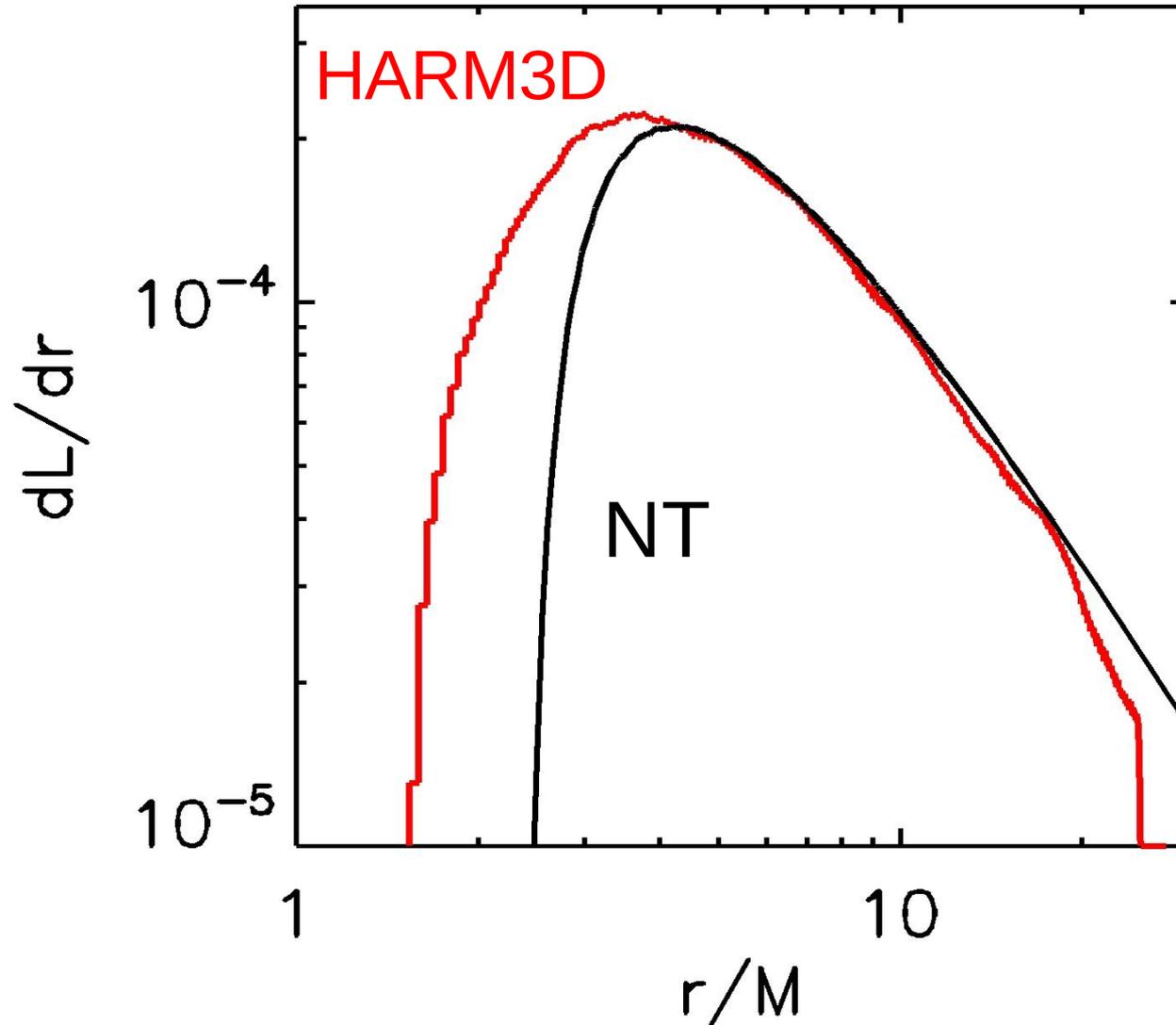
$$\frac{d\mathcal{I}}{d\lambda} = \mathcal{J} - \mathcal{A}\mathcal{I}$$

$\mathcal{J}$   $\mathcal{A}$   $\mathcal{I}$

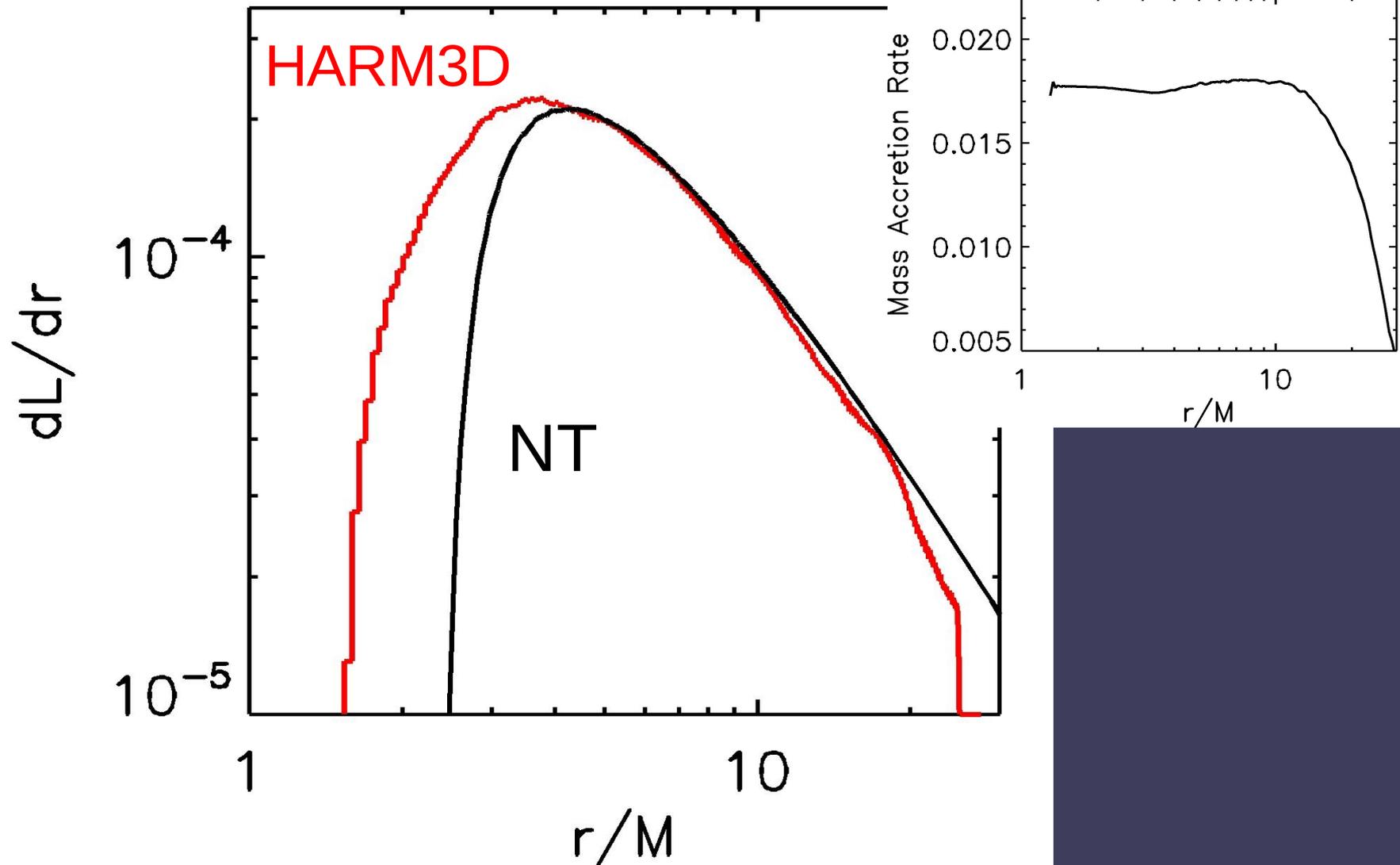


(objects not shown to scale)

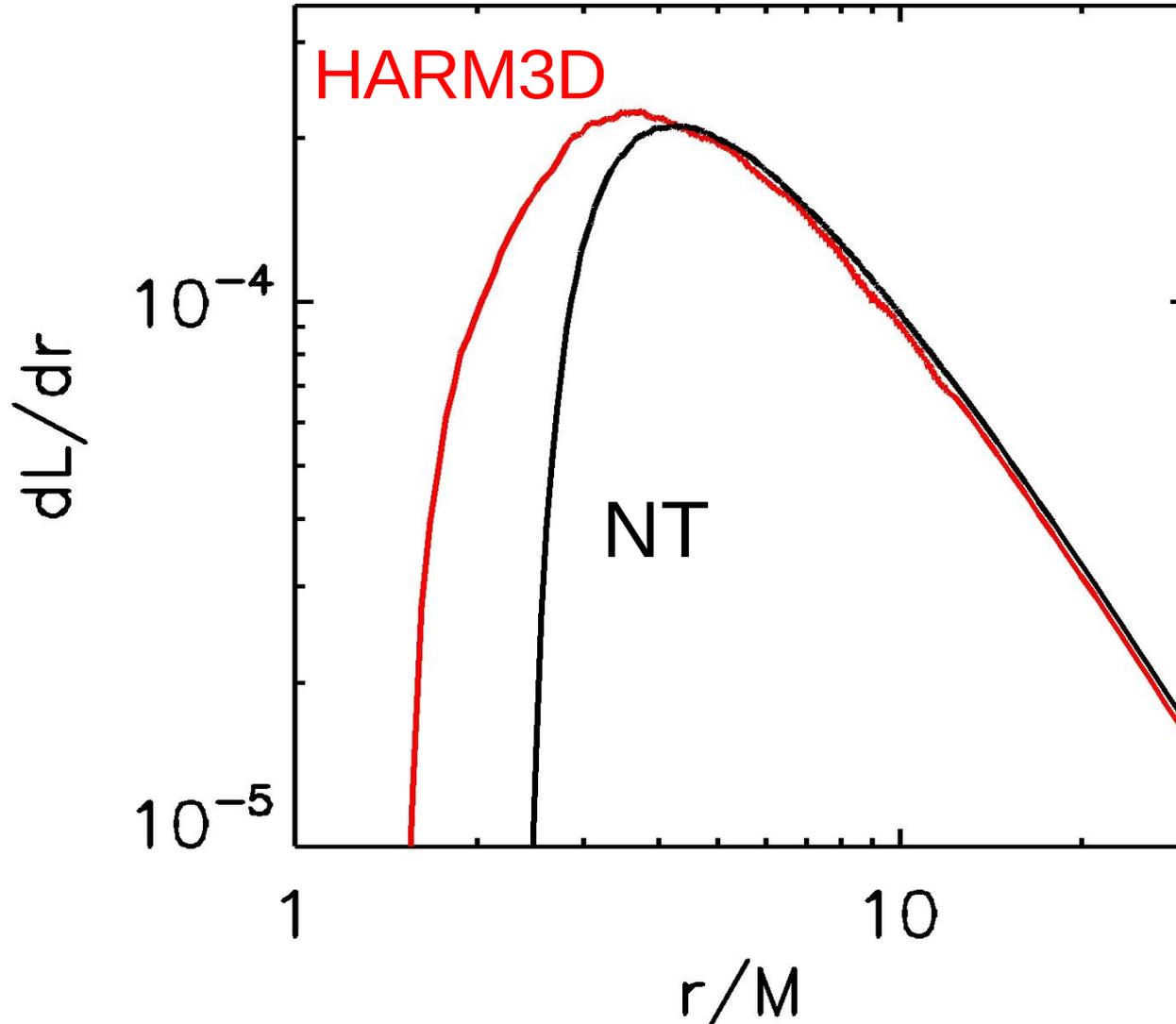
# Observer Frame Luminosity: Angle/Time Average



# Observer Frame Luminosity: Angle/Time Average



# Observer Frame Luminosity: Angle/Time Average



Assume NT profile  
for  $r > 12M$ .

$$\Delta L = 4\% L$$