Critical Phenomena and Driven Neutron Star Collapse

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with

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Outline

- Introduction to Critical Phenomena
- Example: Primordial Black Holes and Crit. Phen.
- Theoretical Model of Non-equilibrium Neutron Stars
- Parameter Space Survey and Dynamical Scenarios
- Type I & II Critical Behavior
- Conclusions

A Decade of Critical Phenomena

- M. W. Choptuik “Universality and Scaling in Gravitational Collapse of a Massless Scalar Field”, PRL **70**, 1, January 4, 1993.
- Crit. Phen. observed anywhere you have (BH)/(No BH);
- General feature of gravitational collapse, observed in many different matter models (even w/o matter: Gravitational Waves!)
- “Tuning” of initial data to Critical Solution
  $\longrightarrow$ eliminate the 1 unstable mode from solution;
- Some Crit. Solutions are “Naked Singularities”!
- Reviews: C. Gundlach, Physics Reports, **376**, 339 (2003),
  C. Gundlach, *Living Reviews*, lrr-1999-4

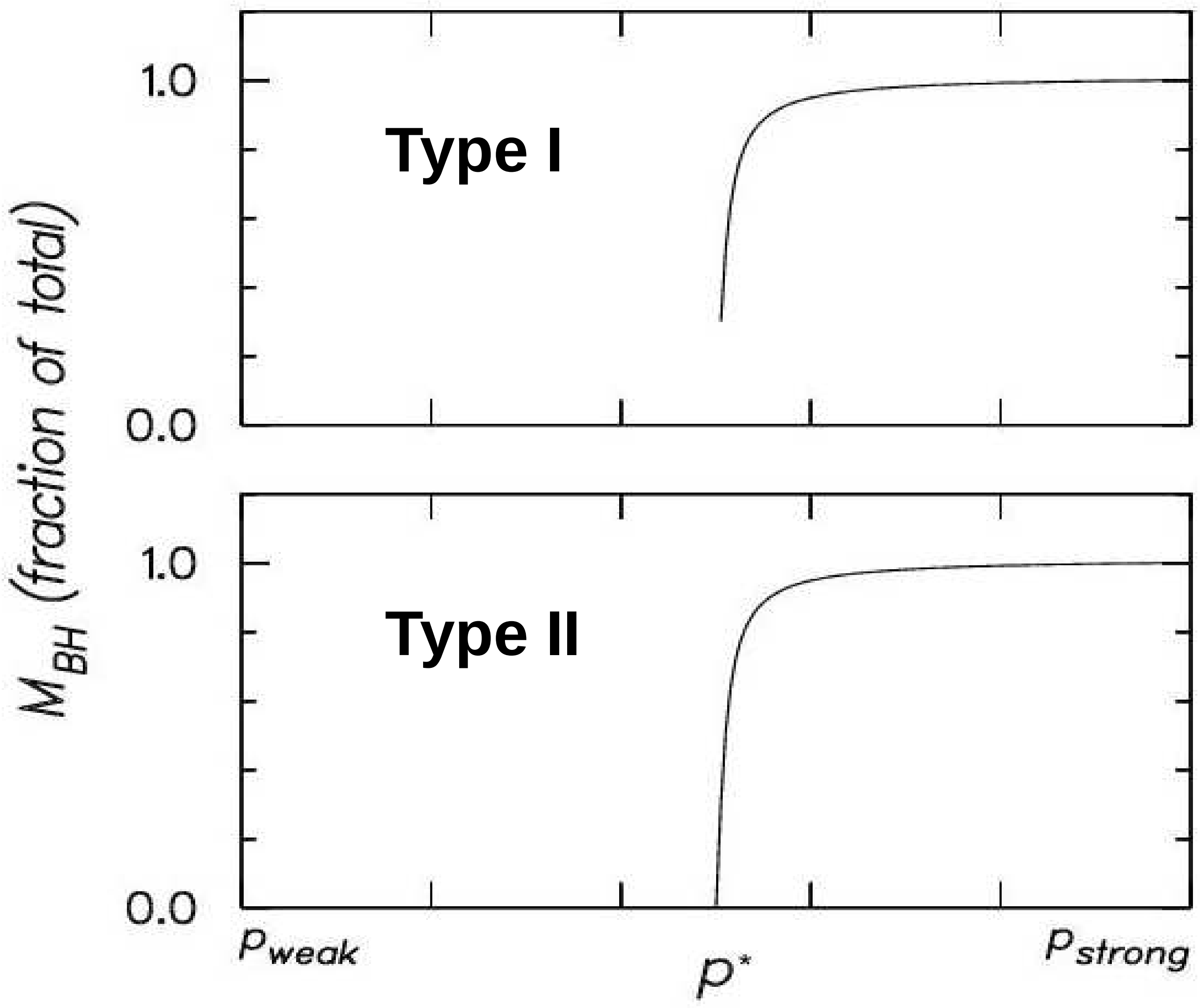
Systems Exhibiting Critical Phenomena

Matter	Type	Collapse simulations	Critical solution	Perturbations of crit. soln.
Perfect fluid $p = k\rho$	II	[69, 142]	CSS [69, 138, 142]	[138, 128, 93, 97]
Real scalar field:				
– massless, min. coupled	II	[47, 48, 49]	DSS [89]	[90, 139]
– massive	I	[32]	oscillating [165]	
	II	[49]	DSS [104, 99]	[104, 99]
– conformally coupled	II	[48]	DSS	
– 4+1	II	[16]		
– 5+1	II	[77]		
Massive complex scalar field	I (II)	[110]	[165]	[110]
Massless scalar electrodynamics	II	[117]	DSS [99]	[99]
2-d sigma model				
– complex scalar ($\kappa = 0$)	II	[50]	DSS [90]	[90]
– axion-dilaton ($\kappa = 1$)	II	[101]	CSS [67, 101]	[101]
– scalar-Brans-Dicke ($\kappa > 0$)	II	[136, 133]	CSS, DSS	
– general κ including $\kappa < 0$	II		CSS, DSS [115]	[115]
$SU(2)$ Yang-Mills	I	[53]	static [12]	[131]
	II	[53]	DSS [92]	[92]
	"III"	[55]	colored BH [17, 173]	[168, 172]
$SU(2)$ Skyrme model	I	[19]	static [19]	[19]
	II	[22]	static [22]	
$SO(3)$ Mexican hat	II	[134]	DSS	
Vlasov	I?	[160, 148]	[141]	

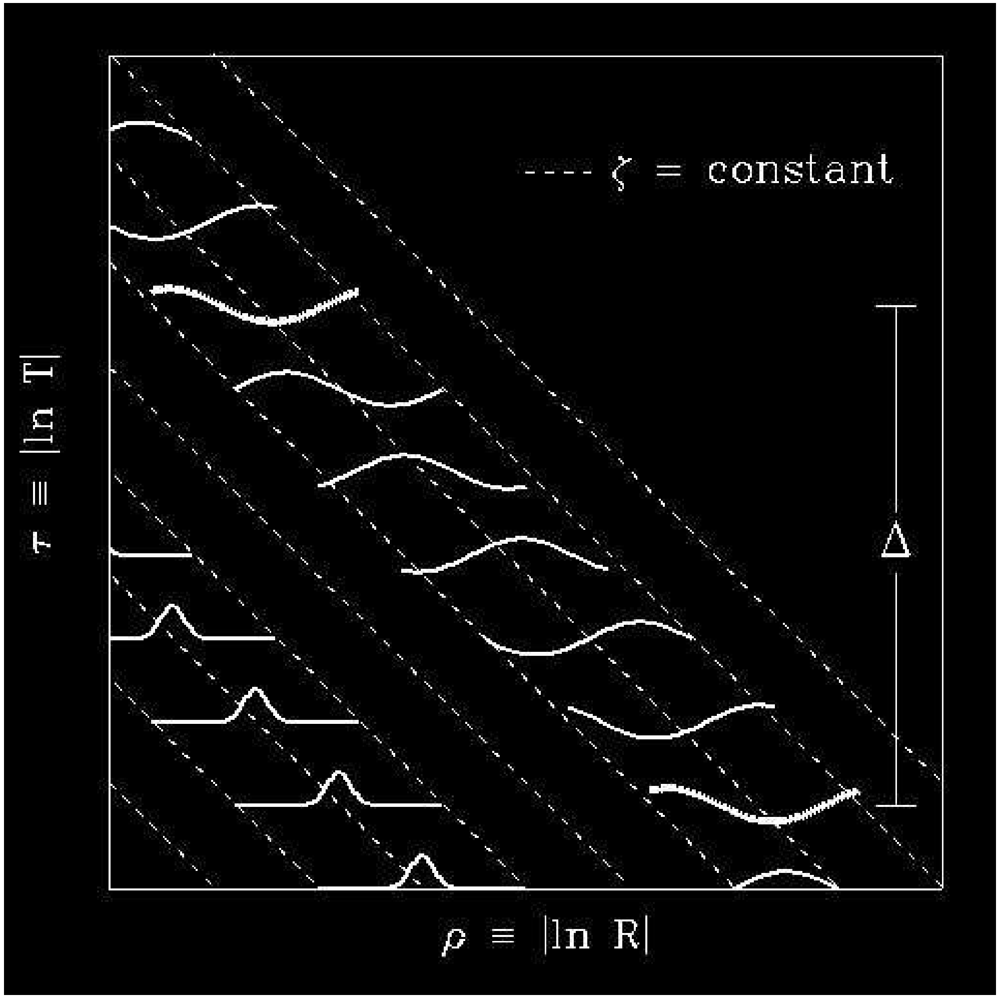
Types of Critical Phenomena

Type I	Type II
Discontinuous "Phase" Transition $M_{BH} \to M^* > 0$	Continuous "Phase" Transition $M_{BH} \to 0$
Static or Oscillatory	Cont. or Discretely Self-similar
$t_{\rm hang} \propto \left\|p - p^*\right\|^{-\gamma}$	$M_{\rm BH} \propto \left\|p - p^*\right\|^{\gamma}, T_{\rm max} \propto \left\|p - p^*\right\|^{-2\gamma}$

Black Hole Mass Scaling



Cont. and Discrete Self-Similarity of Type-II



$\mathcal{X} = \ln(R/T)$

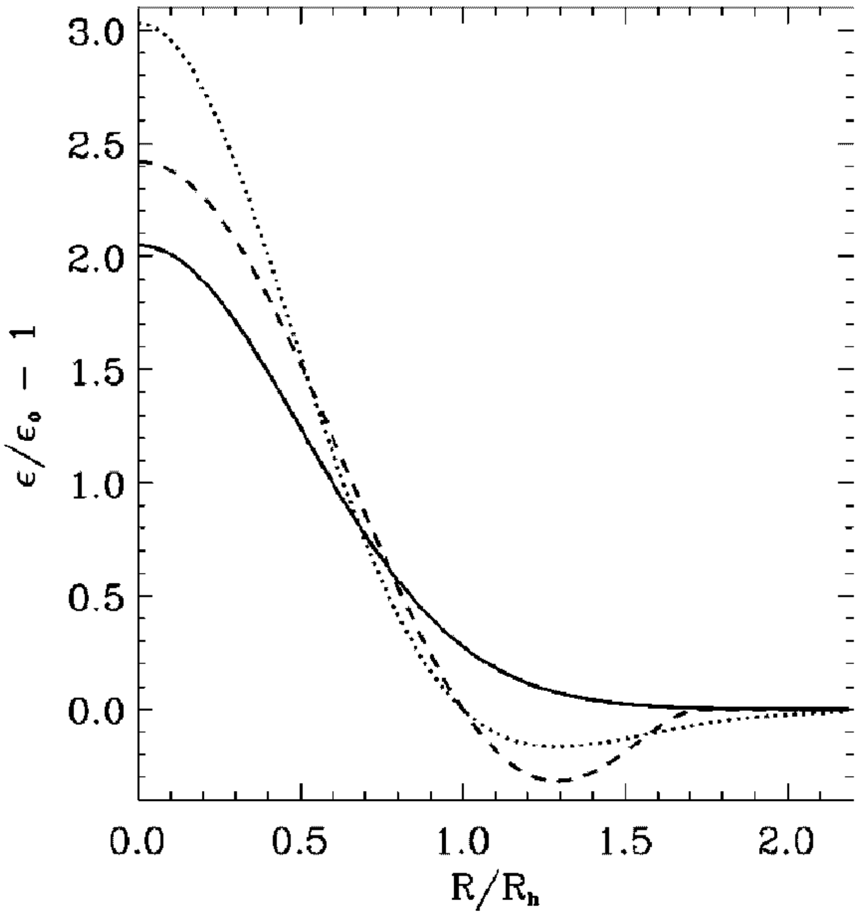
- CSS:
  - $Z^\star(\mathcal{X}, \tau) = Z^\star(\mathcal{X})$
- DSS:
  - $Z^\star(\mathcal{X}, \tau) = Z^\star(\mathcal{X}, \tau + \Delta)$

$l \propto |p - p^\star|^\gamma \quad , \quad \gamma = 1/\omega$

$\Rightarrow M_{\mathbf{BH}} \propto R_{\mathbf{BH}} \propto l$

$\Rightarrow \mathcal{R} \propto l^{-2}$

Ex.: Primordial Black Holes (PBH)



Niemeyer & Jedamzik, PRD, **59**, (1999)

- Inhomogeneities in the early universe
  - $(M, R) \sim (M, R)_{\mathrm{horizon}}$, $P = \rho/3$
  - $\delta = \Delta M/M_{\mathrm{h}}$
  - $\delta < \delta_c$ Dispersal
  - $\delta > \delta_c$ Collapse to PBH

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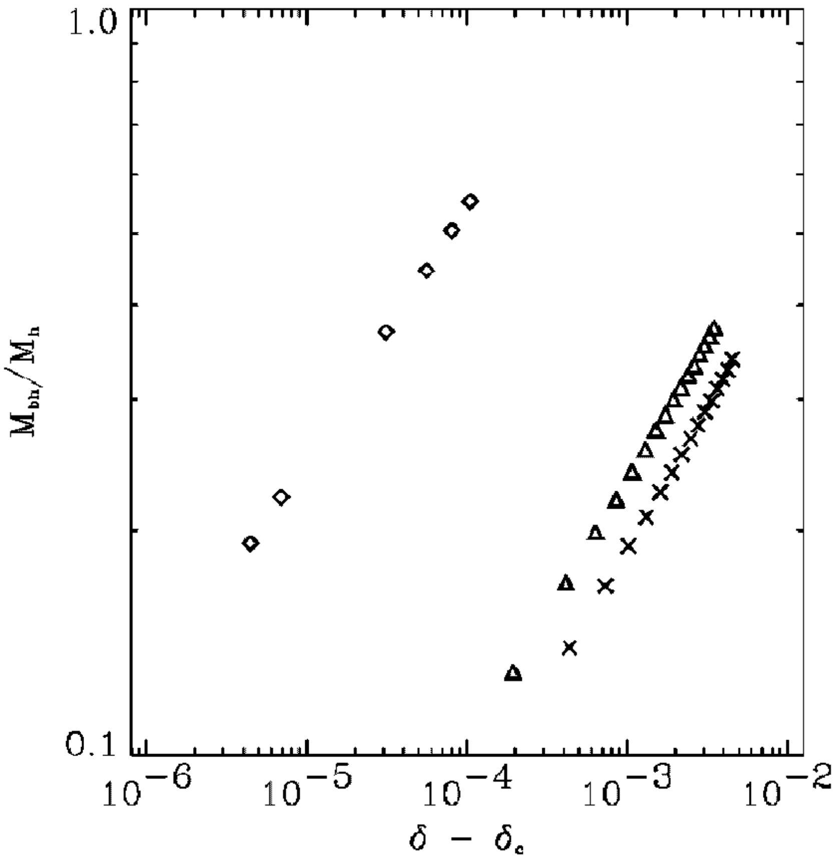

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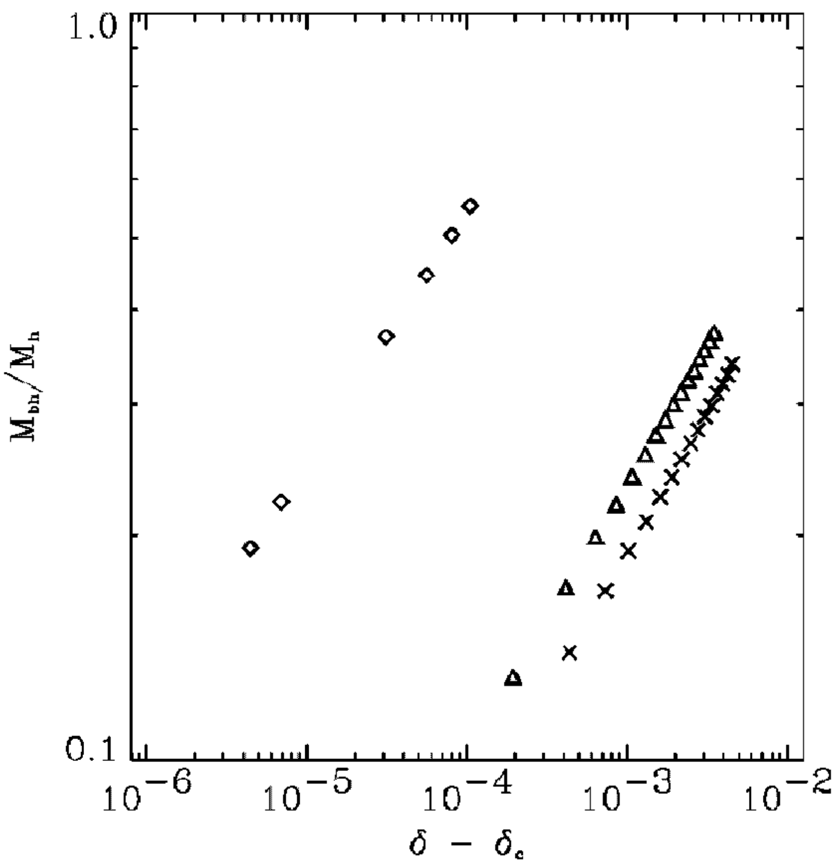

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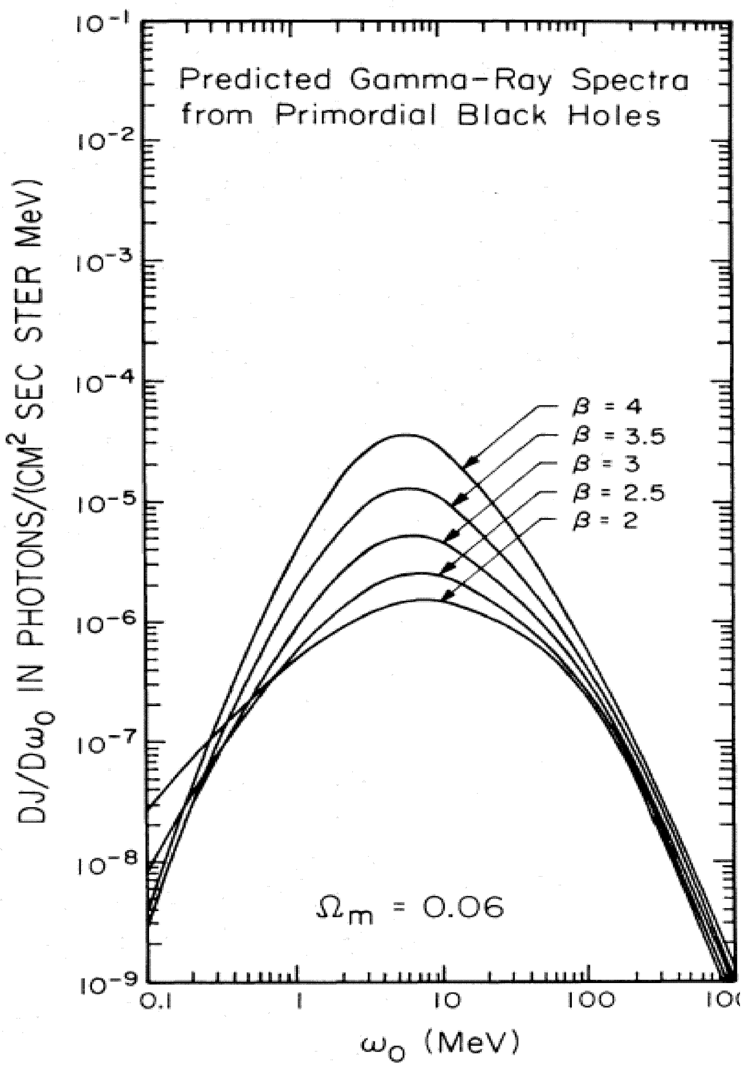
- $M_{\text{PBH}} \propto (\delta - \delta_c)^\gamma$
- $P(\delta) \propto \exp\left(-\delta^2/\sigma^2\right)$

  $\Rightarrow \delta_{\text{PBH}} \simeq \delta_c$

  $\Rightarrow M_{\text{PBH}} \rightarrow$ 0 at any epoch

Diffuse Hawking Radiation



- $T \simeq 6 \times 10^{-8} \left(\frac{M_\odot}{M}\right) K$, $\frac{dM}{dt} = -\frac{\alpha(M)}{M^2}$
- $M_{PBH}^{\rm evap} < 5 \times 10^{14}{\rm g}$ if formed at $t \approx 0$
- Page, PRD **13**, 198 (1976)
  - $M_{\rm PBH} \simeq 5 \times 10^{14}{\rm g}$, $T \simeq 20{\rm MeV}$
  - $L \simeq 2.5 \times 10^{17}{\rm erg/s}$
  - ν's (45%), $e^{\pm}$'s (45%), γ's (9%), gravitons (1%)
  - $\frac{dn}{d(M_i/M_\star)} = 10^4{\rm pc}^{-3}\,(M_i/M_\star)^{-\beta}$
- Page & Hawking, ApJ, **206**, 1 (1976) :
  - $\Omega_{\rm PBH} < 10^{-7}$
- γ-ray excess in Galactic Halo
  (E. L. Wright, ApJ, **459**, 487 (1996))

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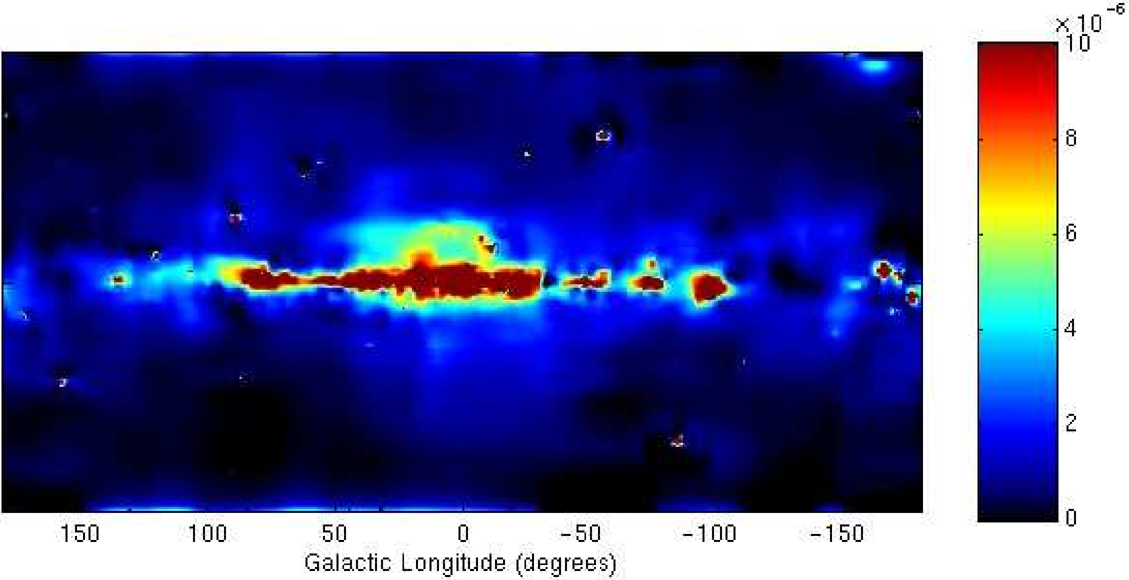

EGRET, Compton γ-ray Obs.,
http://tigre.ucr.edu/halo/halo.html

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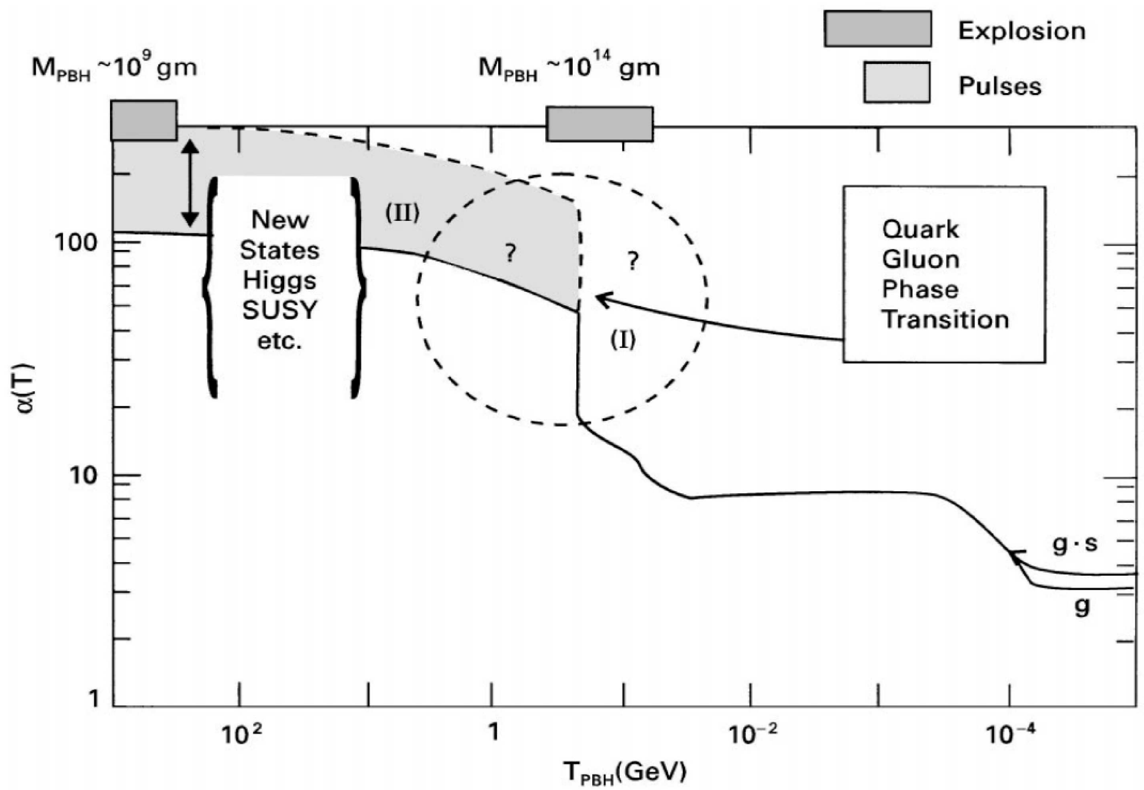
Sudden Hawking Evaporation



At $M \sim 10^{10} - 10^{13}$g, Hawking Radiation $\rightarrow$ Fireball:

(D. B. Cline, Physics Reports, **307**, 173 (1998) ; D. B. Cline, et al., Astropart. Phys., **18**, 531 (2003))

- $E \sim 5 \times 10^{34}$erg , $T \sim T_{\rm QGP} \sim 160$MeV , $t_{\rm rise} \sim 1$ms , $t \sim 100$ms , $R \sim 10^9$cm
- Anisotropy of Short & Hard GRB $\Rightarrow$ Galactic Origin!

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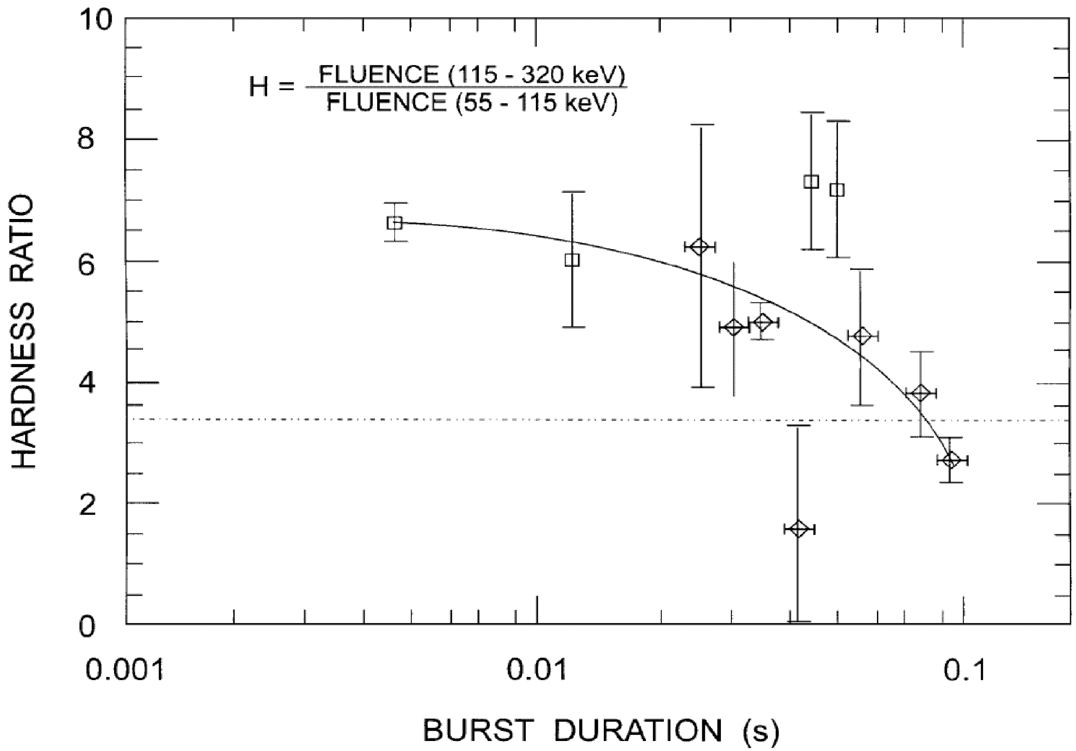

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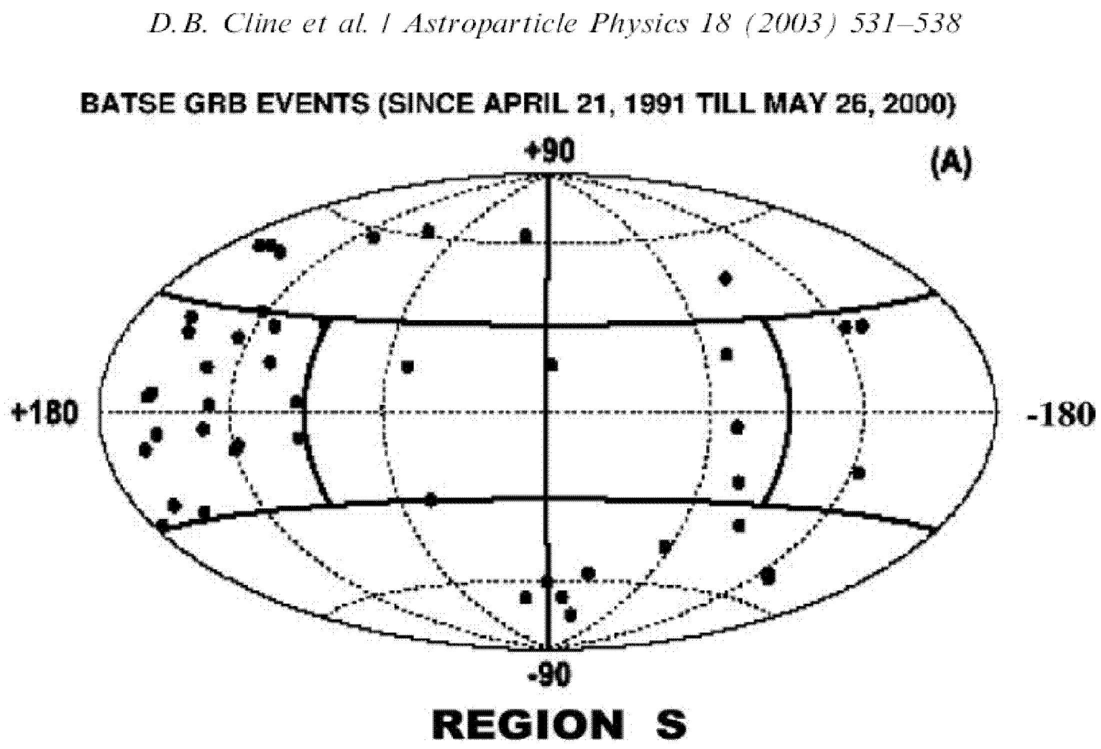

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Fluid Equations of Motion

Local Conservation of Baryons Equation : $\nabla_\mu J^\mu = 0$

Local Conservation of Energy Equation : $\nabla_\mu T^\mu{}_\nu = 0$

$$\frac{\partial}{\partial t}\underbrace{\begin{bmatrix} D \\ S \\ \tau \end{bmatrix}}_{\mathbf{q}} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\alpha}{a} \underbrace{\begin{bmatrix} Dv \\ Sv + P \\ v\,(\tau + P) \end{bmatrix}}_{\mathbf{f}} \right) = \underbrace{\begin{bmatrix} 0 \\ \Sigma \\ 0 \end{bmatrix}}_{\boldsymbol{\psi}}$$

$$v = \frac{au^r}{\alpha u^t}\,, \quad W^2 = \frac{1}{1-v^2}\,, \quad D = a\rho_\circ W\,, \quad S = (\rho + P)\,W^2 v\,, \quad \tau = S/v - D - P$$

- $\Sigma = \Sigma(\alpha, a, \mathbf{q}) \neq \Sigma(\alpha, a, \mathbf{q}, \partial_r \mathbf{q}, \partial_t \mathbf{q}) \Rightarrow$ EOM are hyperbolic!
- Relativistic Ideal gas Equation of State : $P = (\Gamma - 1)\,\rho_\circ \epsilon \quad , \quad \Gamma =$ constant

Metric Equations

Slicing Condition :

$$\frac{\alpha'}{\alpha} = a^2 \left[4\pi r \left(Sv + P \right) + \frac{1}{2r} \left(1 - 1/a^2 \right) \right]$$

Hamiltonian Constraint :

$$\frac{a'}{a} = a^2 \left[4\pi r \left(\tau + D \right) - \frac{1}{2r} \left(1 - 1/a^2 \right) \right]$$

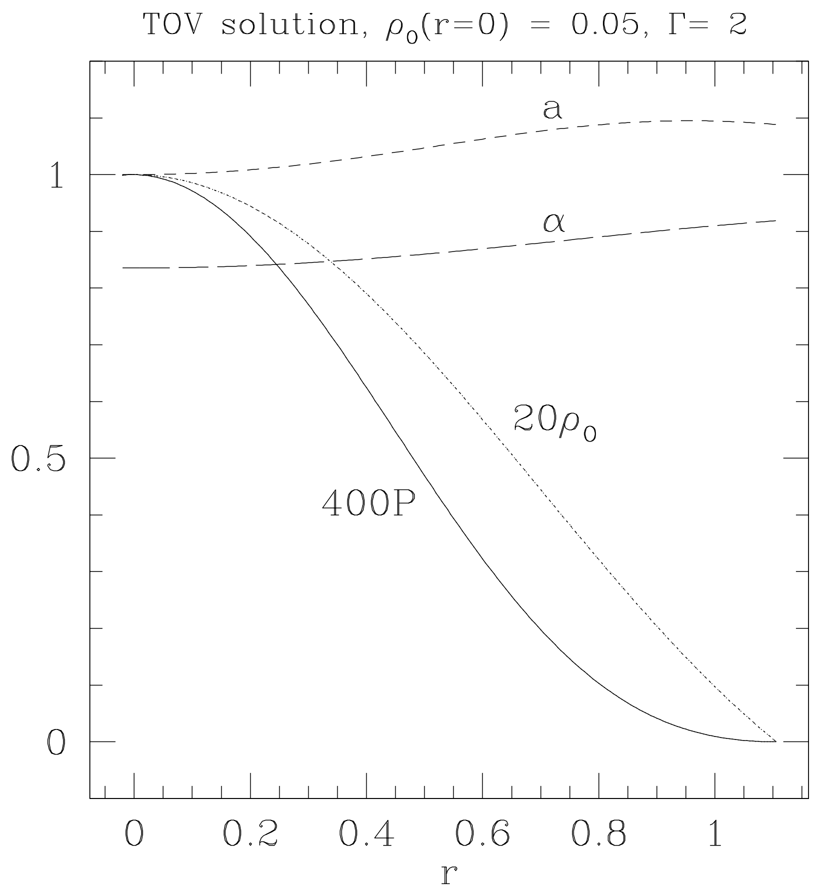
Mass Aspect Function :

$$m(r,t) = \frac{r}{2} \left(1 - 1/a^2 \right)$$

Mass of Spherical Shell :

$$\frac{dm}{dr} = 4\pi r^2 \left(\tau + D \right)$$

Neutron Star Model



- Tolman-Oppenheimer-Volkoff (TOV) solutions:
  Static, spherical solutions to Einstein's Eq. w/ perfect fluid;

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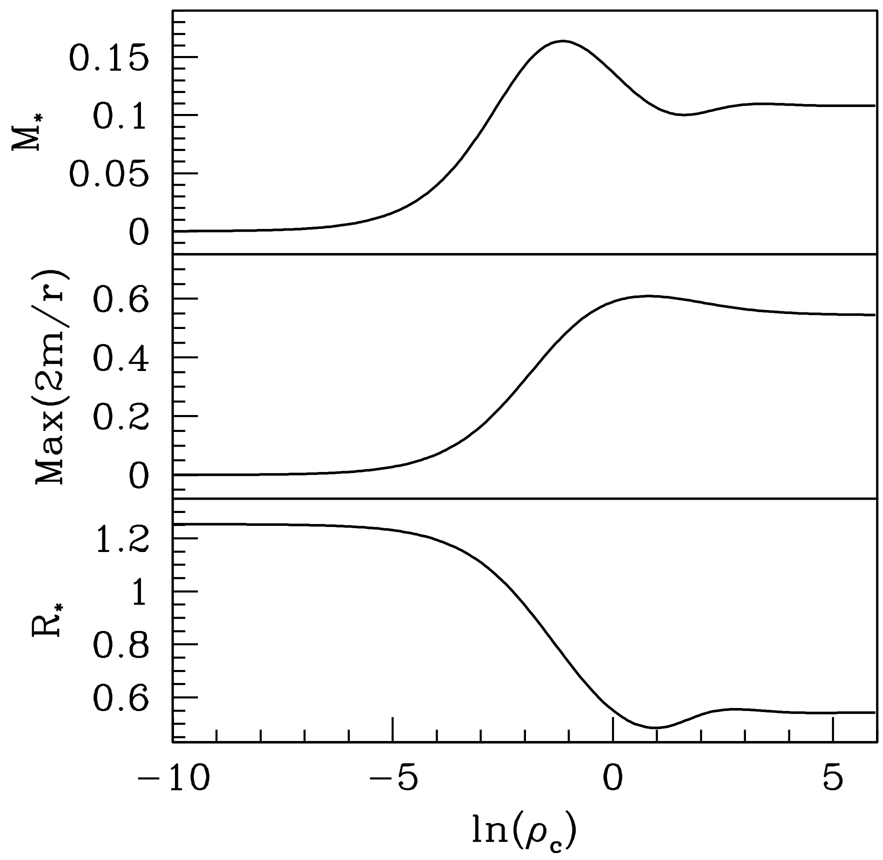

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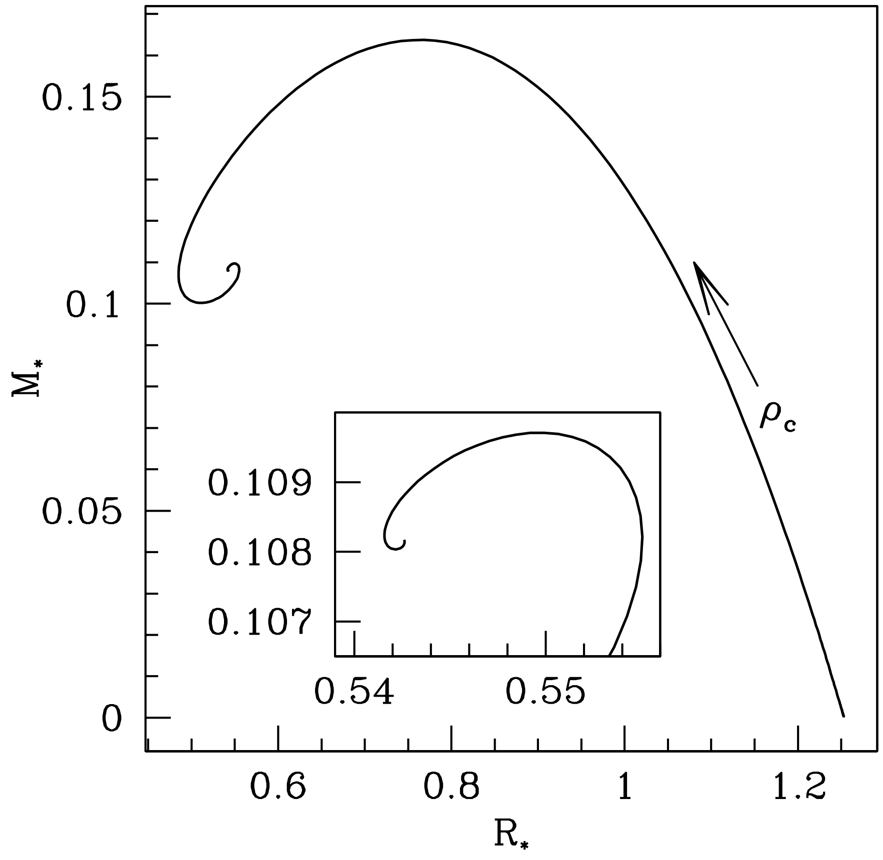

- Tolman-Oppenheimer-Volkoff (TOV) solutions:
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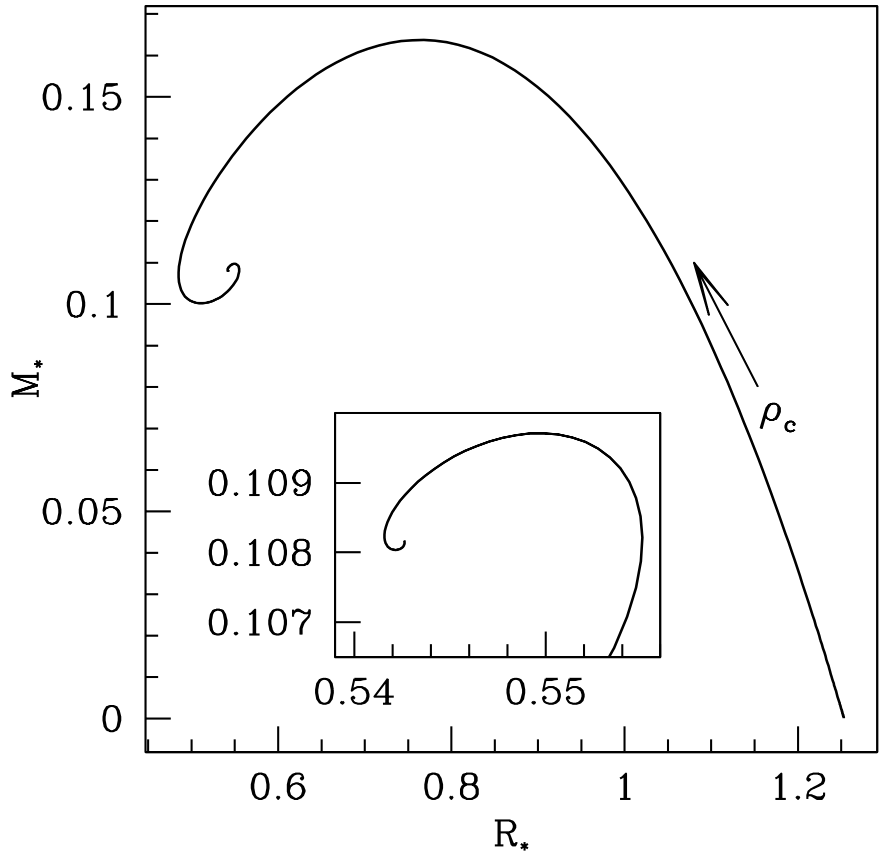
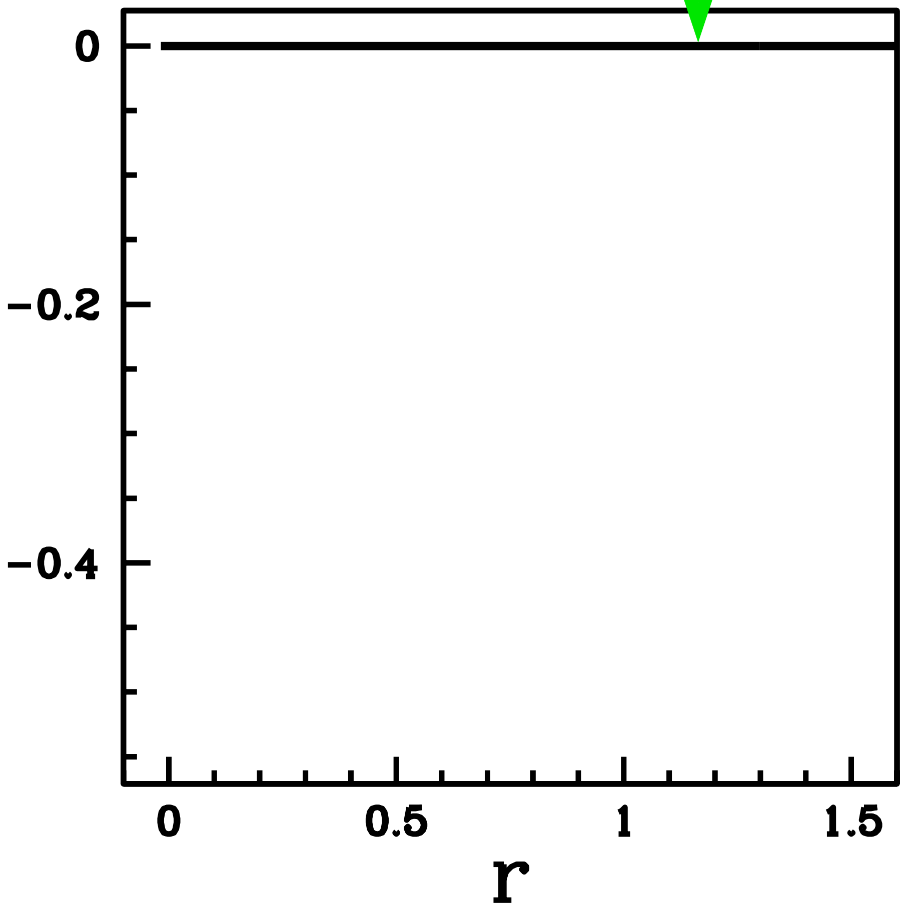
- Stable & Unstable Solutions

Neutron Star Model



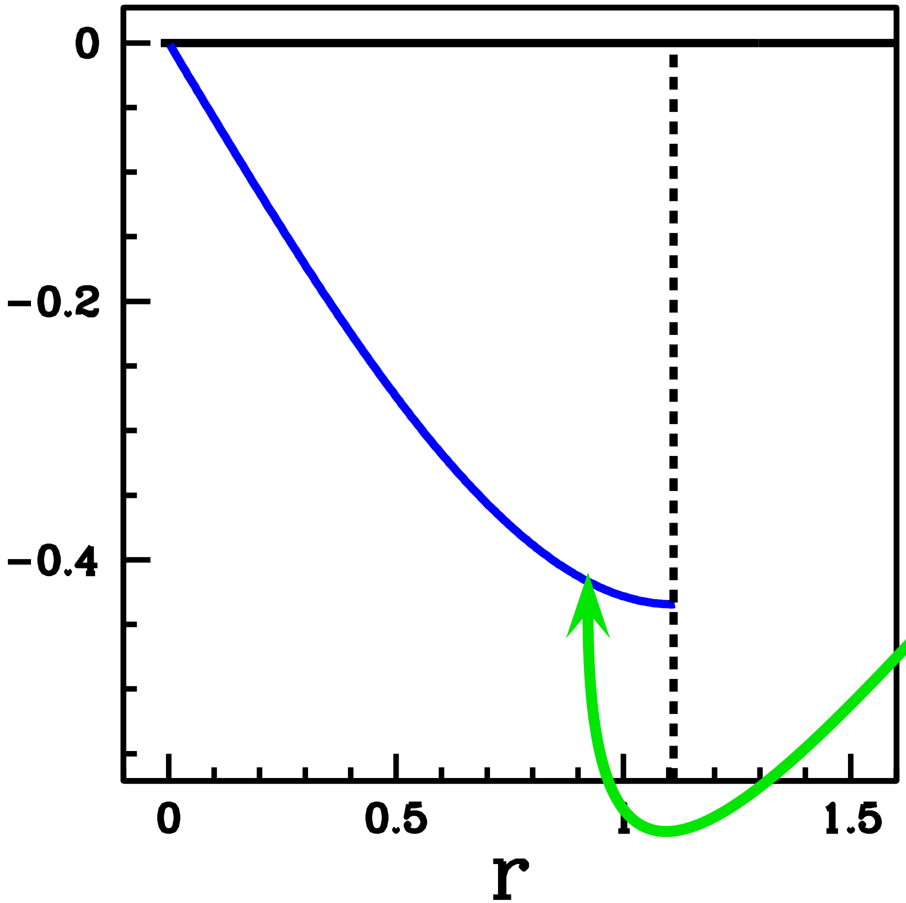
- Tolman-Oppenheimer-Volkoff (TOV) solutions:
  Static, spherical solutions to Einstein's Eq. w/ perfect fluid;
- Parameterized by $\rho_c = \rho_\circ(0,0)$
- Stable & Unstable Solutions
- Isentropic State Equations:
  $$P = K\rho_\circ^\Gamma \ , \ P = (\Gamma - 1)\,\rho_\circ\epsilon$$
  $$\Gamma = 2$$

Velocity-Driven Collapse



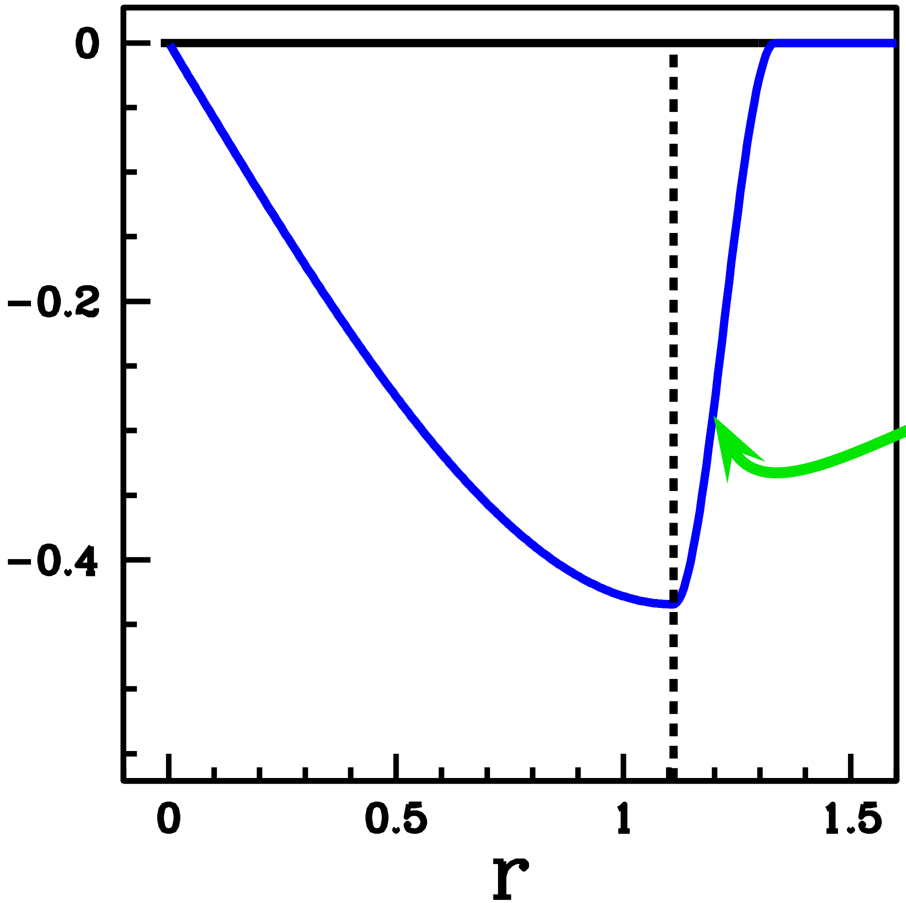
- Solve TOV Eq.'s ($\dot{g}_{\mu\nu} = \dot{T}_{\mu\nu} = v = 0$)

Velocity-Driven Collapse



- Solve TOV Eq.'s $(\dot{g}_{\mu\nu} = \dot{T}_{\mu\nu} = v = 0)$
- Add in-going coordinate velocity: $U(\tilde{r} = r/R_*) = \frac{u^r}{u^t} = p\,\tilde{r}\left(\tilde{r}^2 - b\right)$

Velocity-Driven Collapse



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- Match to U = 0

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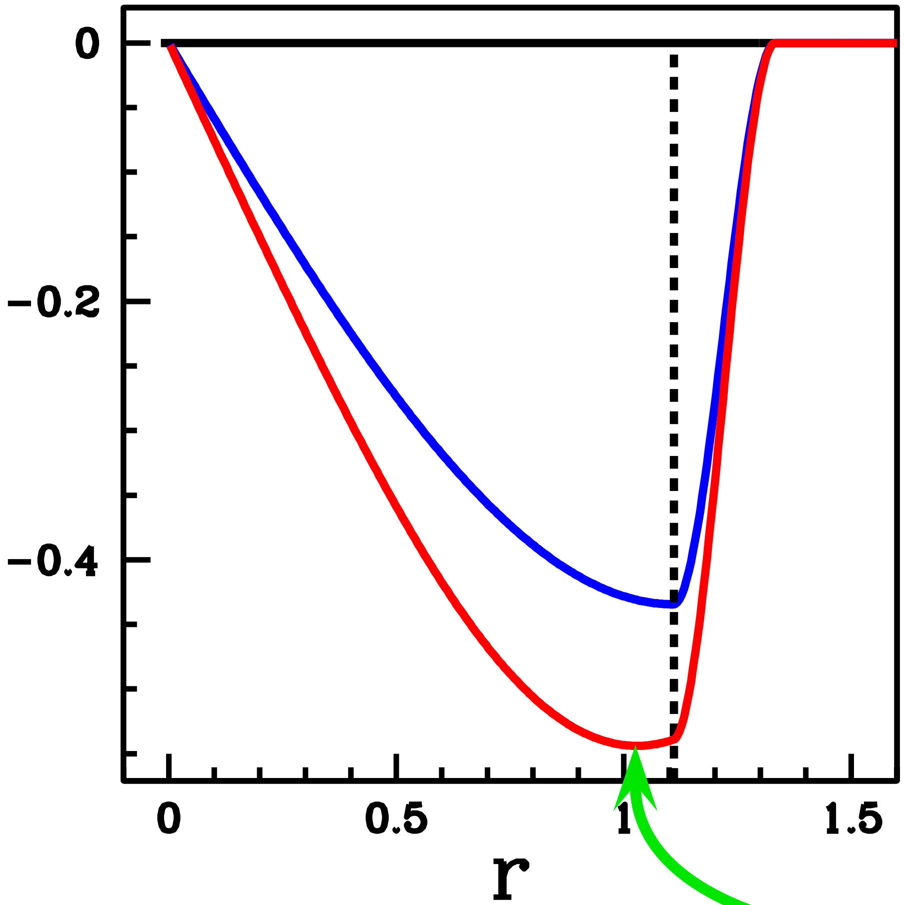

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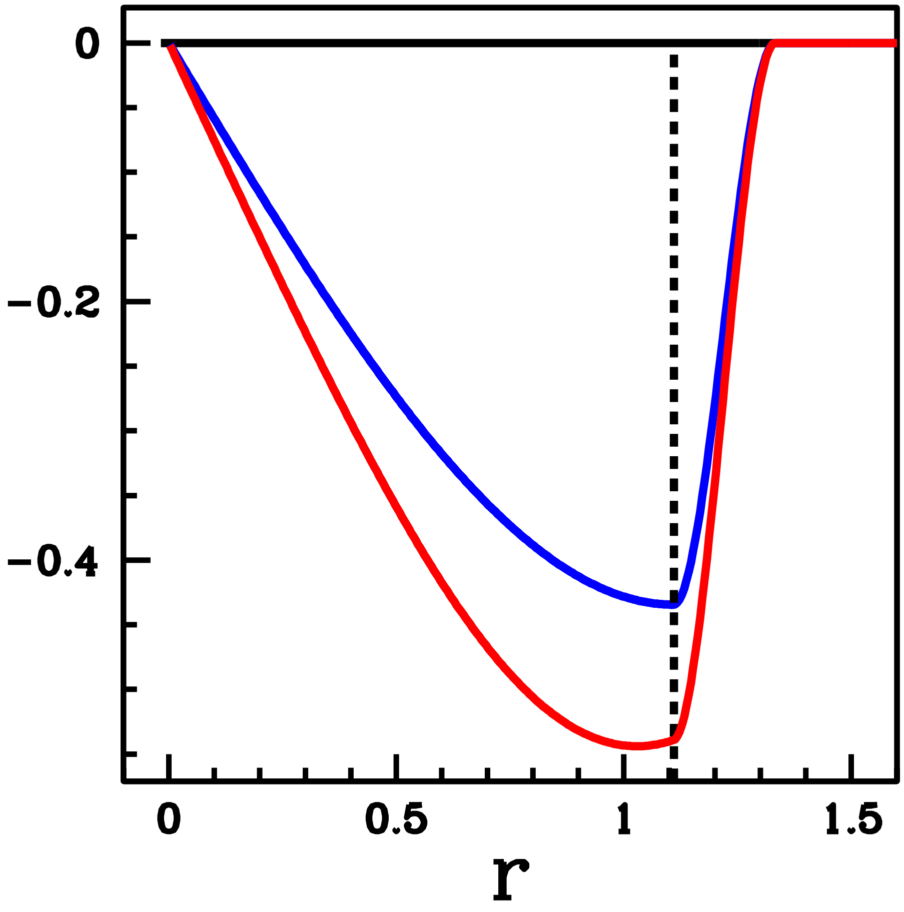
- Solve $(\alpha' = \ldots)$ and $(a' = \ldots)$
  and find $v = aU/\alpha$

Velocity-Driven Collapse



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- Match to U = 0
- Solve $(\alpha' = \ldots)$ and $(a' = \ldots)$
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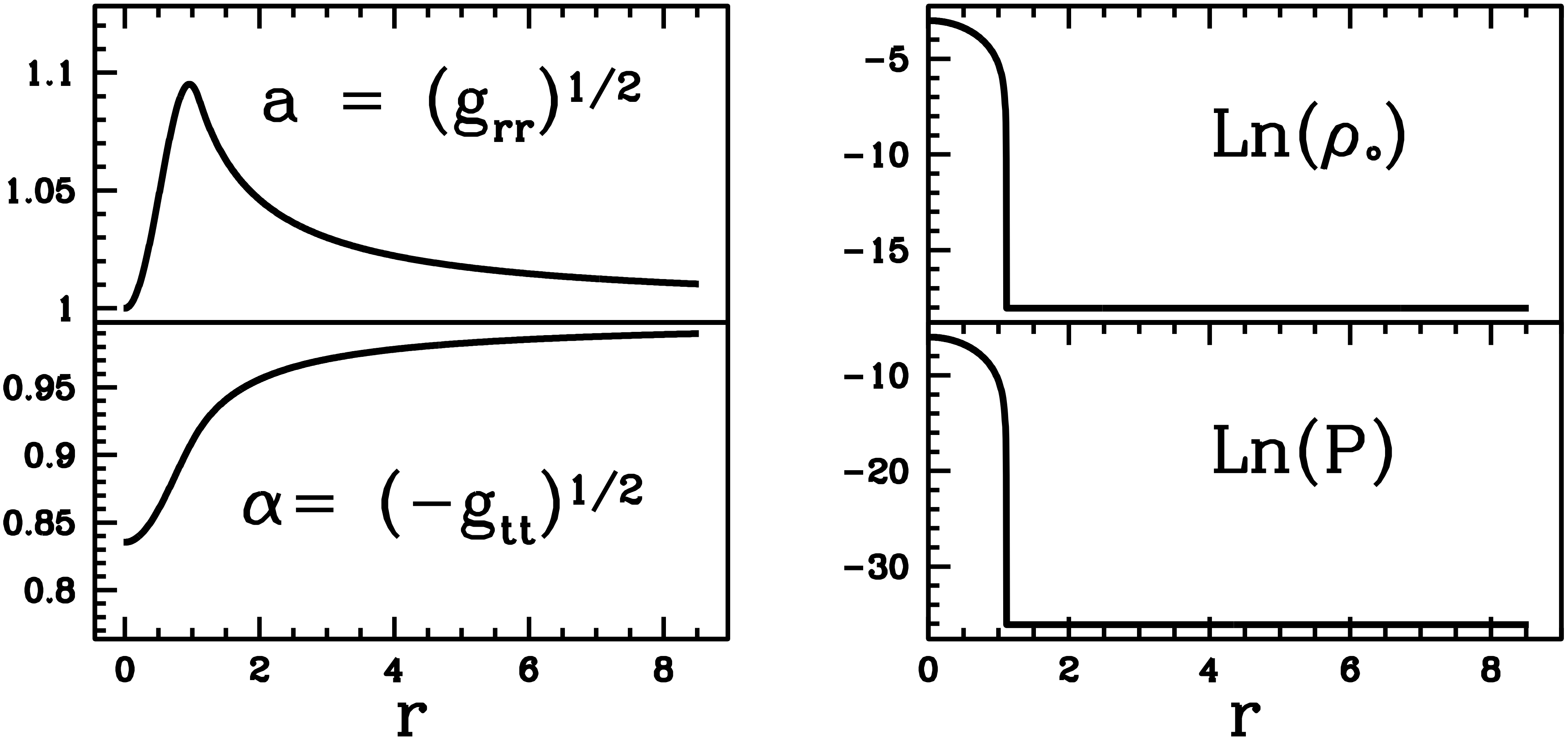
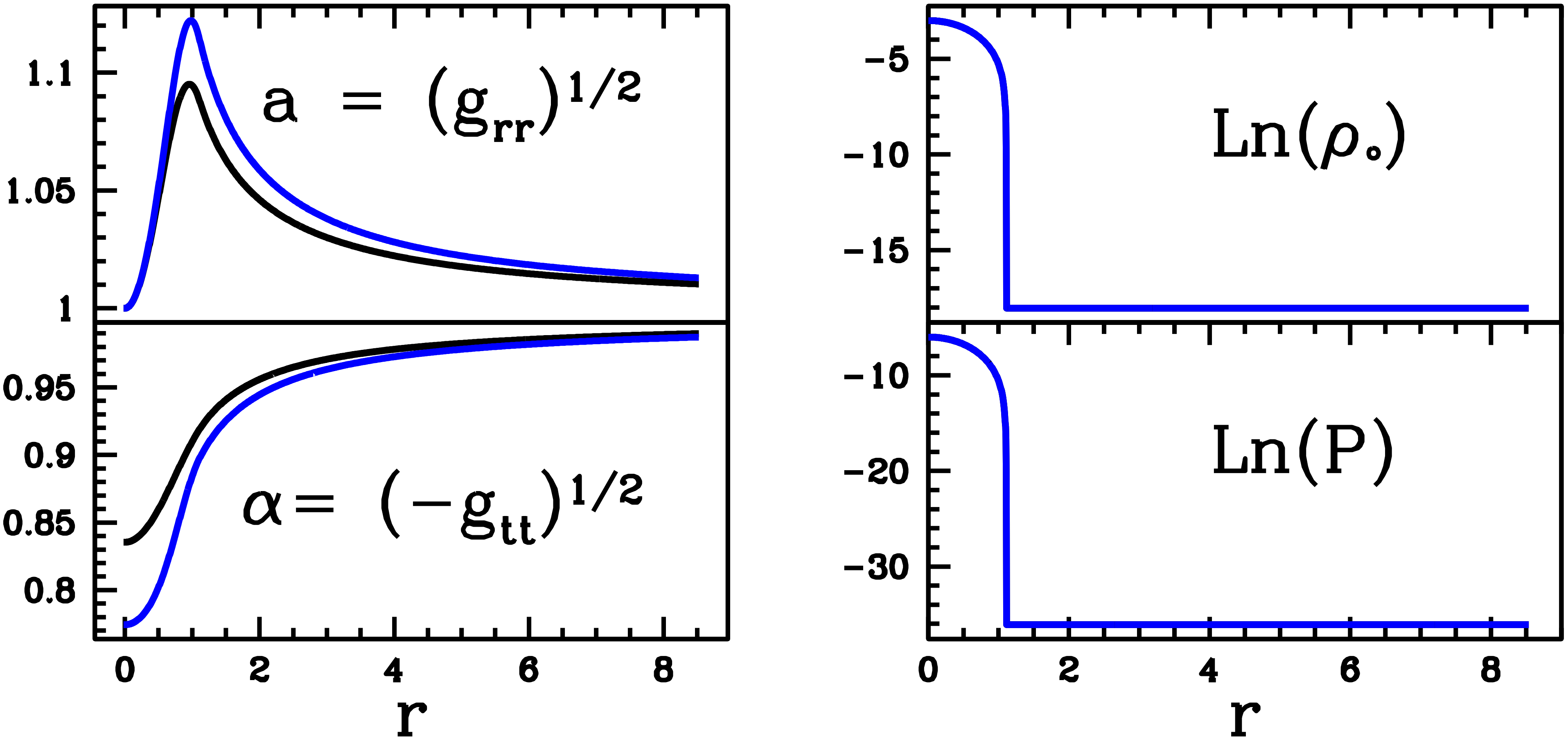
- Tune to vary amount of kinetic energy

Initial Data : TOV Solution



Initial Data : TOV + In-going Velocity



Minimally-Coupled Massless Scalar Field

- Coupled only through the geometry ("poor man's gravitational wave"):

$$T_{ab} = T_{ab}^{\rm scalar} + T_{ab}^{\rm fluid} \quad , \quad G_{ab} = 8\pi T_{ab}$$

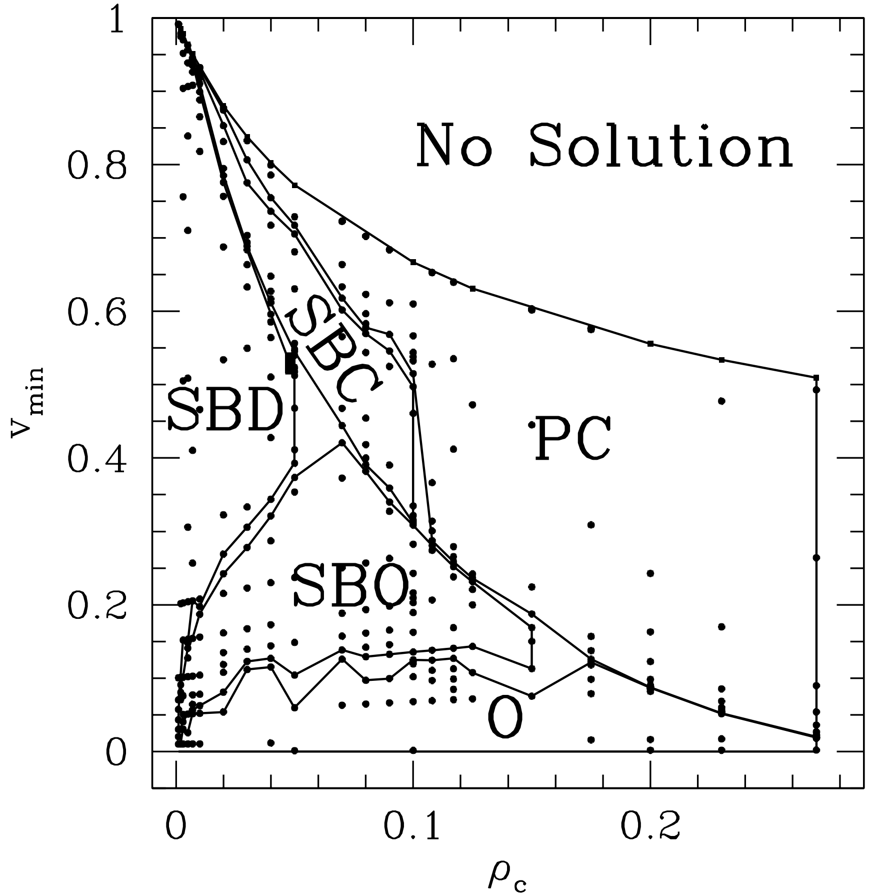
$$\frac{dm}{dr} = \frac{dm_{\rm scalar}}{dr} + \frac{dm_{\rm fluid}}{dr}$$

- Einstein-massless-Klein-Gordon (EMKG) scalar field:

$$T_{ab}^{\rm scalar} = \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \left(\nabla_c \phi \nabla^c \phi \right)$$

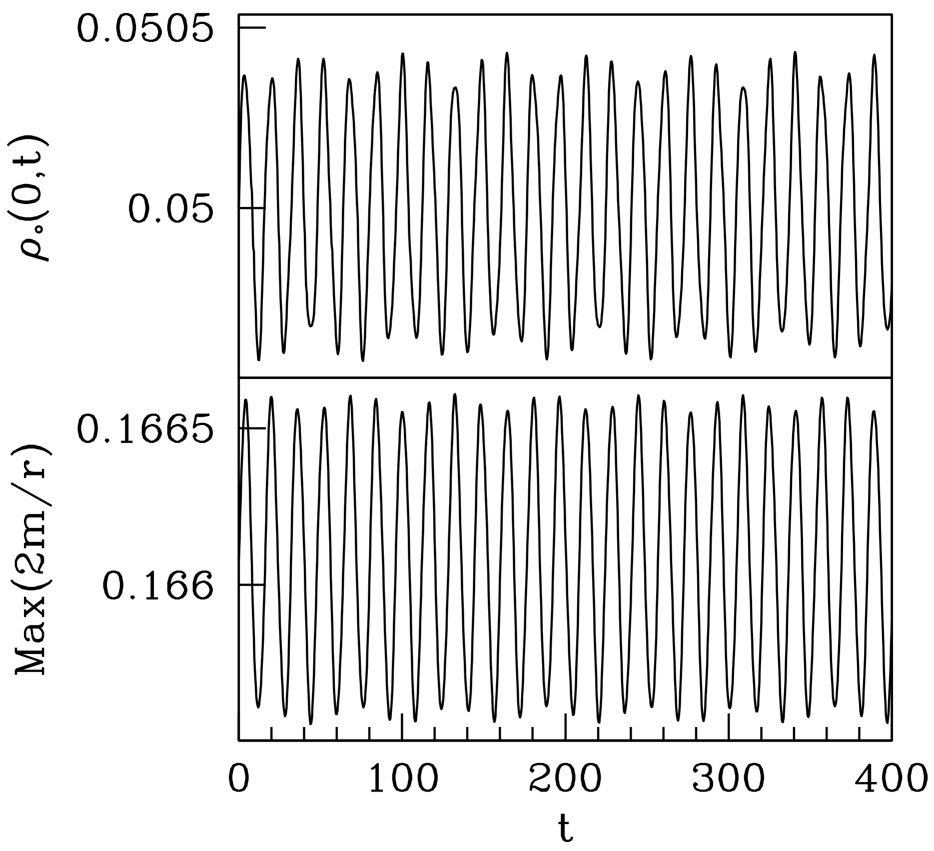
$$\nabla^a \nabla_a \phi = 0$$

Parameter Space Survey



- Previous work:
  - S. Shapiro and Teukolsky (1980)
  - Gourgoulhon (1992)
  - Novak (2001)
- Parameterized by $v_{\min}$ and ρ_c
- Dynamical scenarios:
  - Normal **O**scillations (**O**)
  - **S**hock/**B**ounce/**O**scillations (**SBO**)
  - **S**hock/**B**ounce/**D**ispersal (**SBD**)
  - **S**hock/**B**ounce/**C**ollapse (**SBC**)
  - **P**rompt **C**ollapse (**PC**)

Normal Oscillations (O)



- All stars, small $v_{\rm min}$
- Perturbed stable solution
- Normal, radial oscillations

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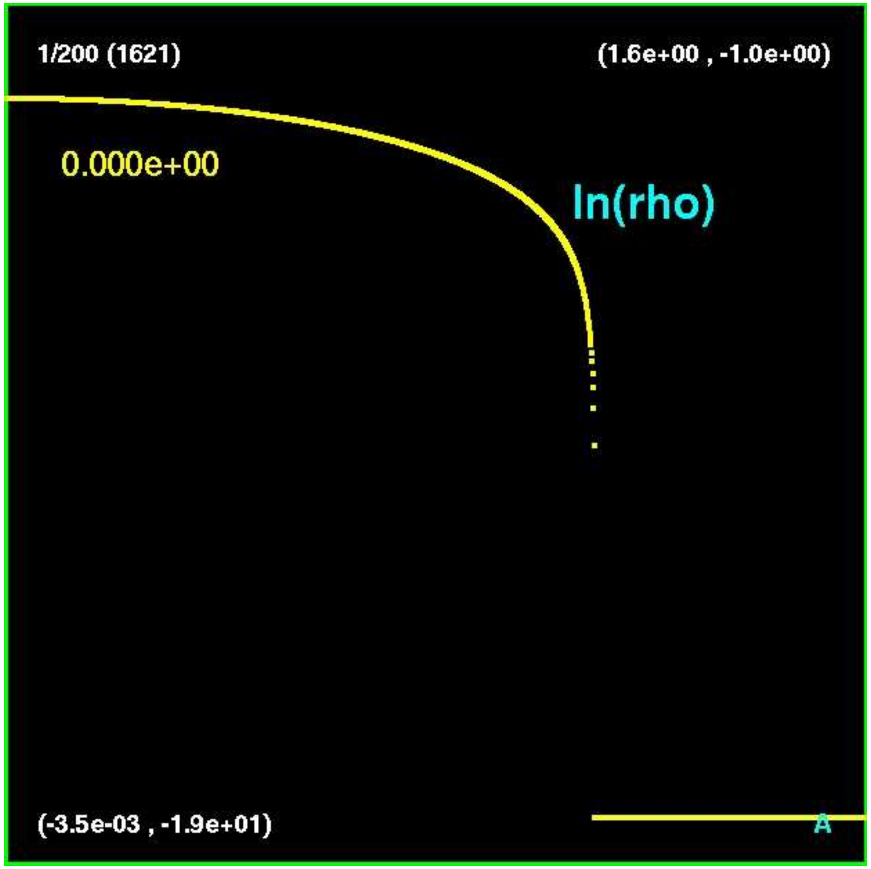
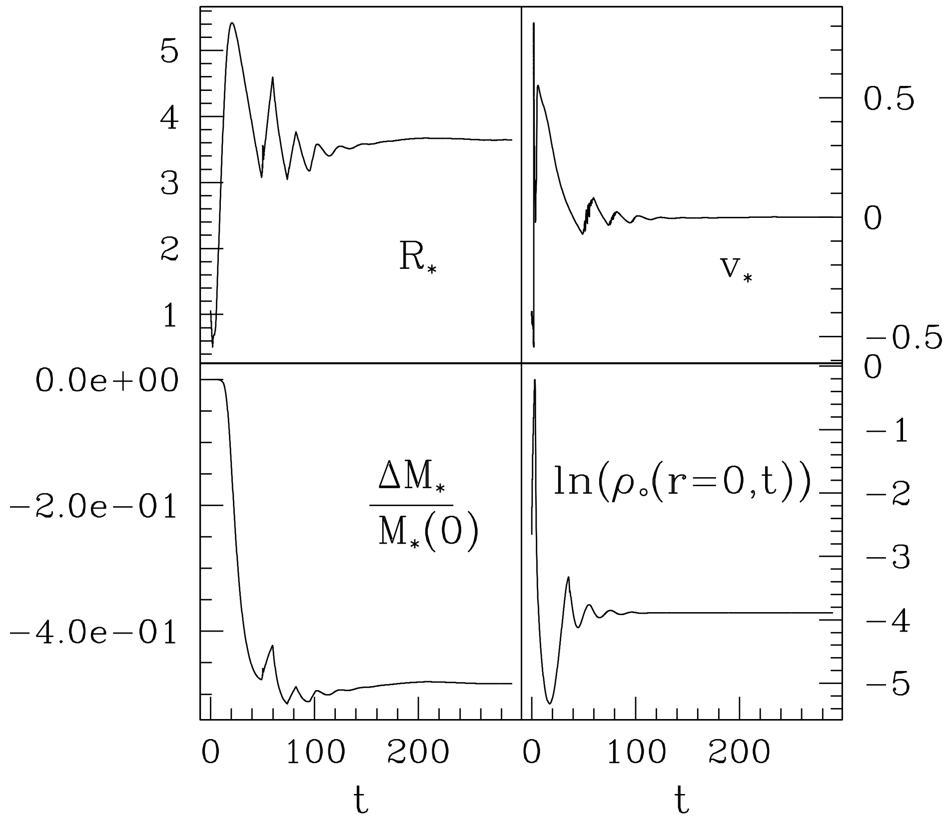


- All stars, small $v_{\min}$
- Perturbed stable solution
- Normal, radial oscillations
- Movies:

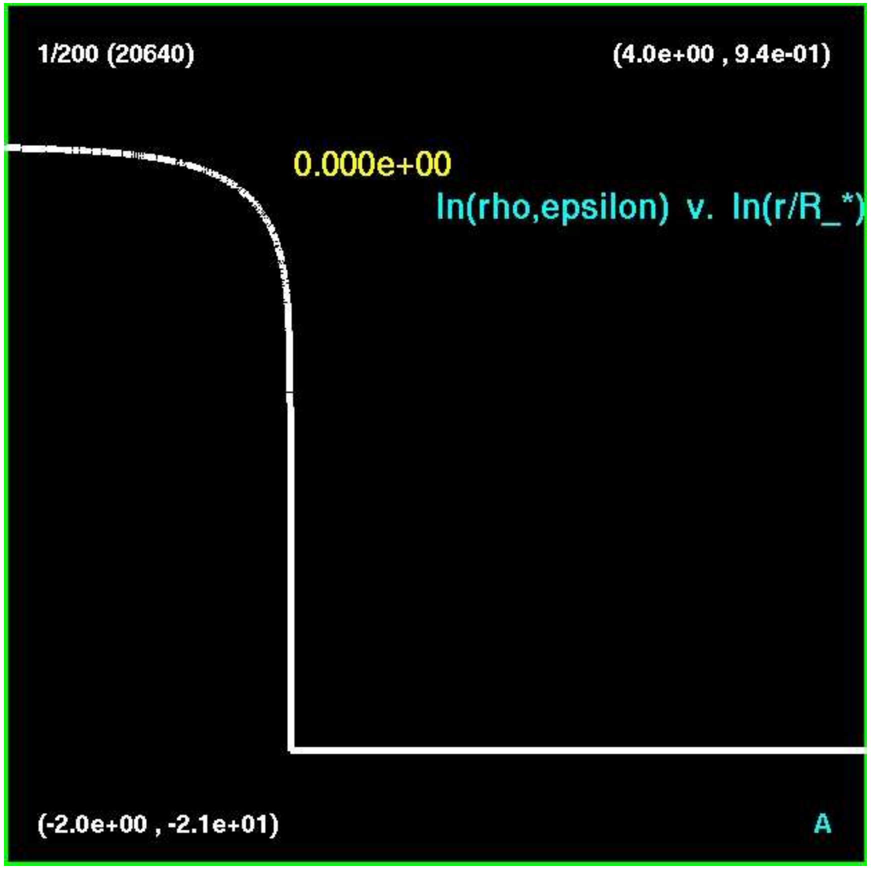
  $\ln\left(\rho_\circ(r,t)\right), \quad \rho_\circ(r,t), \quad v(r,t)$

Shock/Bounce/Oscillations (SBO)



- Moderately compact stars, intermediate v_{min}
- Bounce, Core's Rebound $\rightarrow$ Mass Ejection
- Highly-damped oscillations about sparser star

Shock/Bounce/Oscillations (SBO)



- Moderately compact stars, intermediate $v_{\rm min}$
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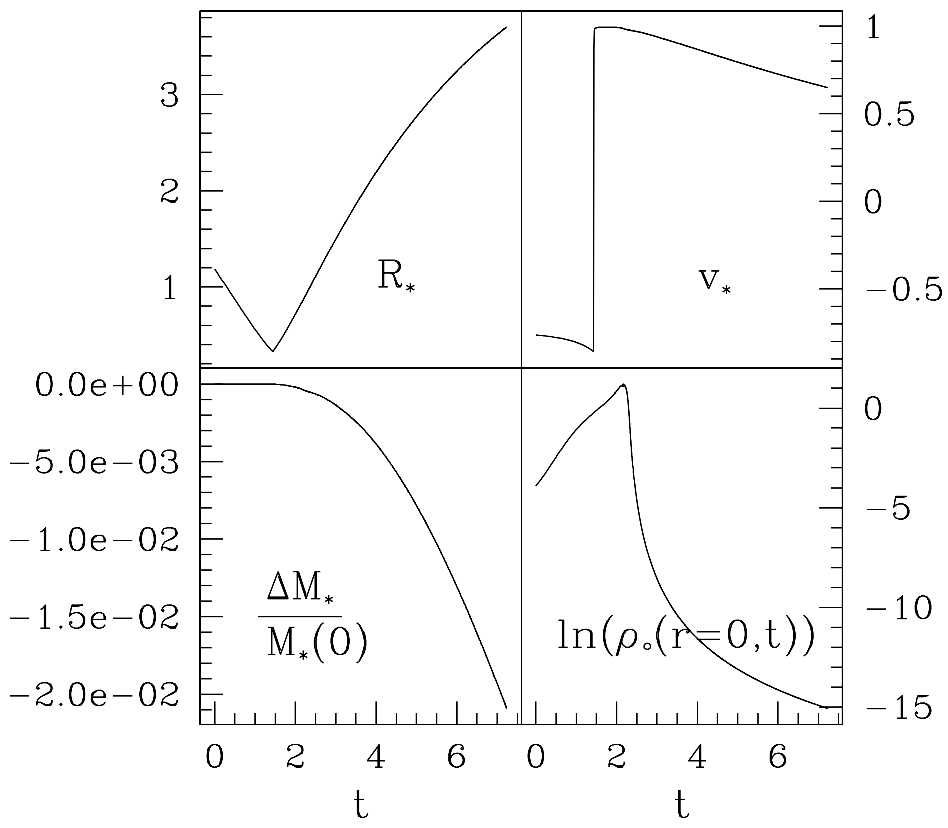
- Highly-damped oscillations about sparser star
- Movies:

$$\ln\left(\rho_\circ\right) \;\&\; \ln\left(\epsilon\right) \quad \text{vs.} \quad \{\ln(r/R_*), t\}\,,$$
$$v \quad \text{vs.} \quad \{\ln\left(r/R_*\right), t\}$$

Shock/Bounce/Dispersal (SBD)



- Sparse stars, small—to—large $v_{\min}$
- Bounce, Core's Rebound $\to$ Dispersal
- Negligible mass left behind

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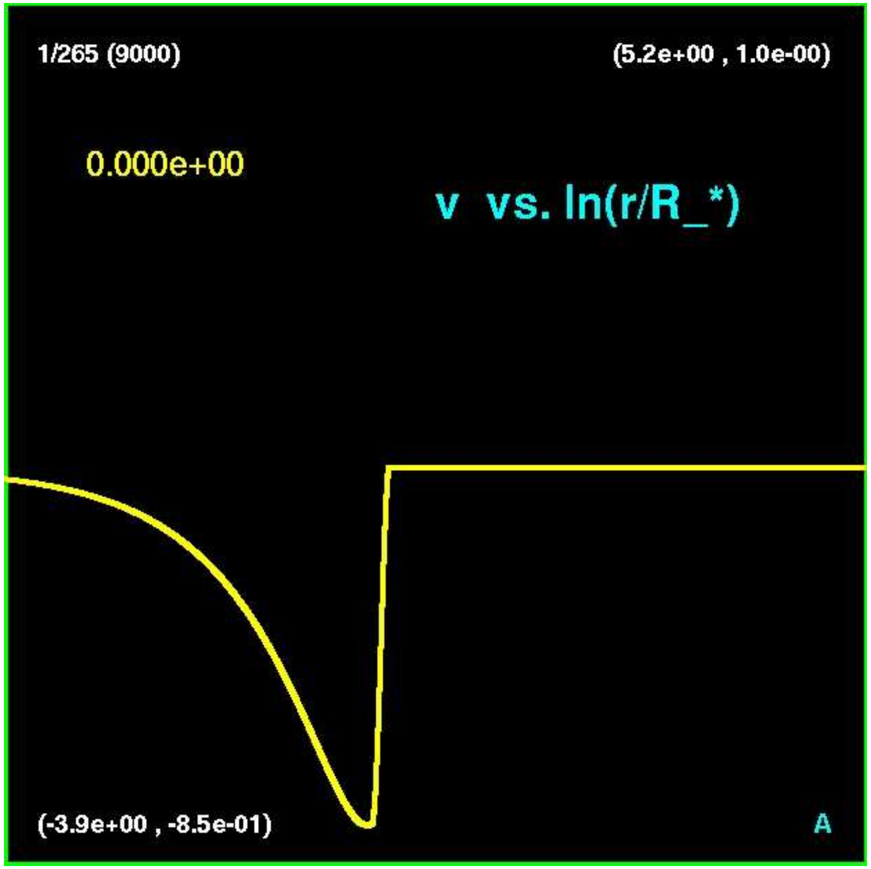

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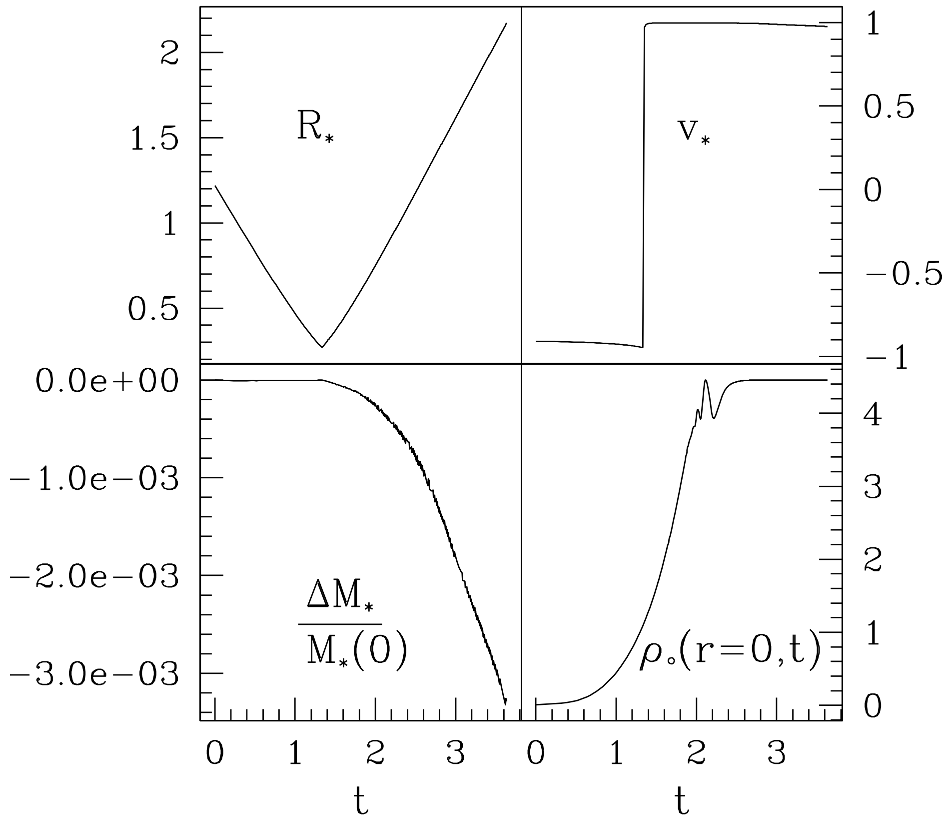
Shock/Bounce/Collapse (SBC)



- Sparse—to—semi-dense stars, medium—to—large $v_{\rm min}$
- Bounce $\rightarrow$ Mass Ejection
- Black hole formation, $M_{\rm BH} < M_*$

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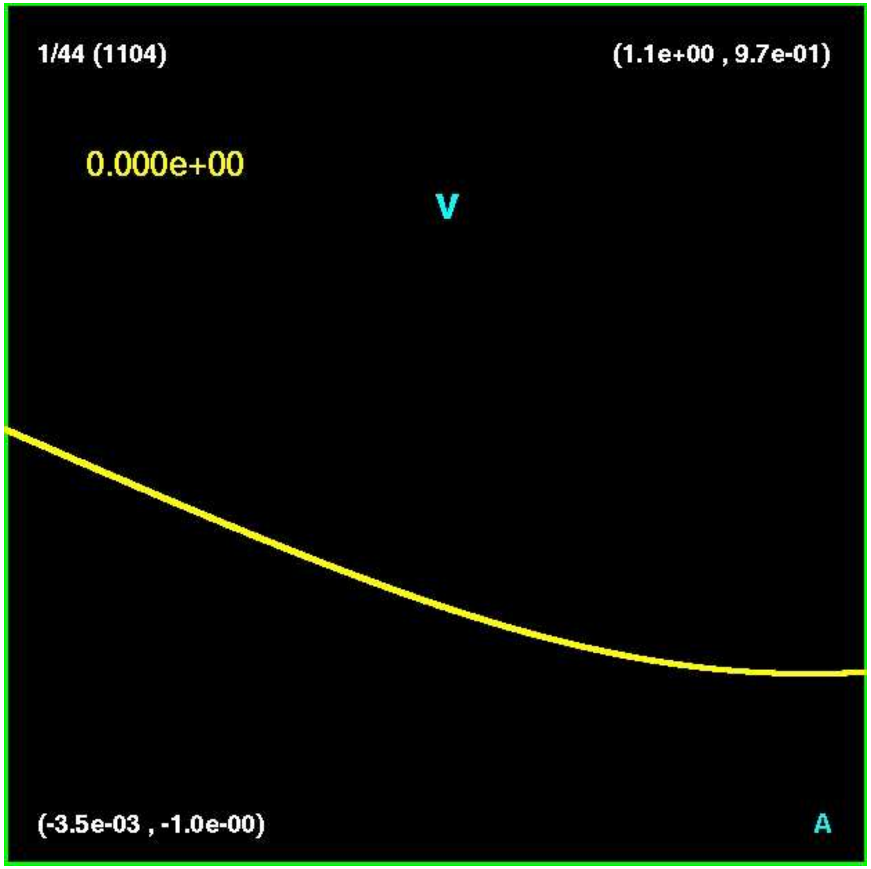

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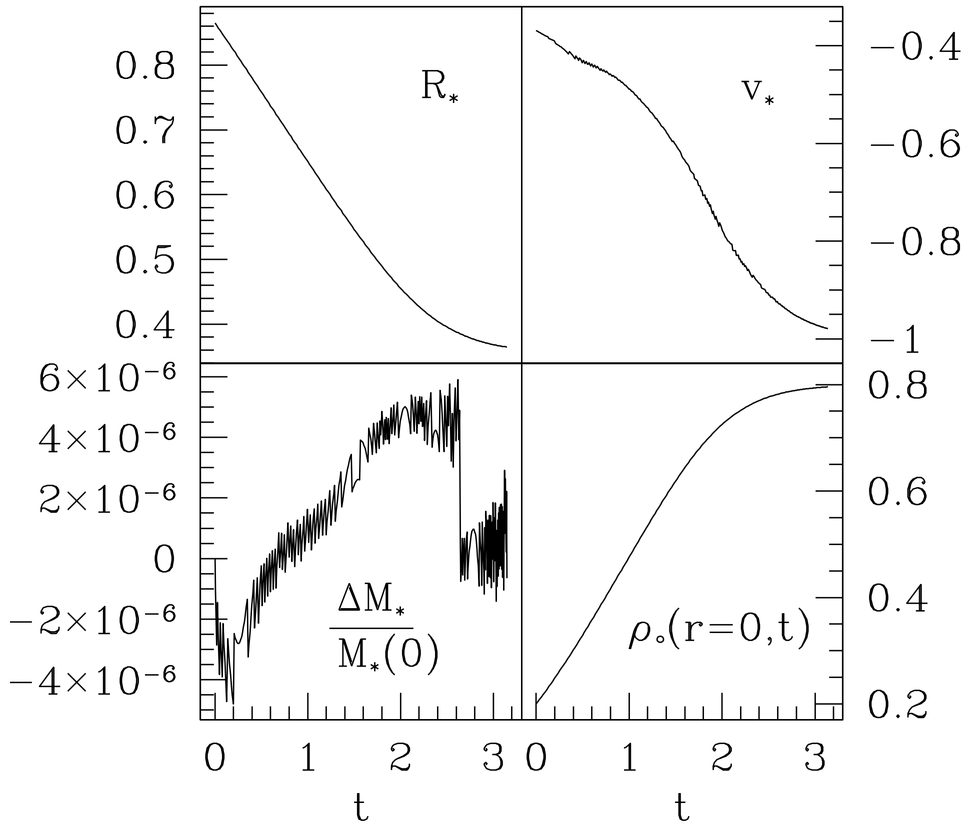
$$a(r,t) \;\; , \; \alpha(r,t) \;\; , \; \rho_\circ(r,t) \;\; , \; v(r,t)$$

$$r \in [0, R_*]$$

Prompt Collapse (PC)



- Nearly all stars, large $v_{\rm min}$
- No mass ejection
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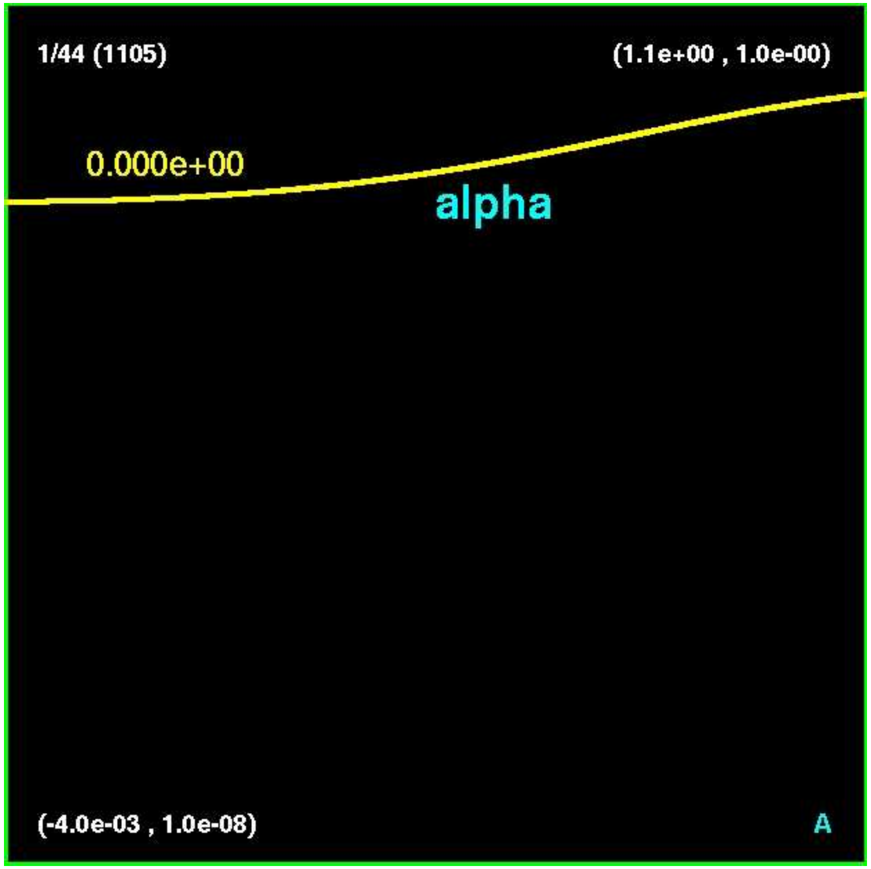

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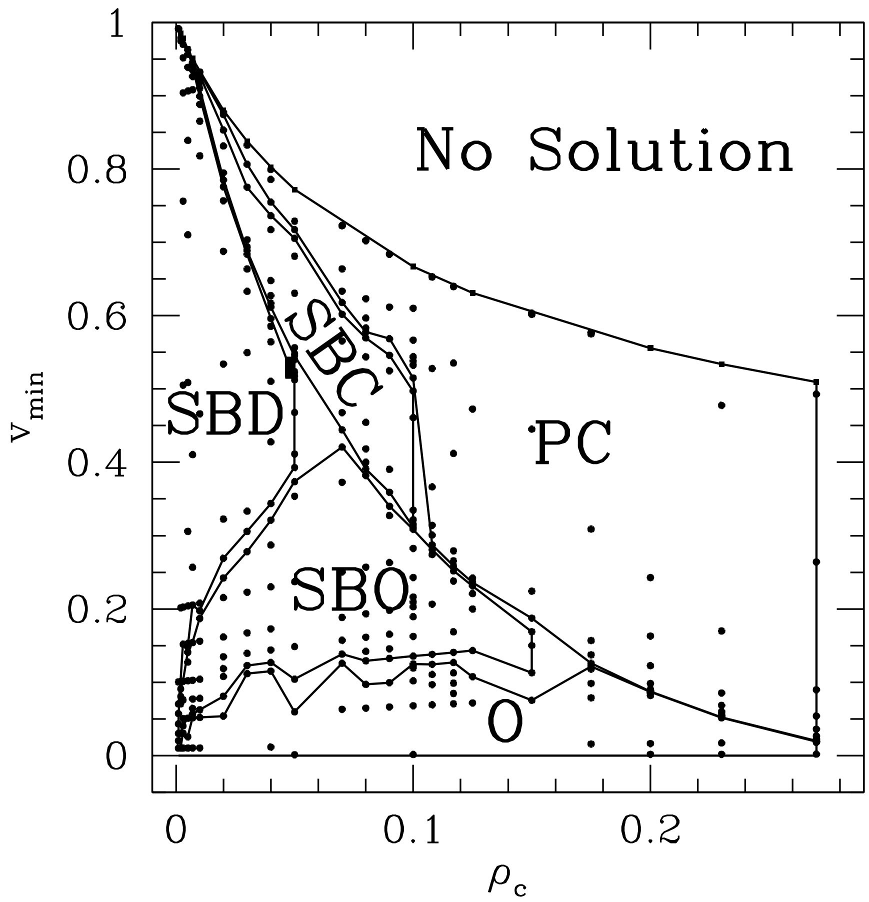
Parameter Space Survey



- $\min(\rho_c^{\mathrm{BH}}) \sim 0.007$;
- $\min(M_{\mathrm{BH}}) \lesssim 0.017$;
- Arbitrarily small BH's for $\rho_c \lesssim 0.05343$, $M_\star \lesssim 0.09$;
- Dynamical scenarios:
  - Normal **O**scillations (**O**)
  - **S**hock/**B**ounce/**O**scillations (**SBO**)
  - **S**hock/**B**ounce/**D**ispersal (**SBD**)
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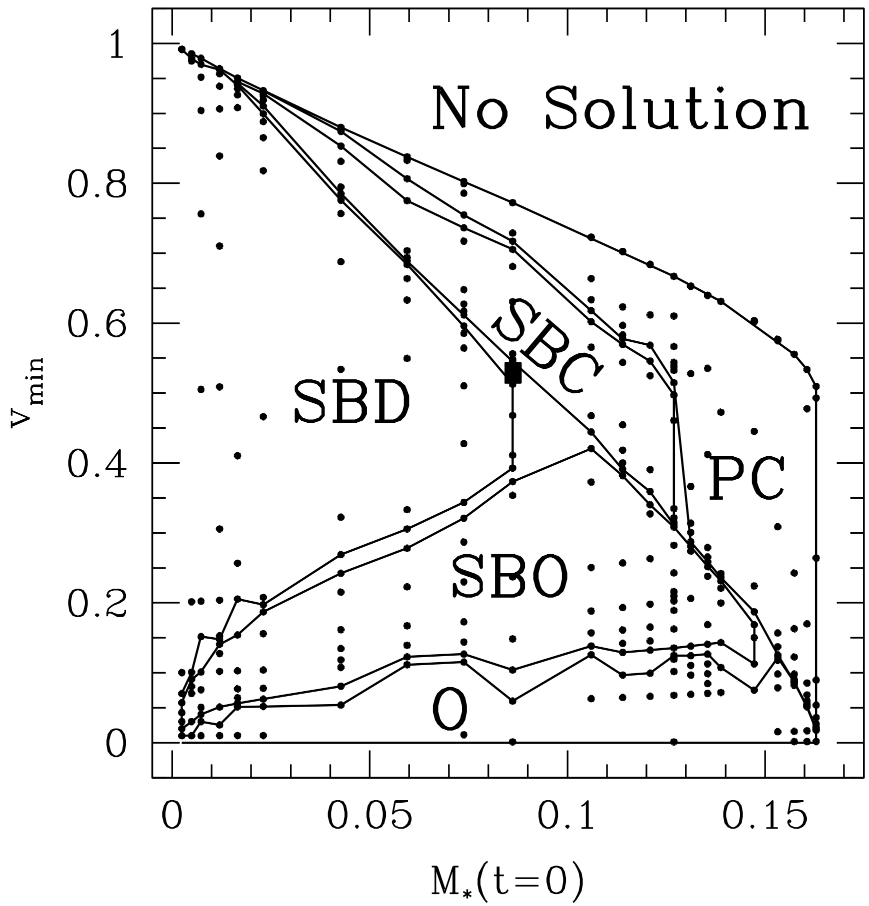

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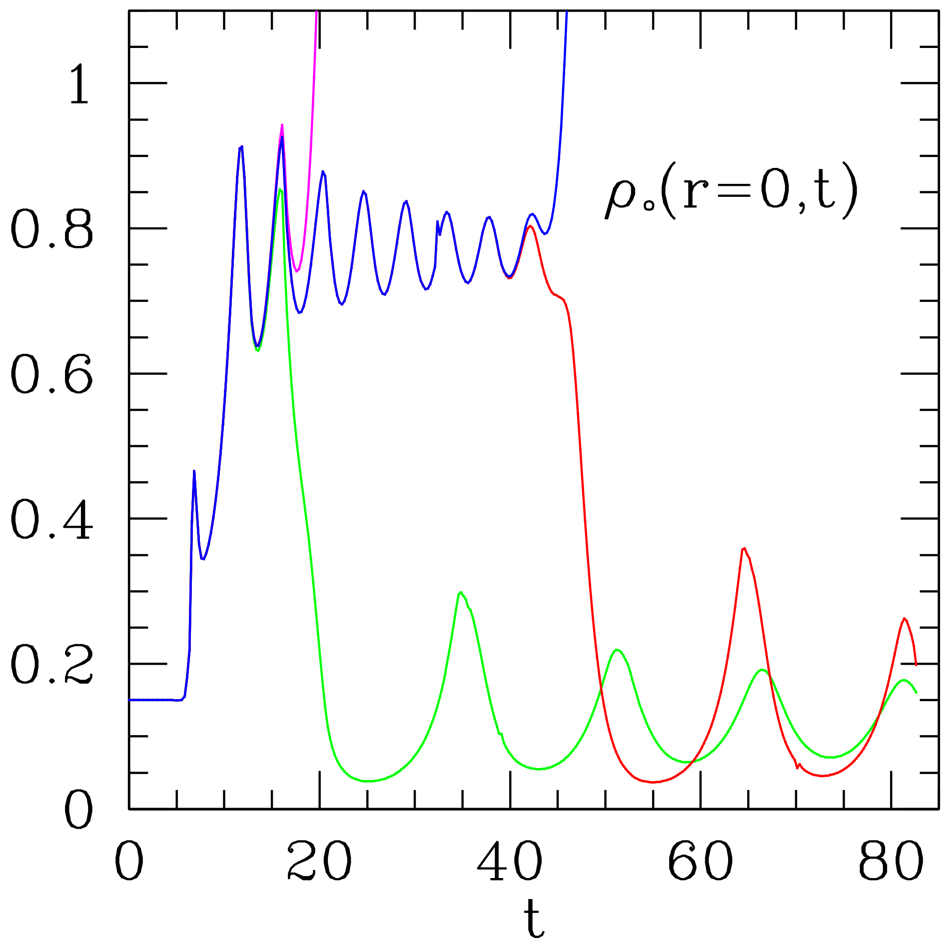
Type I Critical Phenomena

$\rho_c = 0.15$



- Hawley & Choptuik (2000): Boson Stars
- Vary p:
  $\phi(r, 0) = p\,\mathtt{exp}\left(-\left[r - r_\circ\right]^2 / \Delta^2\right)$
- Large p $\rightarrow$ BLACK HOLE
- Small p $\rightarrow$ NO BLACK HOLE (e.g. perturbed star)
- Tuning away the only unstable mode

$$\Rightarrow T_\circ \propto \frac{1}{\omega_{Ly}} \ln |p - p^*|$$

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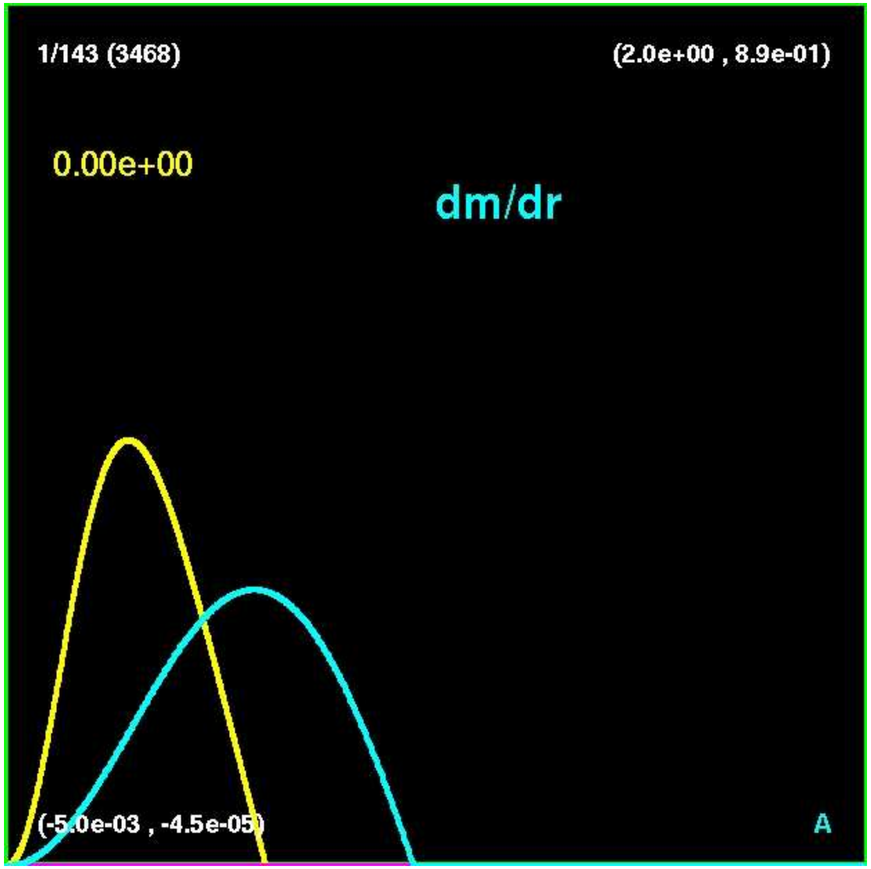

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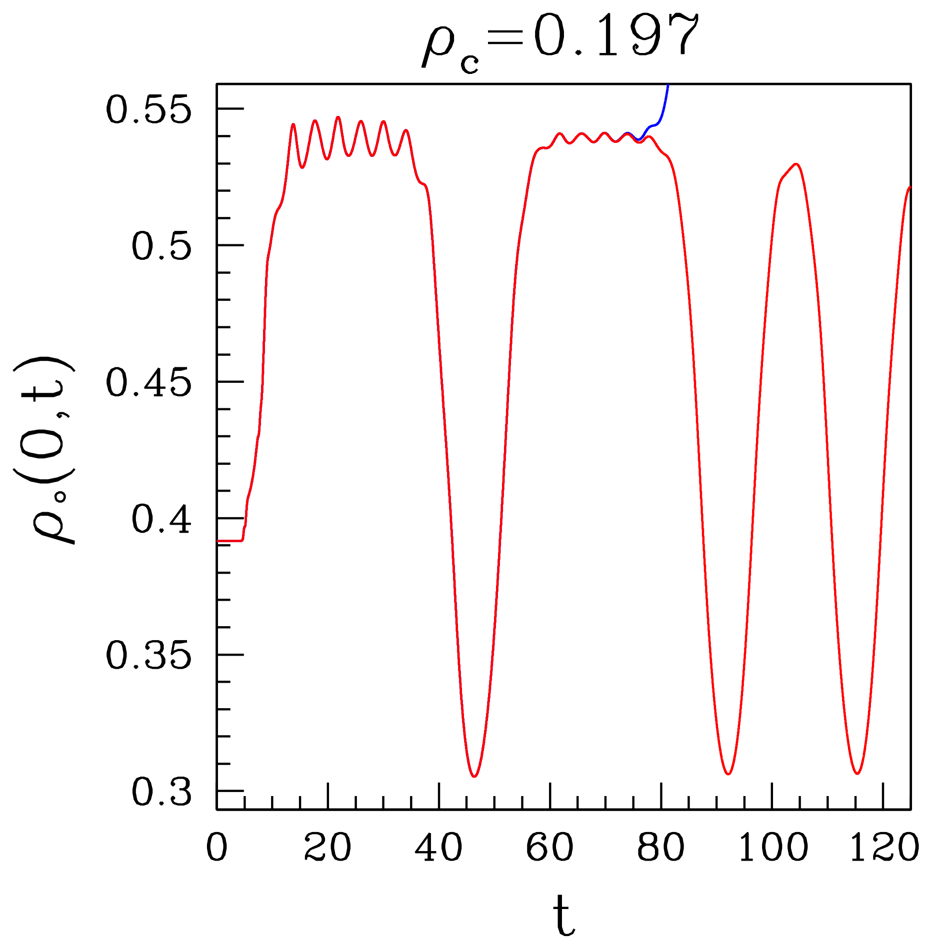
- Movies:

  dm/dr , $\ln(dm/dr)$ (wide view) ,
  $\ln(dm/dr)$ (closeup)

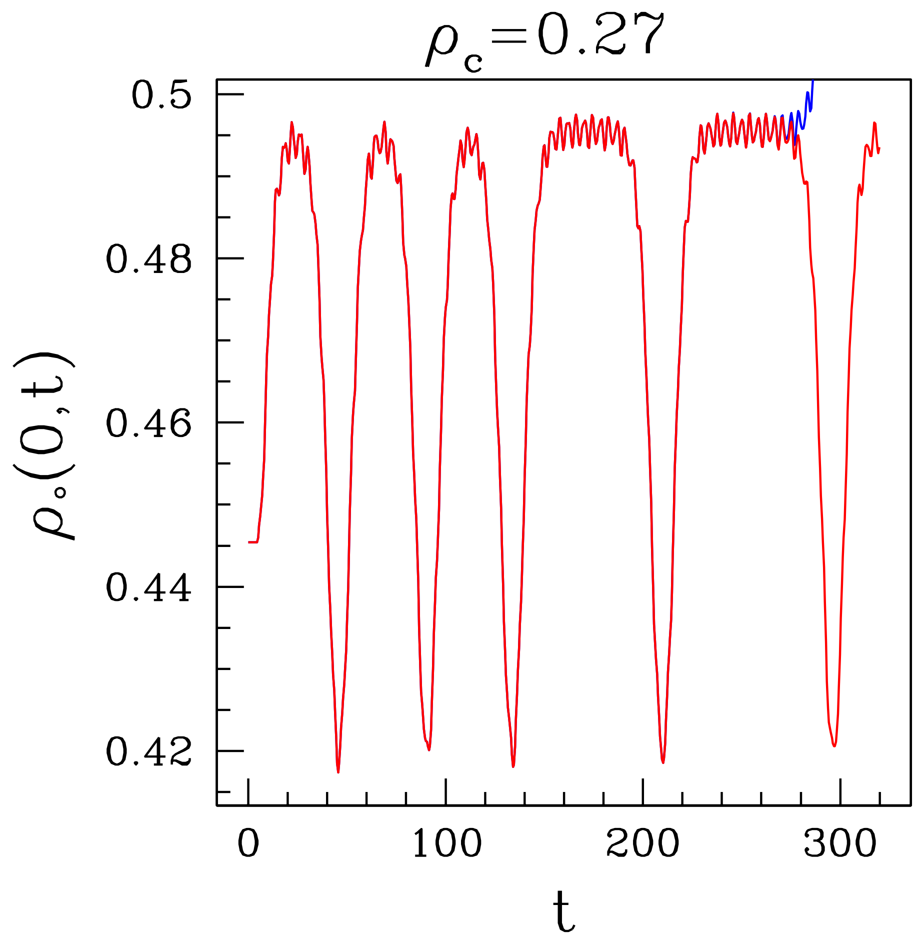
Type I: Anomalous Case $\rho_c = 0.197$



- Movie:

dm/dr

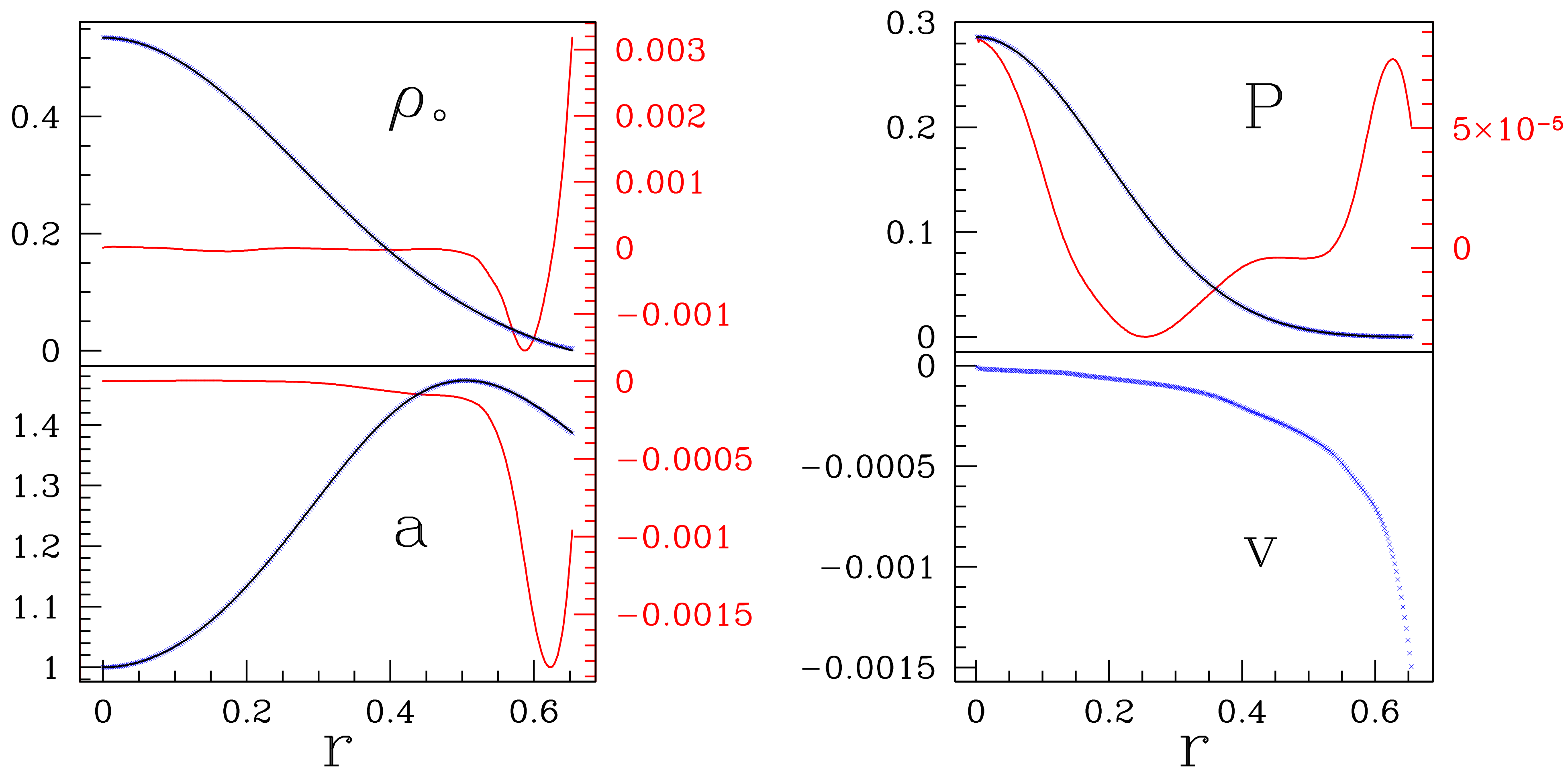
Type I: Anomalous Case $\rho_c = 0.27$



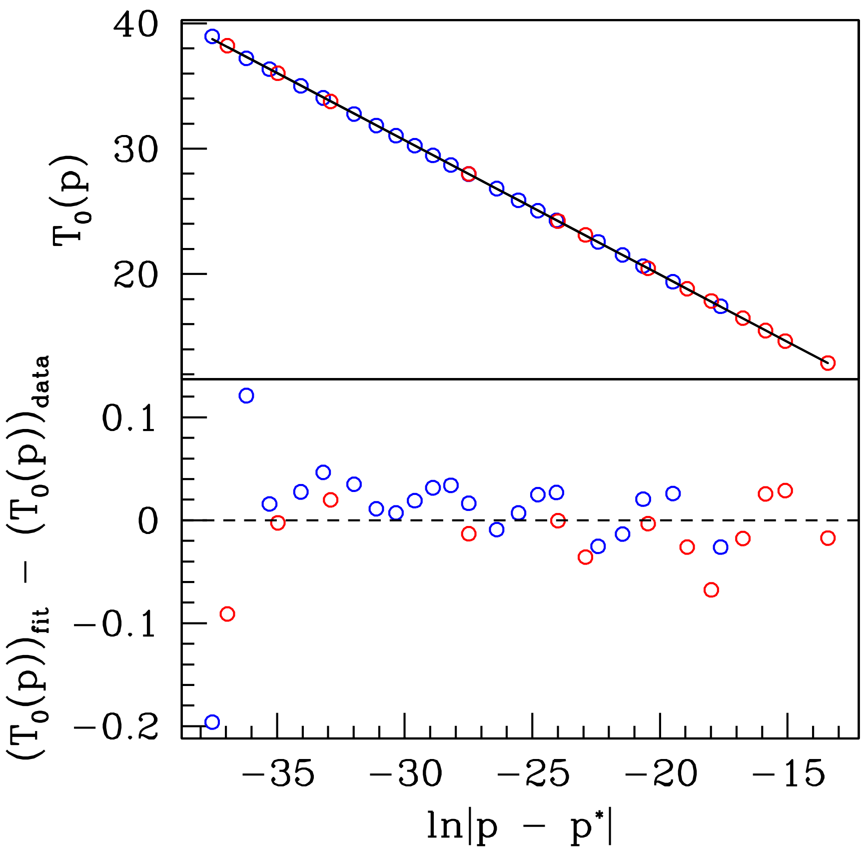
- Movies:

  dm/dr , $\ln(dm/dr)$

Critical Solution $\stackrel{?}{=}$ Unstable TOV $(\rho_c = 0.197)$



Scaling Behavior



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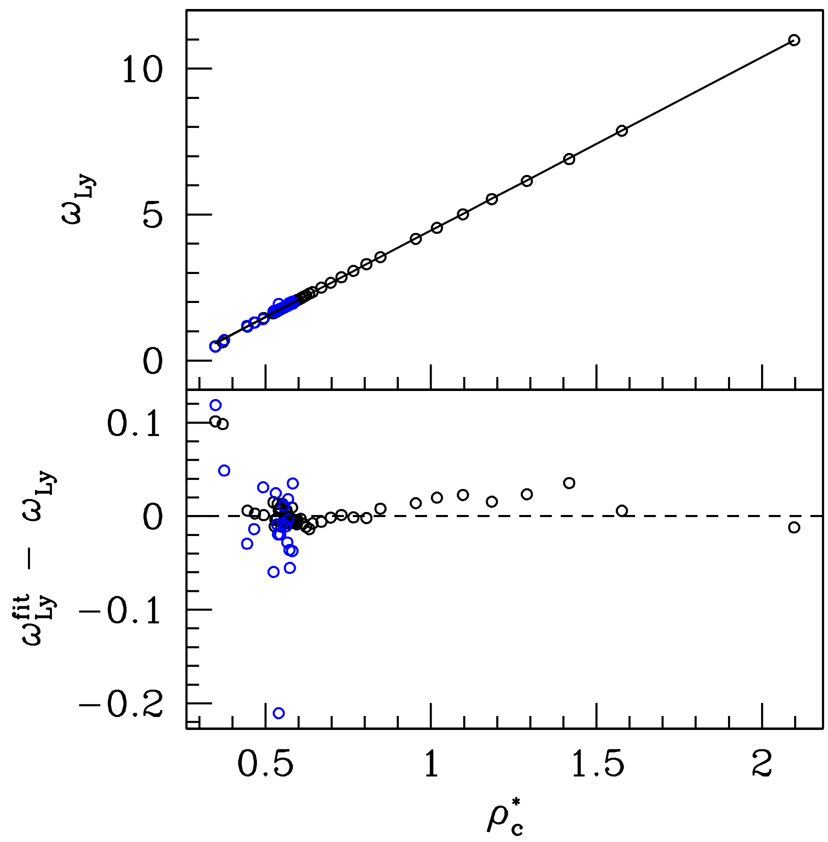

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- $\omega_{Ly} \propto \rho_c^*$

Type II Critical Phenomena: Motivation

- J. Novak (2001):
  - "Ideal-gas" EOS: $P = (\Gamma - 1)\,\rho_\circ \epsilon$, $\Gamma = 2$
  - Tuning star's init. vel. $\rightarrow$ Type II critical behavior;
  - $M_{BH} \propto |p - p^*|^\gamma$ with $\gamma \simeq 0.52$
- Neilsen and Choptuik (2000), Brady et al. (2002)
  - Studied ultra-relativistic fluid collapse;
  - A limit of "ideal-gas" case where $\rho \equiv (1+\epsilon)\,\rho_\circ \simeq \rho_\circ \epsilon$
  - $P = (\Gamma - 1)\,\rho$, only EOS to admit CSS soln's;
  - For $\Gamma = 2$, $\gamma \simeq 0.95 \pm 0.02$
- Neilsen and Choptuik (2000)
  - For $\Gamma = 1.4$: Ideal-gas Type II Sol'n. $=$ Ultra-rel. Type II Sol'n.

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??

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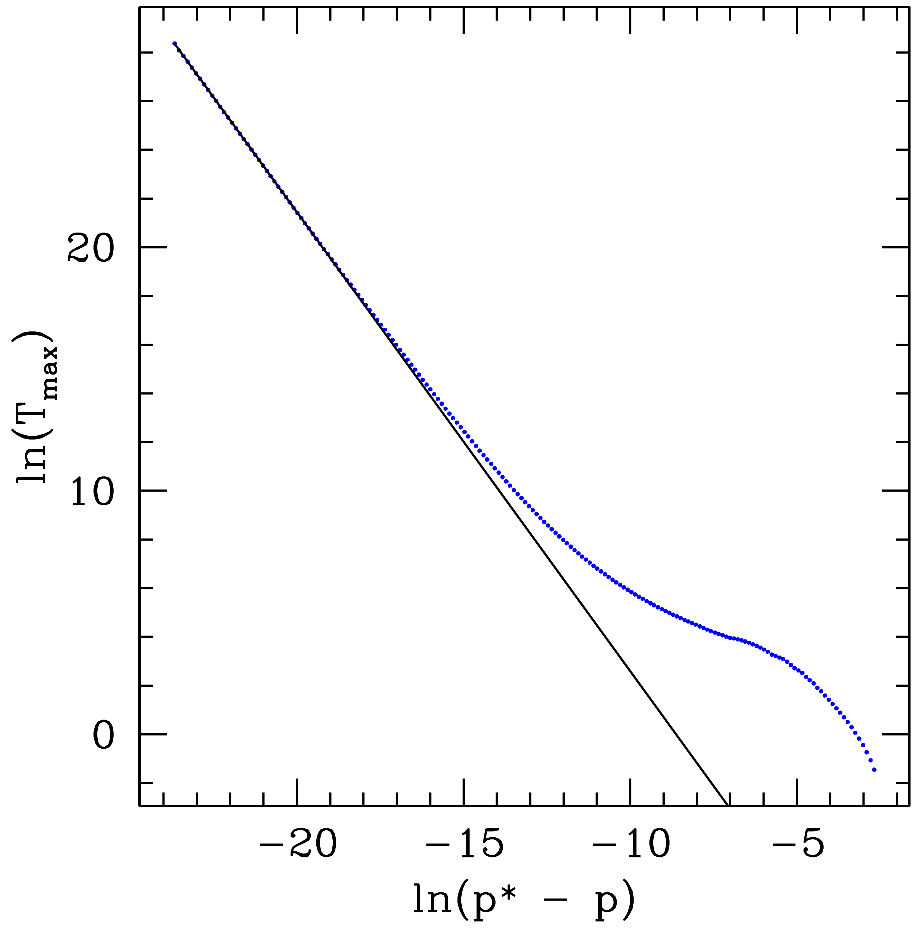
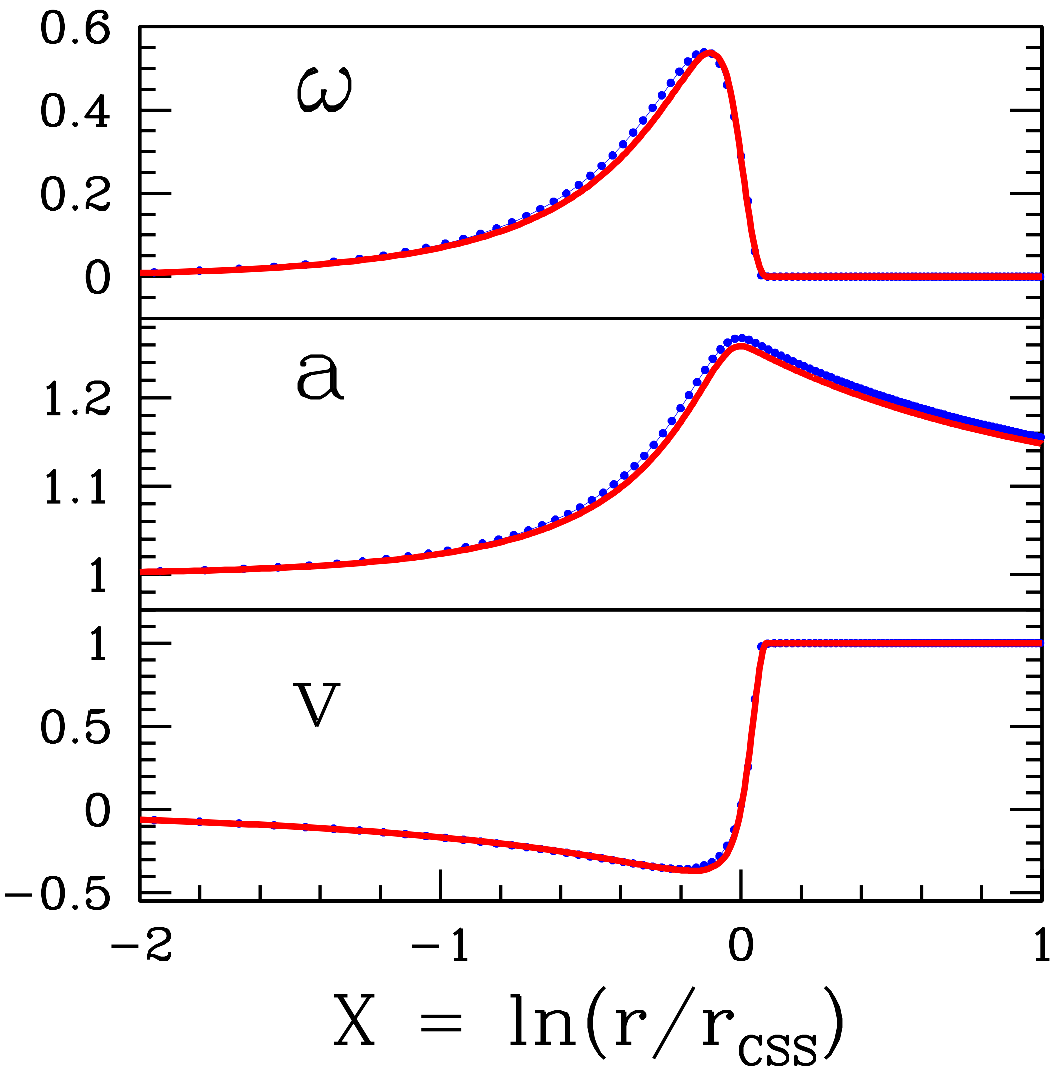
Critical Regime of Parameter Space



- $T_{\max} \equiv \text{Global Max.}(T^a{}_a)$
- $T^a{}_a = 3P - (\rho_\circ + \rho_\circ \epsilon)$
- Anticipated subcritical scaling behavior: $T_{\max} \propto |p - p^*|^{-2\gamma} \quad \gamma = 1/\omega_{Ly}$
- Novak tuned to $\ln|p^* - p| \simeq -7$

CSS Solutions of Ideal-gas and Ultra-rel.

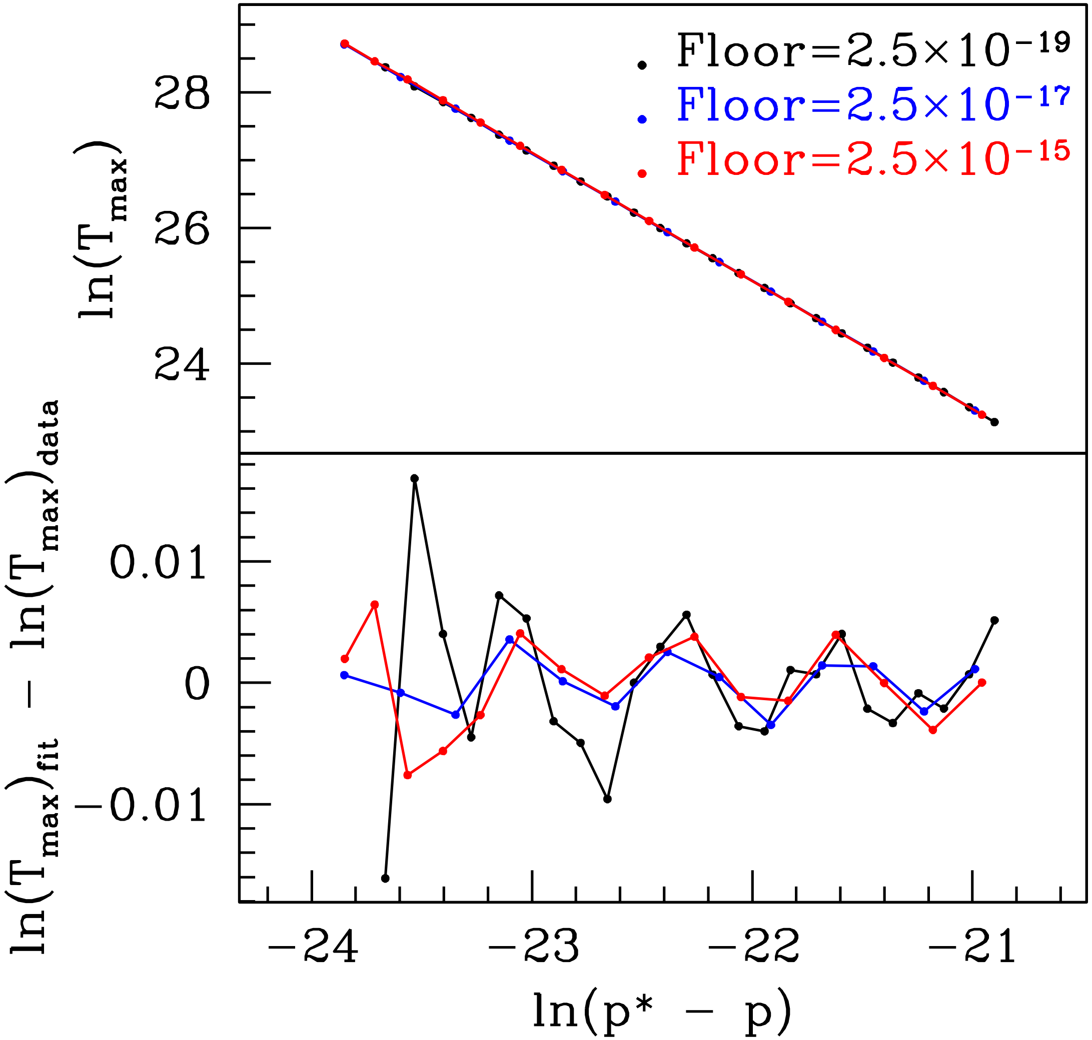


- Comparison of dimensionless quantities:
  - $\omega \equiv 4\pi r^2 a^2 \rho$
  - $a = \sqrt{g_{rr}}$
  - $v = \frac{au^r}{\alpha u^t}$ = Eulerian Velocity

    (u^μ = Fluid's 4-velocity)
- Star: $\rho_c = 0.05$
- Ultra-relativistic fluid:

  Initial profile = Gaussian

Scaling of $T_{\max}$: Dependence on Fluid's Floor

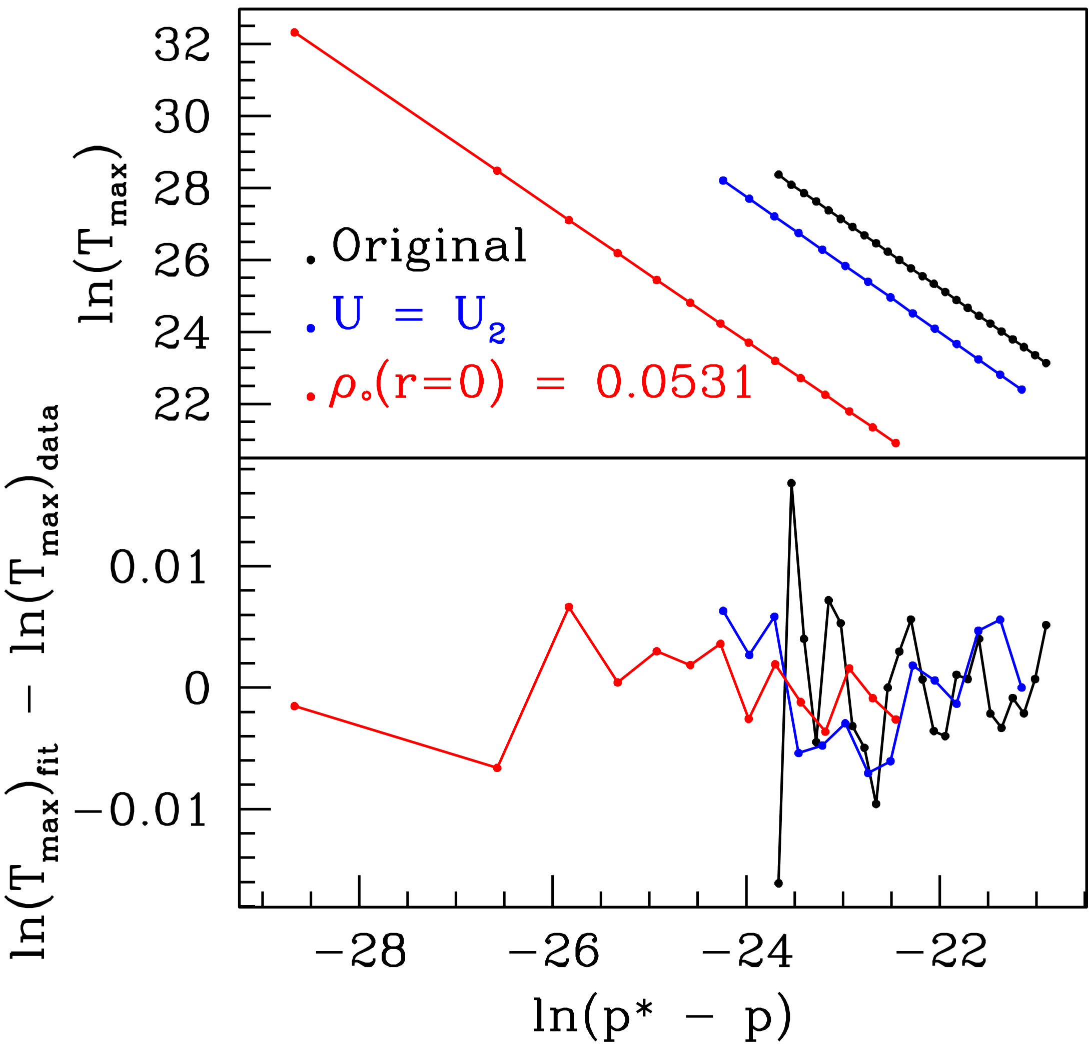


γ	p^*
0.9427	0.46875367383
0.9436	0.46875350285
0.9470	0.4687516089

- Floor used to prevent

  $v \geq 1 \quad , \quad P, \rho_\circ < 0$
- No significant effect;

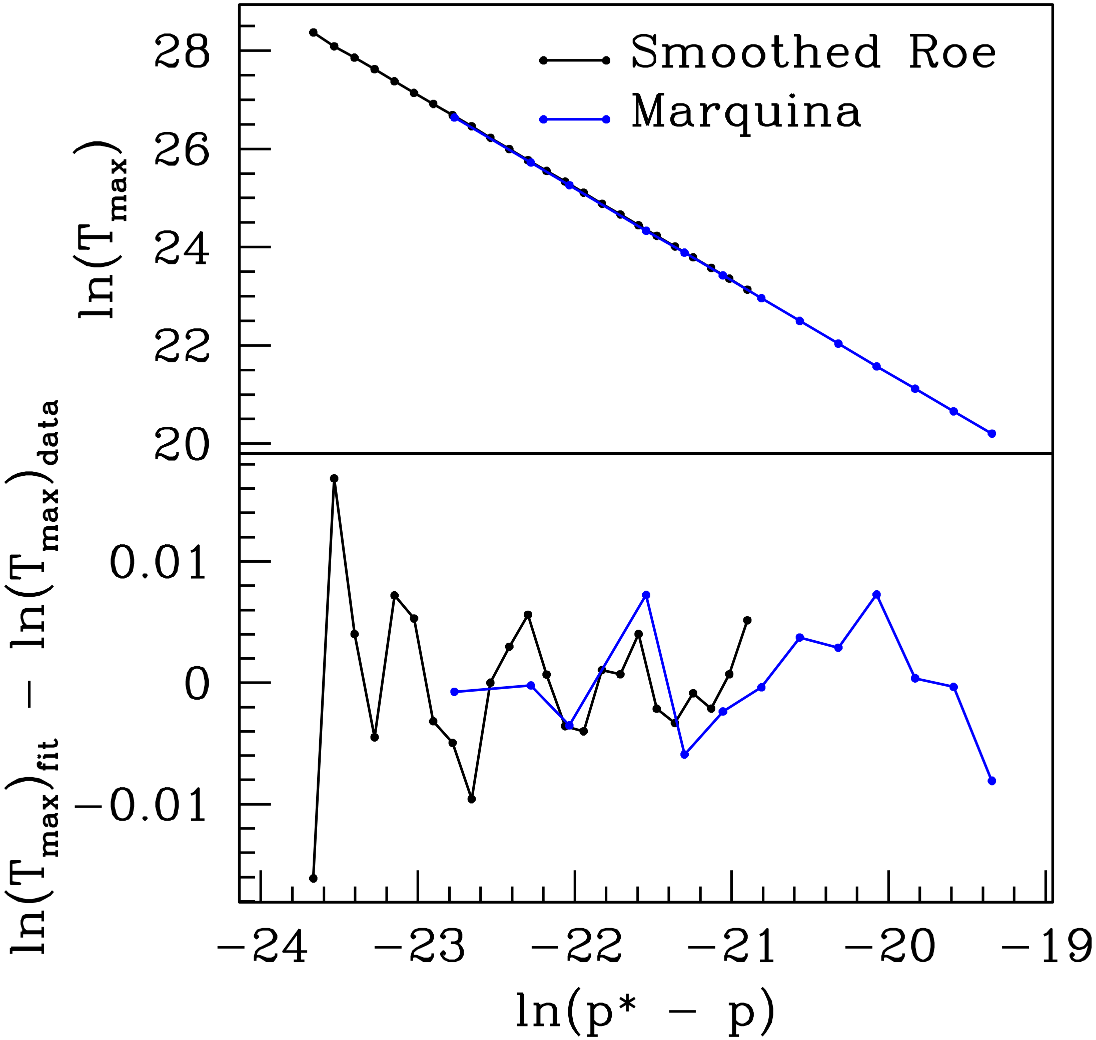
Scaling of $T_{\max}$: Different "Families"



γ	p^*
0.9427	0.46875367383
0.9423	0.42990315097
0.9187	0.4482047429836

- Suggests scaling is fairly independent of:
  - Functional form of perturbation;
  - Initial star configuration;

Scaling of $T_{\max}$: Different Flux Functions



γ	p^*
0.9427	0.46875367383
0.9399	0.46876822118

- Suggests scaling is independent of flux formula;
- Able to tune further with "Smoothed" Roe solver;

Comparison of Scaling Parameters

Noble and Choptuik	Ideal gas	$\gamma = 0.94 \pm 0.01$
Noble and Choptuik	Ultra-relativistic fluid	$\gamma = 0.9747$
Neilsen and Choptuik (2000) and Brady et al. (2002)	Ultra-relativistic fluid	$\gamma = 0.95 \pm 0.02$
Novak (2001)	Ideal gas	$\gamma \simeq 0.52$

Conclusion

- Parameter Space Survey:
  - Illuminated possible dynamical scenarios
  - Provided a backdrop for critical phenomena studies
- Type I Behavior:
  - Critical solutions $\simeq$ perturbed unstable TOV solutions
  - Found anticipated scaling behavior $T_\circ \propto \frac{1}{\omega_{Ly}} \ln |p - p^*|$
  - $\omega_{Ly} \propto \rho_c^*$
- Type II Behavior:
  - Ideal gas critical solution $\simeq$ ultra-relativistic critical solution
  - $\gamma_{\mathrm{ideal}} \simeq \gamma_{\mathrm{ultra-rel}}$

Future Work

- Type I Phenomena:
  - Compare results to ω_{Ly} of unstable TOV growing modes
  - Axially-symmetric collapse, effect of rotation
  - How $\omega_{Ly}(\rho_c^*)$ varies with Γ
  - Dependence on EOS
- Type II Phenomena:
  - Realistic equation of state
  - Axially-symmetric critical behavior
  - Develop general adaptive mesh refinement methods for relativistic fluids