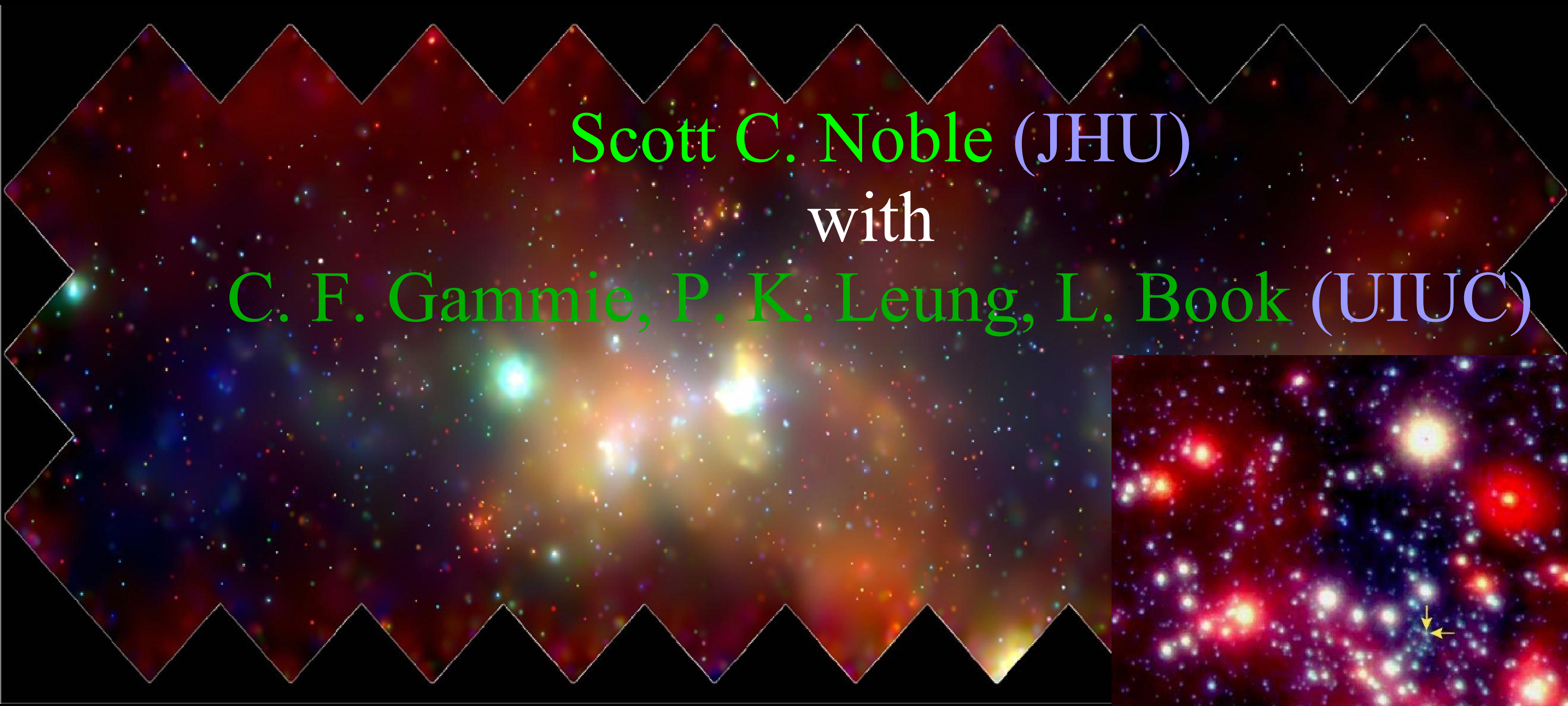
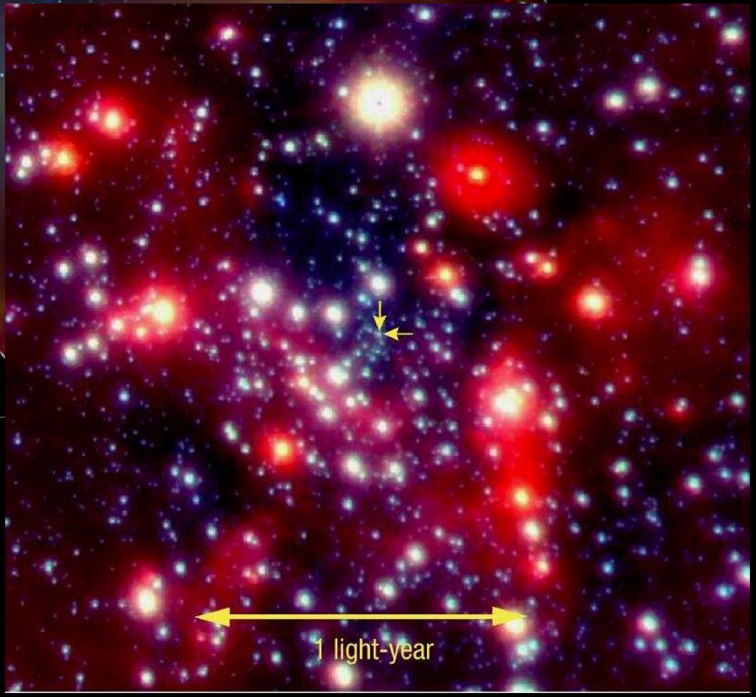
Simulating grMHD Accretion Disks and their Emission



Scott C. Noble (JHU)

with

C. F. Gammie, P. K. Leung, L. Book (UIUC)



Relativity Group, LSU

Nov. 30th, 2006

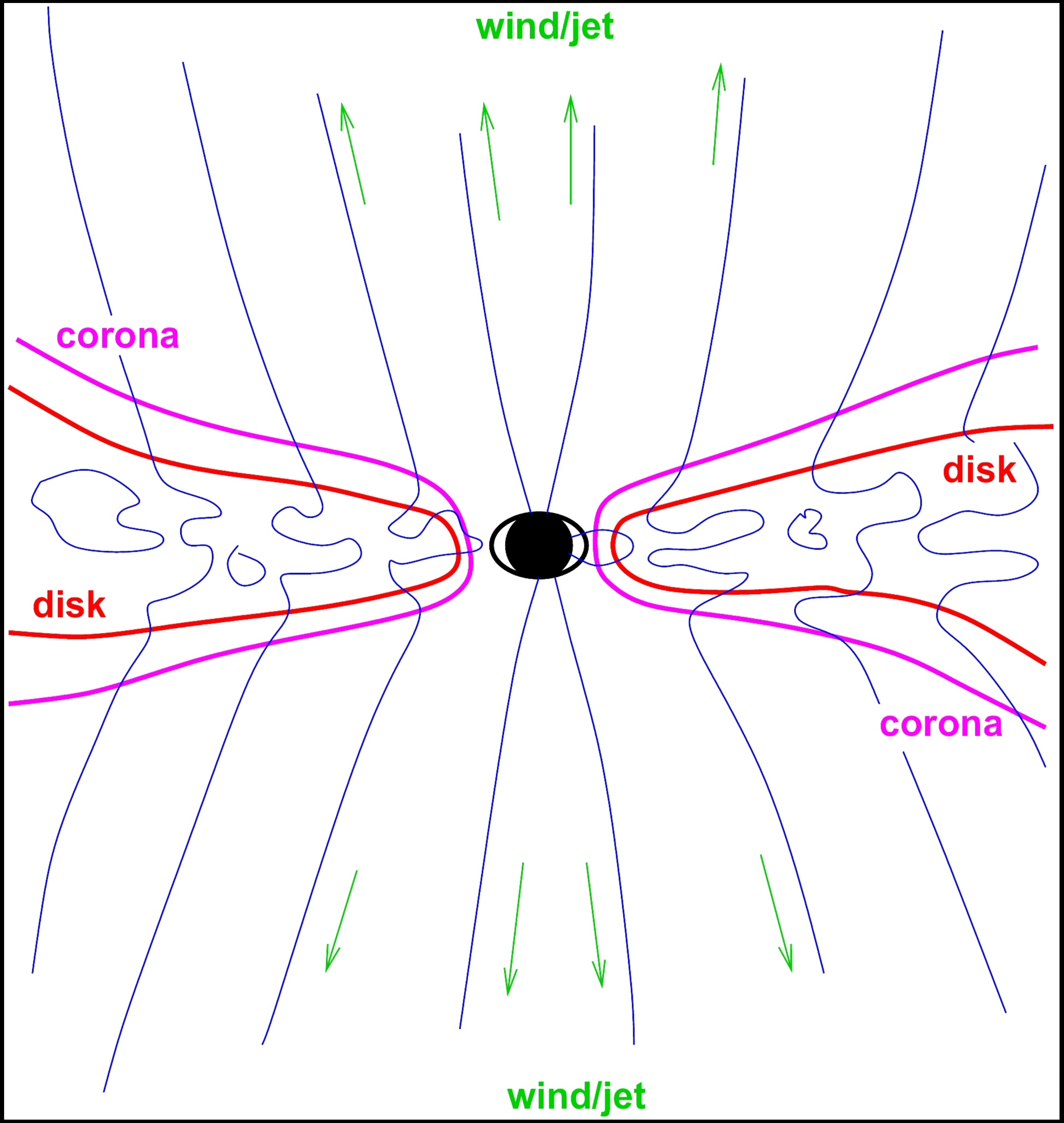
Outline:

- Brief Intro. to Astrophysical Disks
- Disk Simulations
- Radiative Transfer in GR
- The Galactic Center: Sagittarius A* (Sgr A*)
  - Observations and Theory
- Our Calculation of Sgr A*'s Emission
- Summary & Future Work

Astrophysical Disks

Disk Type	Gravity Model
Galaxies, Stellar Disks, Planetary Disks	Newtonian
X-ray binaries, AGN	Stationary metric
Collapsars, SN fall-back disks	Full GR

- MRI explains ang. mom. transfer whenever the disk is ionized!


wind/jet
corona
disk
disk
corona
wind/jet

HARM (Gammie, McKinney, Toth 2003)

- Axi-sym. grMHD on Kerr-Schild
- HLL flux or LF-like (sim. to Kurganov-Tadmor) flux
- MC, minmod slope limiters
- Constrained Transport: flux-CT (Toth 2000)
- "5D" or full inversion for primitive variables
- Simple MPI domain decomposition
- Specialized coordinates focus cells near equator

Download at:
http://rainman.astro.uiuc.edu/codelib/

Primitive Variable Inversion

Noble, Gammie, McKinney, Del Zanna *ApJ* **641** 626 (2006)

$$Q_\mu = -n_\nu T^\nu{}_\mu \qquad \tilde{Q}_\mu = (g_{\mu\nu} + u_\mu u_\nu) Q^\nu$$

$$\tilde{Q}^\mu = \frac{1}{\gamma}(W + B^2)\tilde{u}^\mu - \frac{(u.B)B^\mu}{\gamma}$$

$$\tilde{Q}^2 = v^2 (B^2 + W)^2 - \frac{(Q.B)^2 (B^2 + 2W)}{W^2}$$

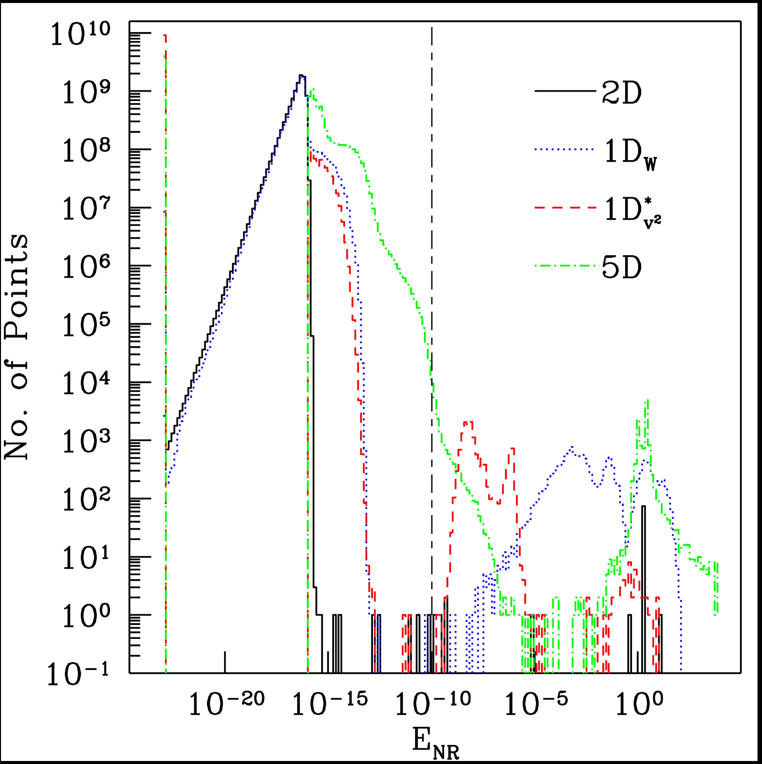
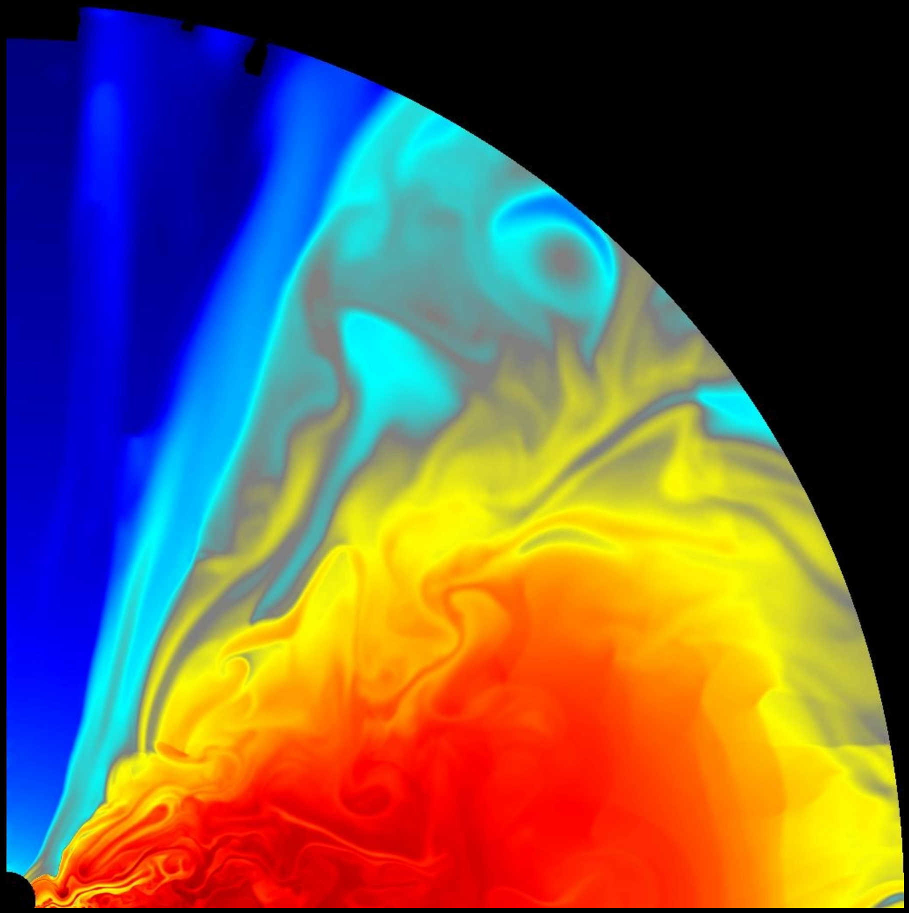
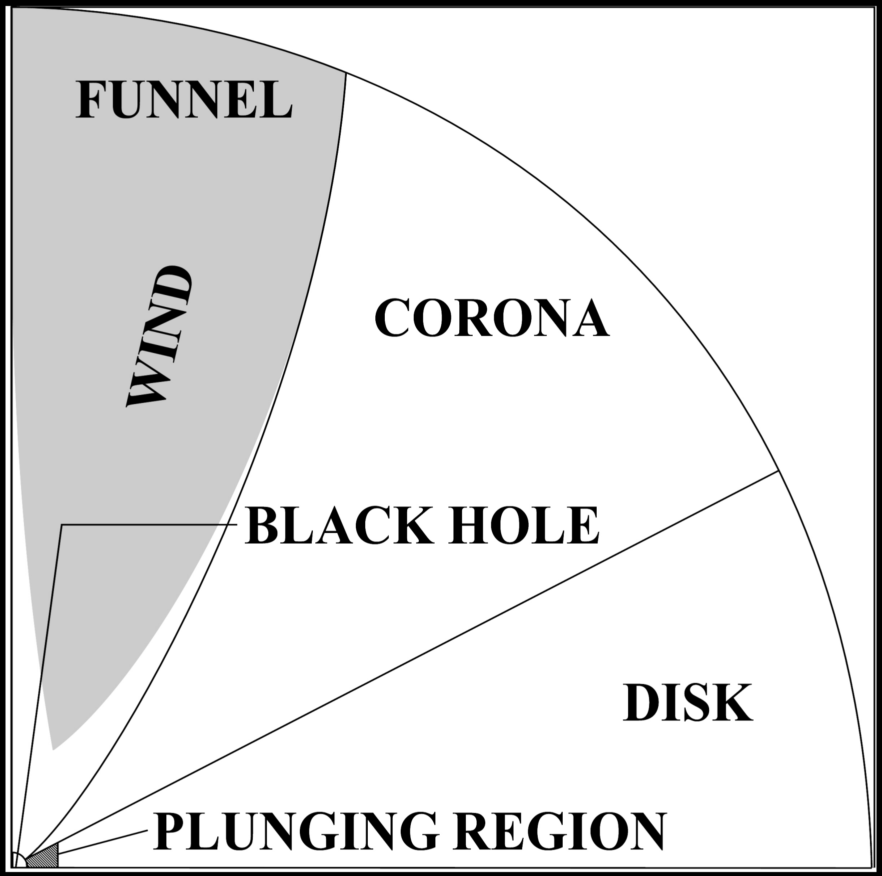


Table 6. Accretion Disk Efficiency Comparison

Method	NR steps per sol.	Zone-cycles/node/sec. [a]	Failure Rate
2D	4.19	24535	9.57×10^{-5}
$1D_W$	4.18	23860	9.33×10^{-5}
$1D^\star_{v^2}$	5.22	20585	9.46×10^{-5}
5D	4.52	14741	9.22×10^{-5}

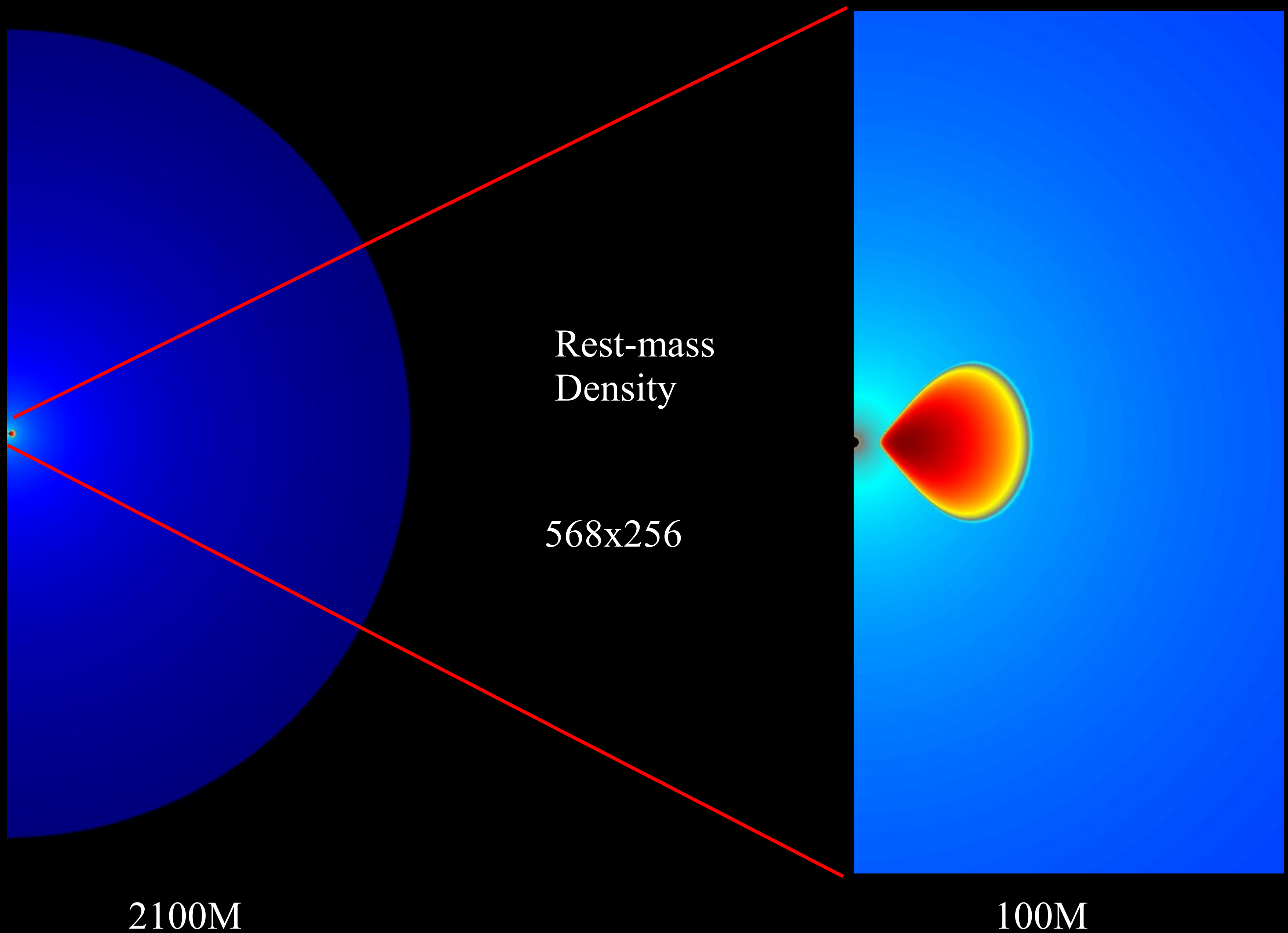
Disk Morphology



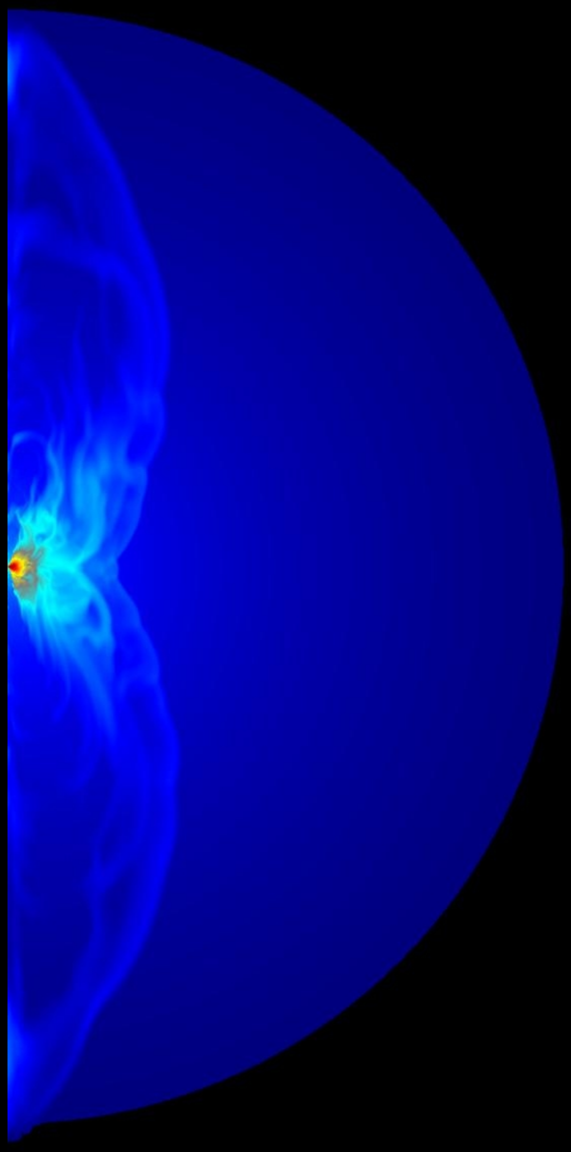


McKinney & Gammie (2004)
Hawley, De Villiers, Krolik, Hirose 2003+

Outflows (Noble, Leung, Gammie, Book, **CQG** submitted)



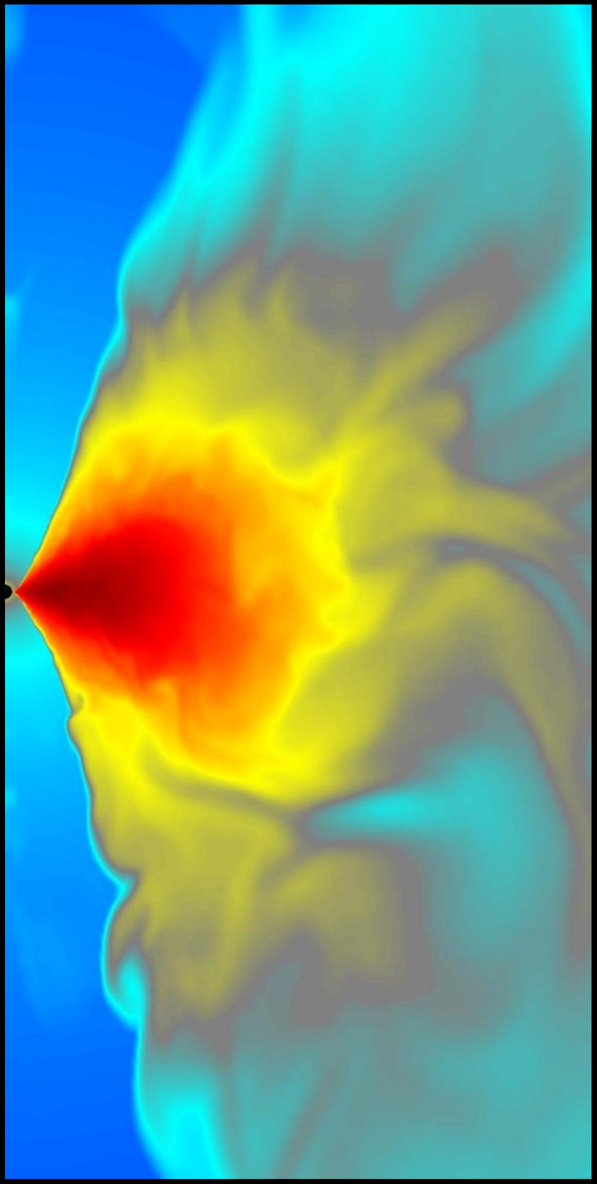
Outflows (Noble, Leung, Gammie, Book, **CQG** submitted)



Rest-mass Density

568x256

t=2900M



2100M

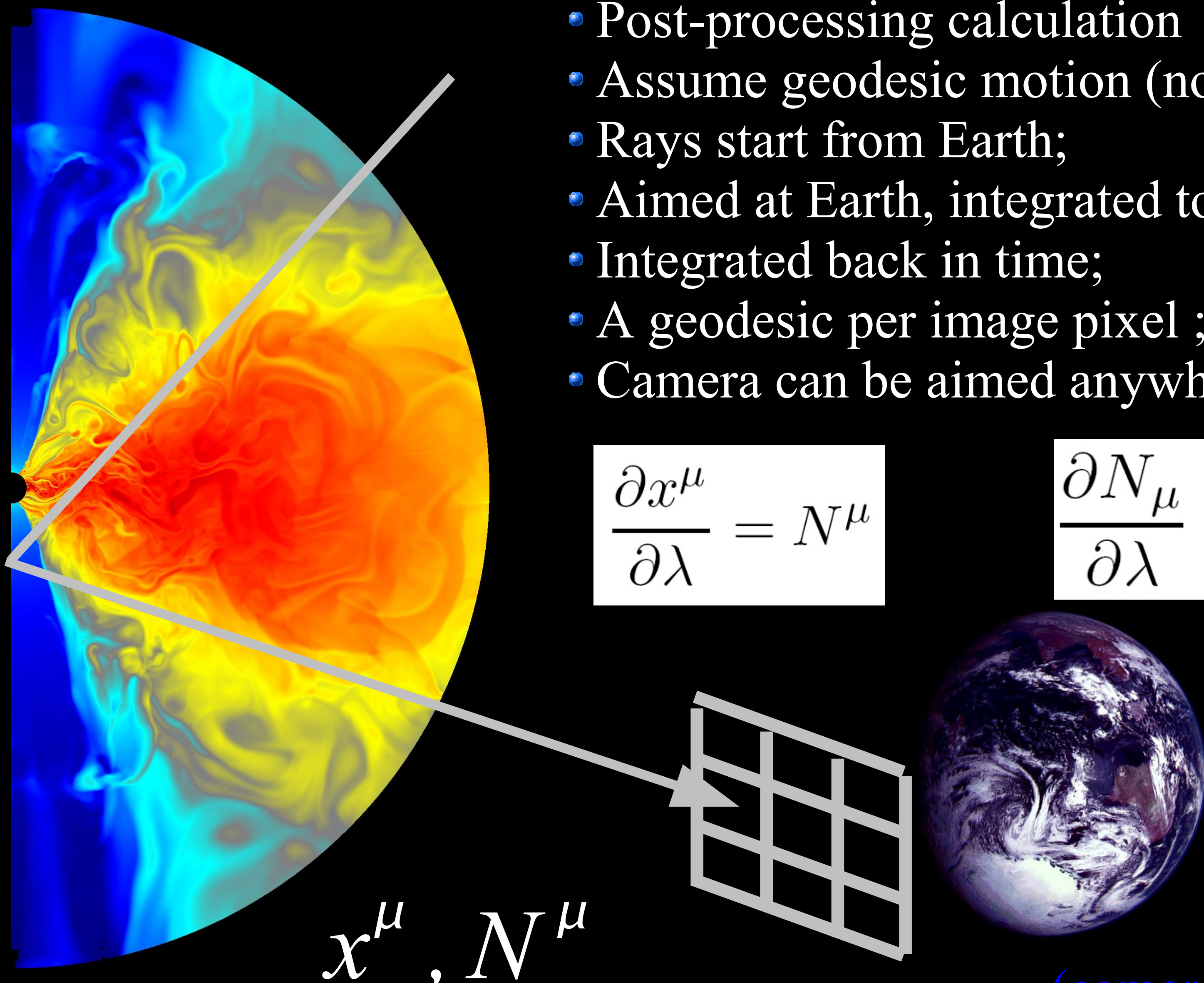
100M

Radiation Transfer in GR: Step #1

- Post-processing calculation
- Assume geodesic motion (no scattering):
- Rays start from Earth;
- Aimed at Earth, integrated toward “SgrA*”;
- Integrated back in time;
- A geodesic per image pixel ;
- Camera can be aimed anywhere at any angle;

$$\frac{\partial x^\mu}{\partial \lambda} = N^\mu$$

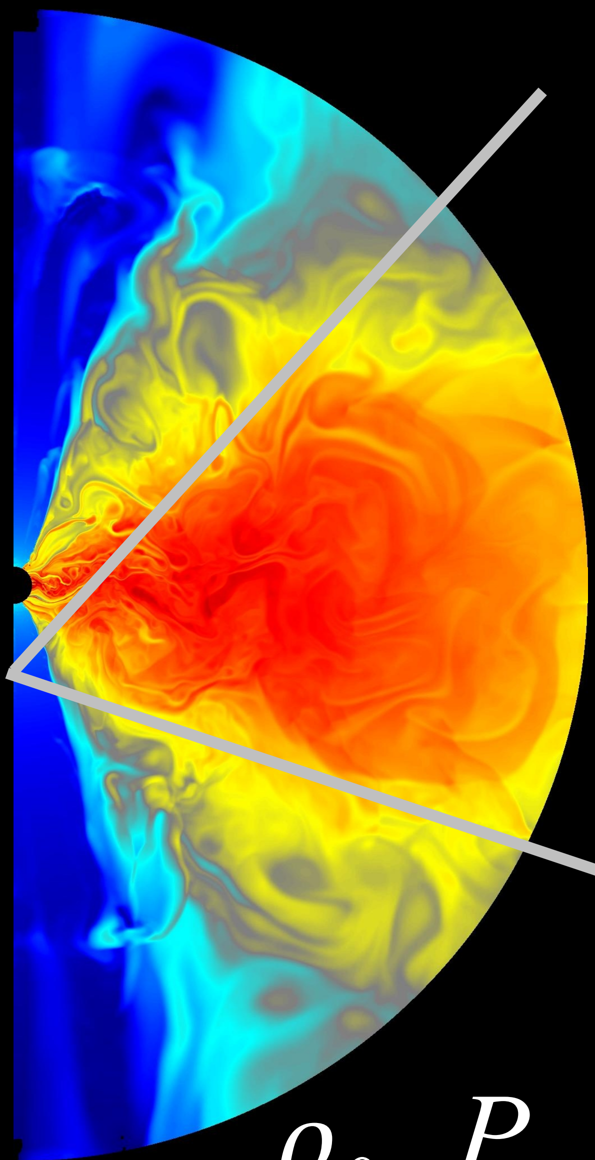
$$\frac{\partial N_\mu}{\partial \lambda} = \Gamma^\nu{}_{\mu\eta} N_\nu N^\eta$$

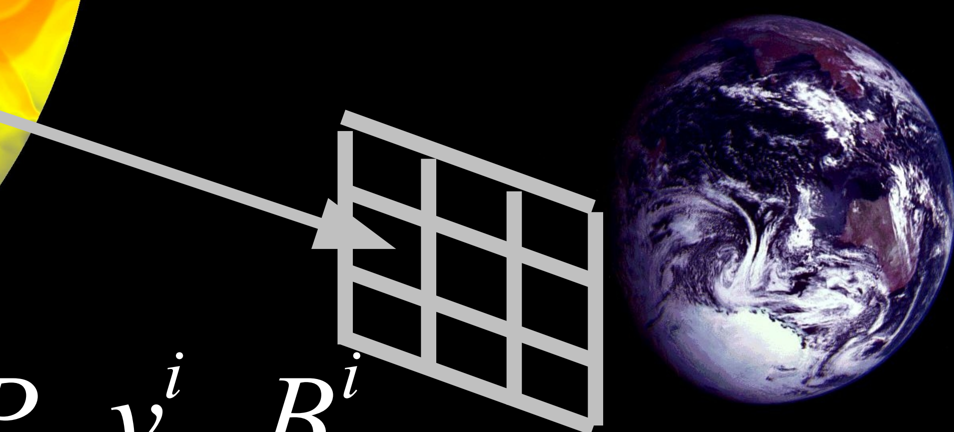


(camera not shown to scale)

Radiation Transfer in GR: Step #2

- Interpolate simulation data along ray
  - 256x256 HARM runs
  - a = 0, 0.5, 0.75, 0.88, 0.93, 0.97
  - R_{out} = 40M, P_{max} at 10-15M
- Spatially interpolate single timeslice per image
  - Assume $t_{dyn} >> t_{crossing}$
- Set units s.t. at 1mm : $Flux_{num} = Flux_{obs}$





ρ_0, P, v^i, B^i

(camera not shown to scale)

Radiation Transfer in GR: Step #3

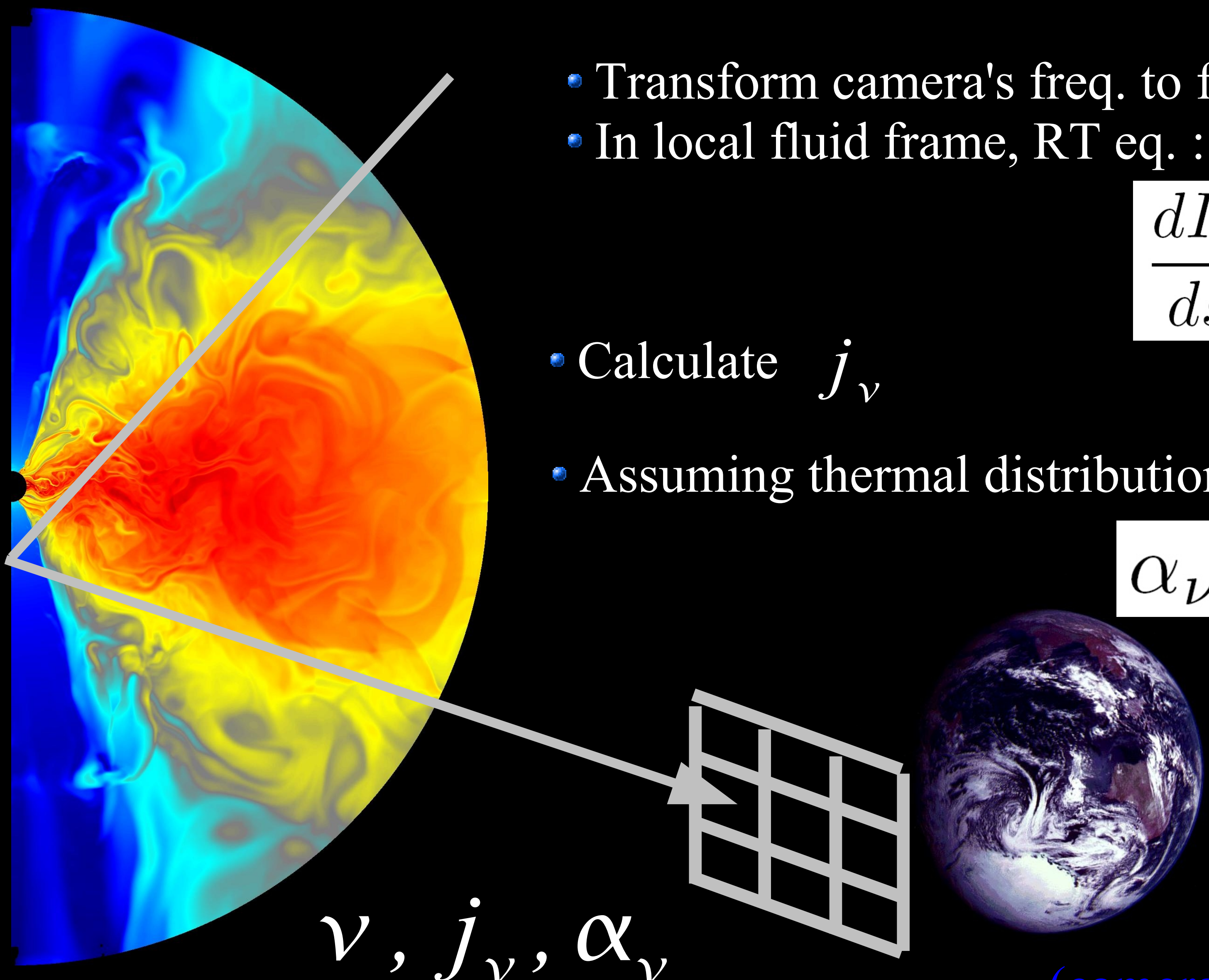
- Transform camera's freq. to fluid frame: ν
- In local fluid frame, RT eq. :

$$\boxed{\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu}$$

- Calculate j_ν

- Assuming thermal distribution of electrons:

$$\boxed{\alpha_\nu = j_\nu / B_\nu}$$



(camera not shown to scale)

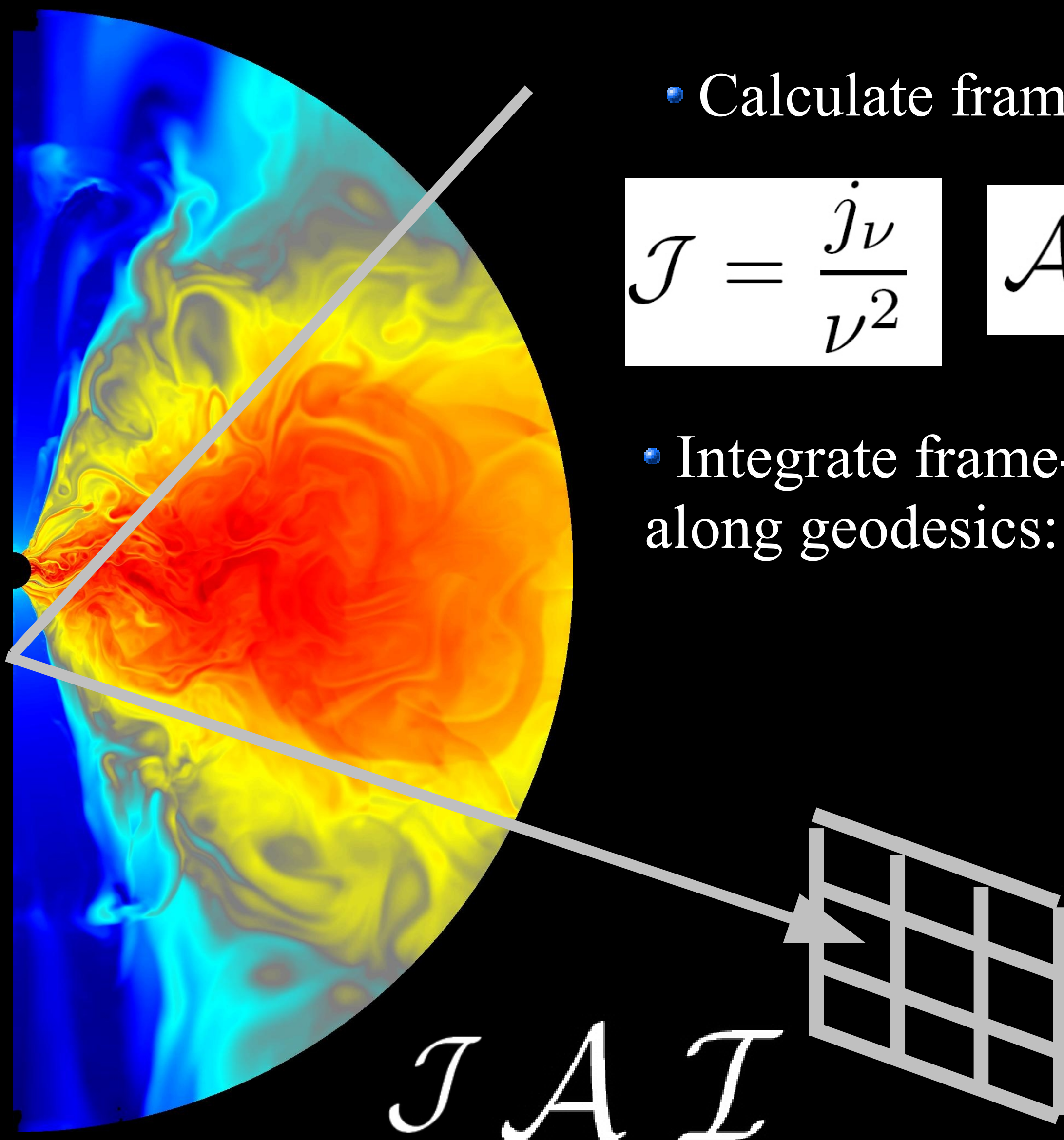
Radiation Transfer in GR: Step #4

- Calculate frame-independent quantities:

$$\mathcal{J} = \frac{j_\nu}{\nu^2} \qquad \mathcal{A} = \nu \alpha_\nu \qquad \mathcal{I} = I_\nu / \nu^3$$

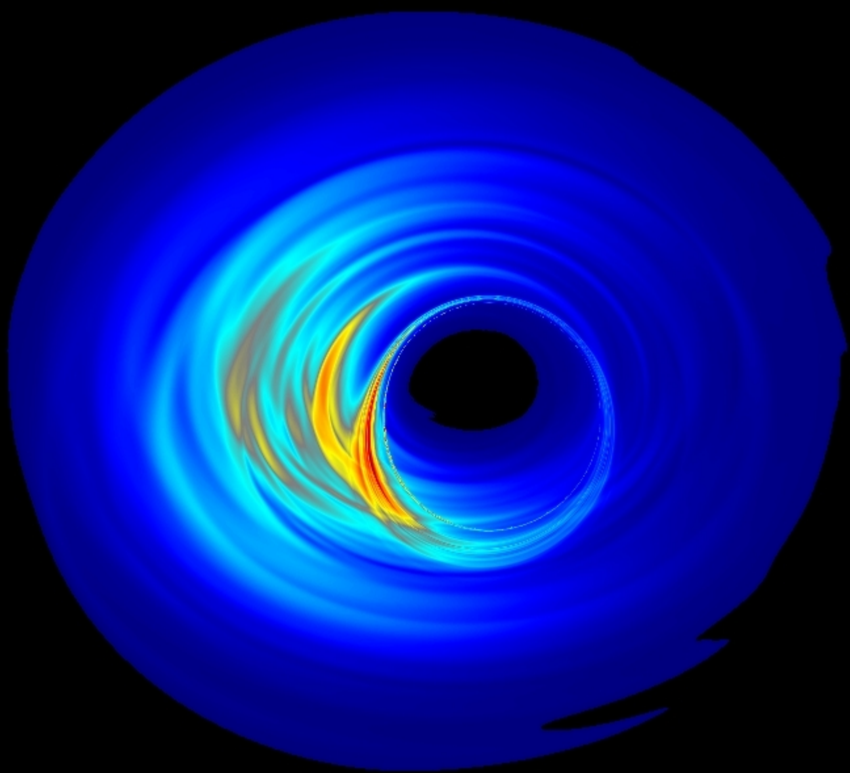
- Integrate frame-independent RT equation along geodesics:

$$\frac{d\mathcal{I}}{d\lambda} = \mathcal{J} - \mathcal{A}\mathcal{I}$$



(camera not shown to scale)

Example Image



1024x1024 pixels

$$\boxed{\nu_{obs}\,,\,\dot{M}\,,\,a\,,\,\theta_{inc}}$$

$$\nu_{obs}=3\text{x}10^{11}\,Hz\,(1\text{mm})$$

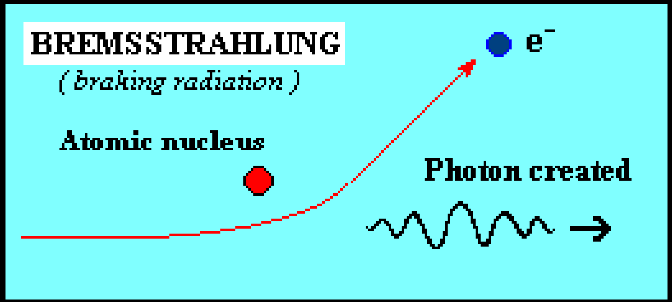
$$\dot{M}=5\text{x}10^{-9}\,M_{sun}\,yr^{-1}$$

$$a=0.94$$

$$\theta_{inc}=30\,degrees$$

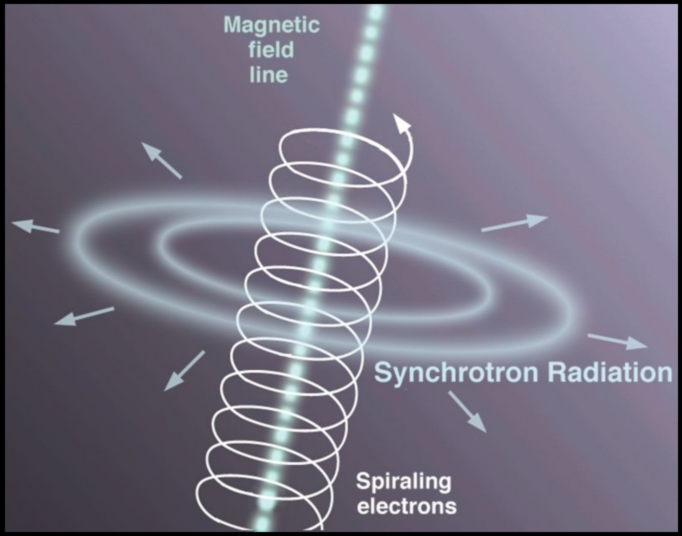
Radiation from Plasmas j_ν

Bremsstrahlung:



- Isotropic in fluid frame
- Easy to calculate from ρ, u^μ, T_e

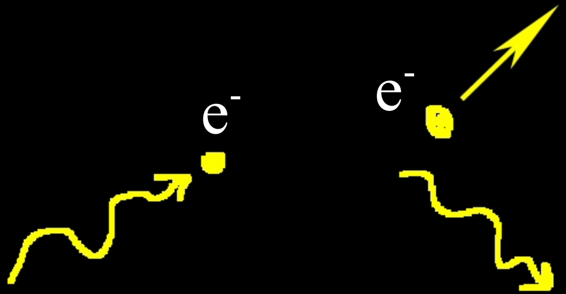
Synchrotron:



- Highly anisotropic
- Hard to calculate exactly
- Approximate methods work well
- Most use angle-averaged equations
- Need: $B^i, \rho, u^\mu, T_e, N^\mu$

Compton Scattering (inverse):

- Anisotropic, up-scatters incident light
- VERY hard to calculate in general (Monte Carlo)

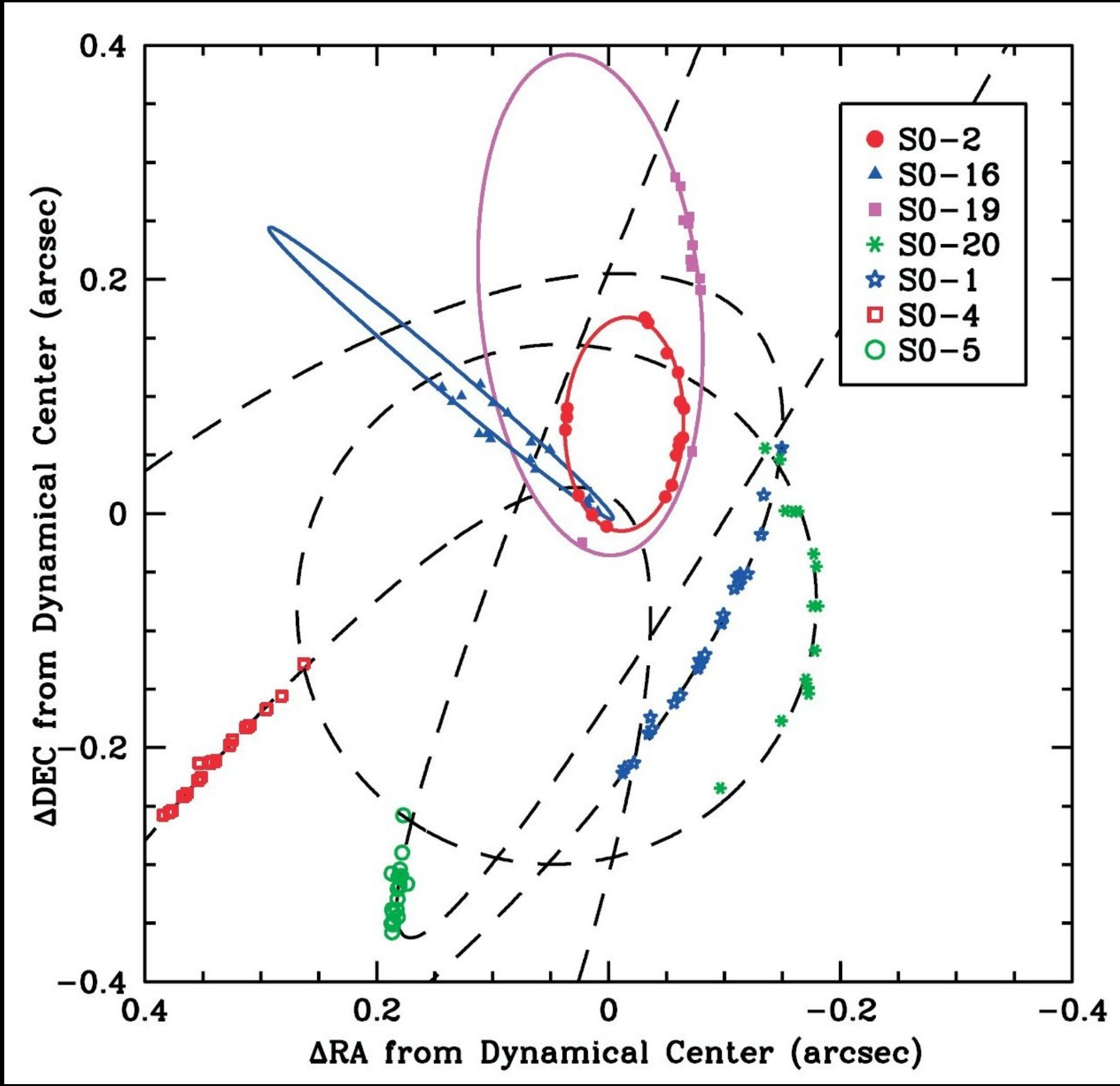


Sagittarius A* (Sgr A*)

NASA/UMass/D.Wang et al. (Chandra)
120x48 arcmin or 900x400 light-year

How Big is it?

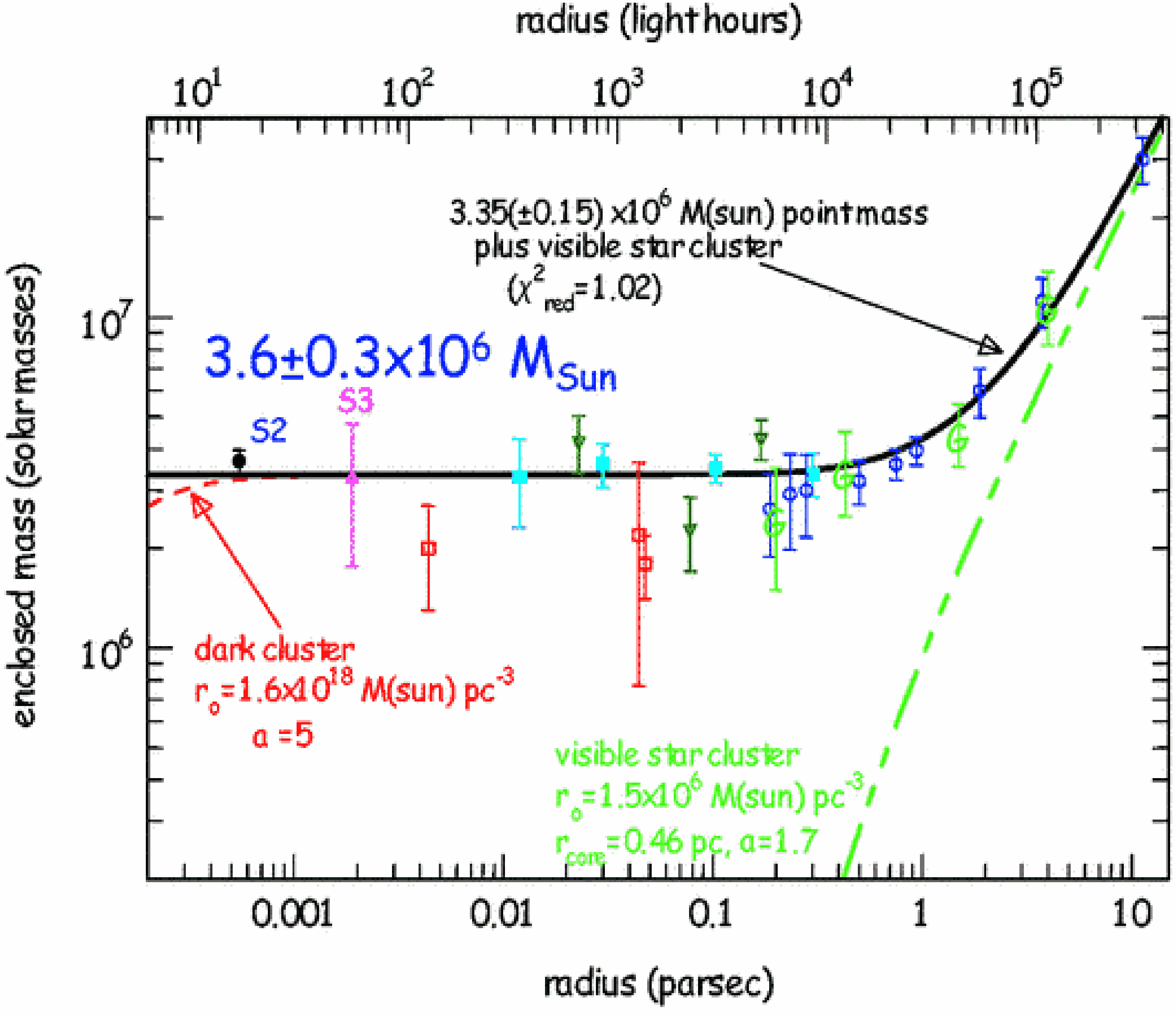
- Ghez et al. 2005 (UCLA)
  - New Keck diffraction limited observation, adaptive optics
  - Simultaneous 6-orbit fit
  - $M_{SgrA*} = 3.7 +/- 0.2 \times 10^6 M_{Sun}$
- Genzel et al., Nature, 2003
- Eisenhauer et al. 2005 (MPE/UCB)
  - ESO/VLT, adaptive optics
  - $M_{SgrA*} = 3.6 +/- 0.3 \times 10^6 M_{sun}$
  - $R_o = 7.6 +/- 0.3$ kpc
  - Biggest black hole on the sky!!!
  - Best chance at seeing horizon!!



Ghez et al. 2005

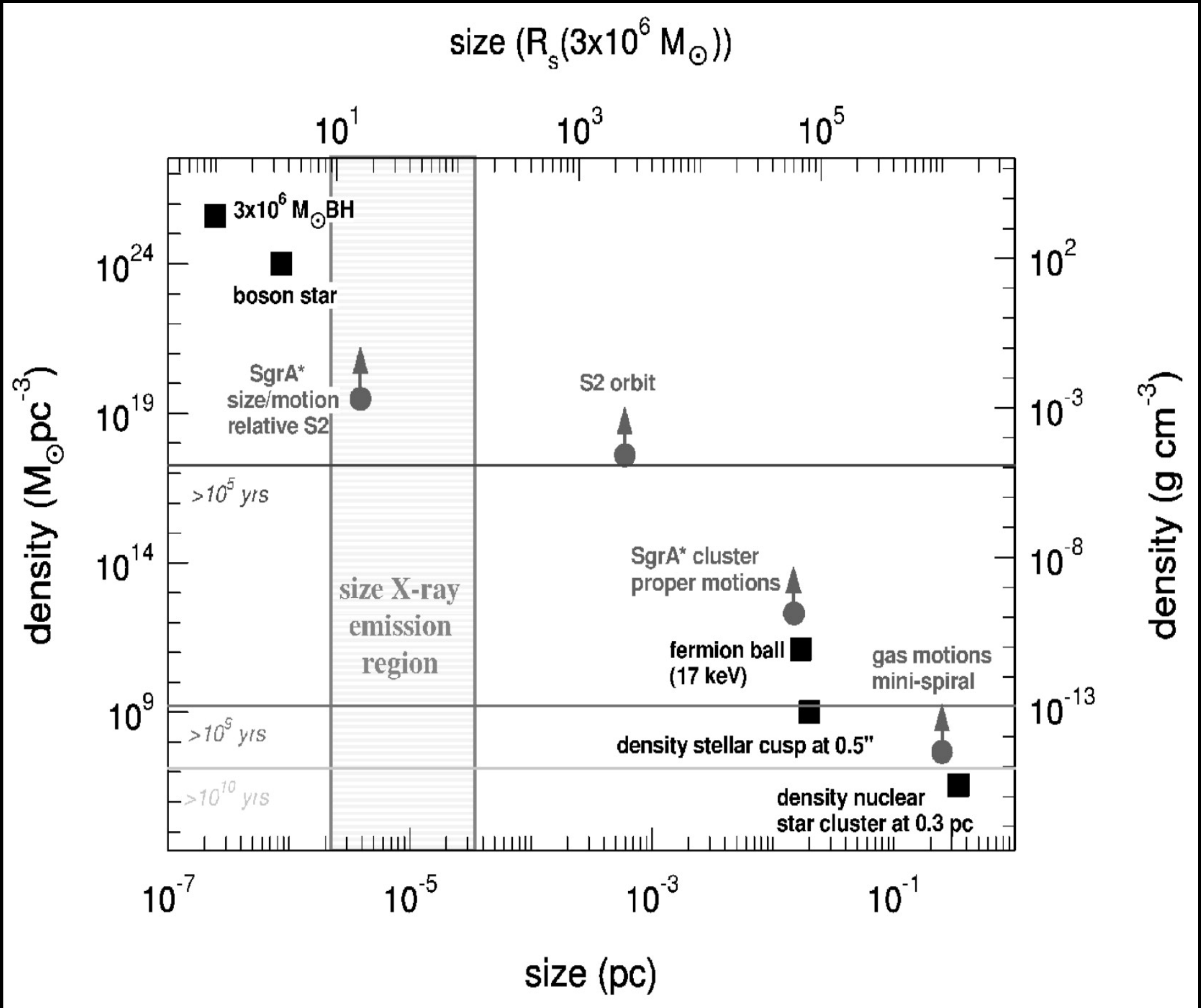
$$r_s = 1\times 10^{12} cm = 3.6\times 10^{-7} pc = 0.07\ AU$$

It's (probably) a Black Hole



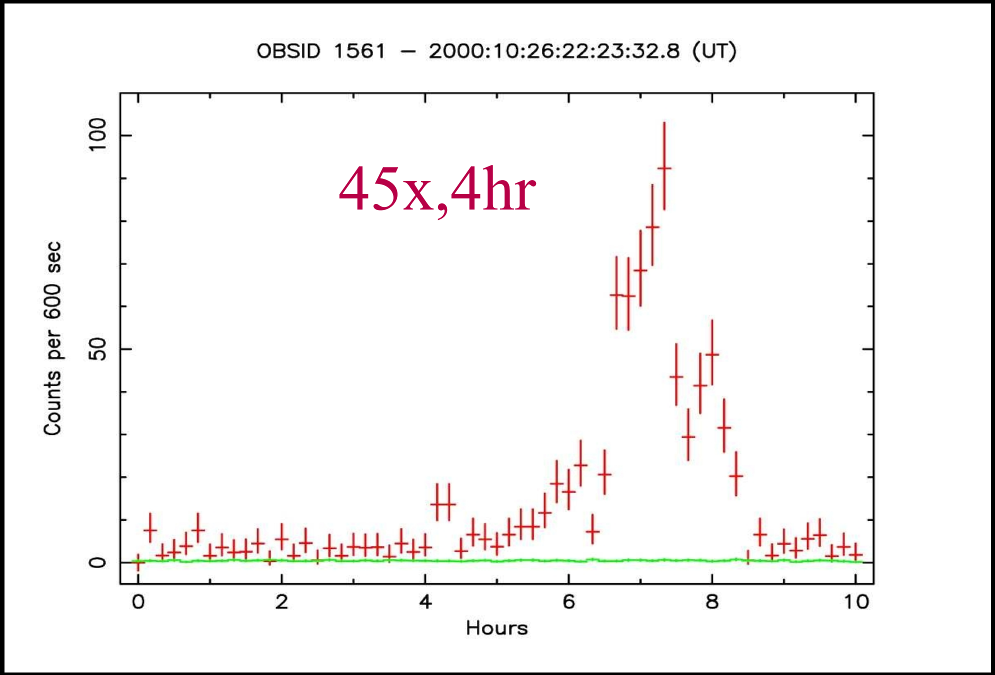
Schödel et al. 2002, 2003, Ghez et al. 2003, Eisenhauer et al. 2003, 2005

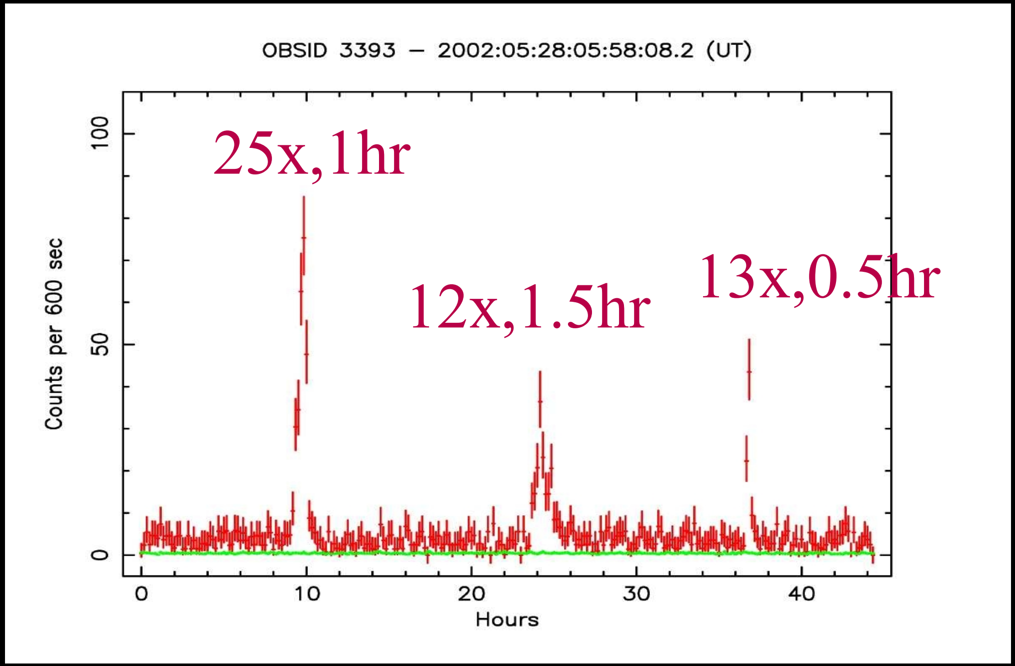
It's (probably) a Black Hole



- Very few possible compact sources
- Who's seen a scalar boson anyway?
- Spectra fits well with jet & accretion models
- Some spectra features seem to indicate variability < 10 R_s
- Dark star clusters are short lived

X-Ray Variability





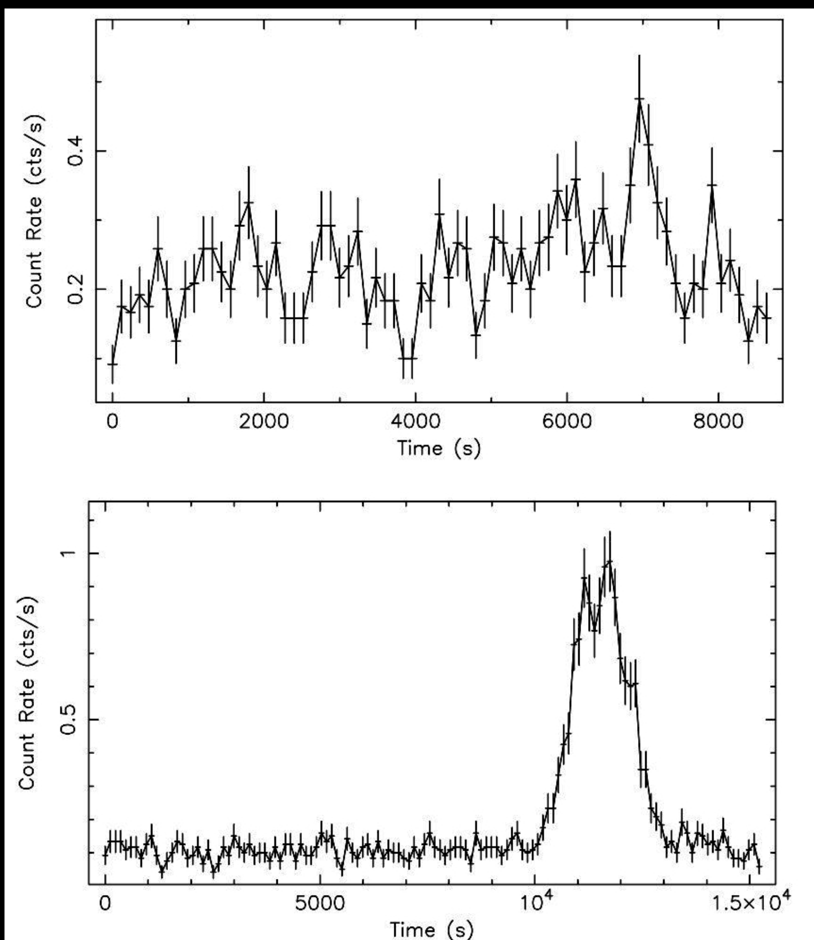
1hr variability --> ~20 R_s

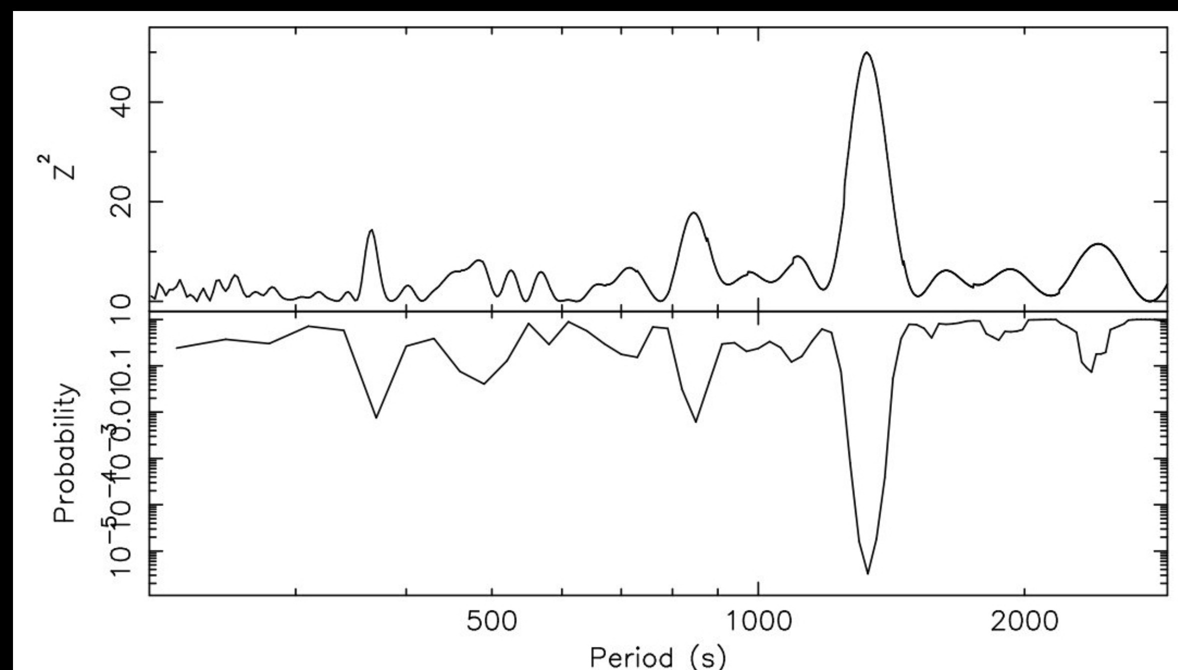
Baganoff et al. (2000-2003)

Sgr A*'s Spin

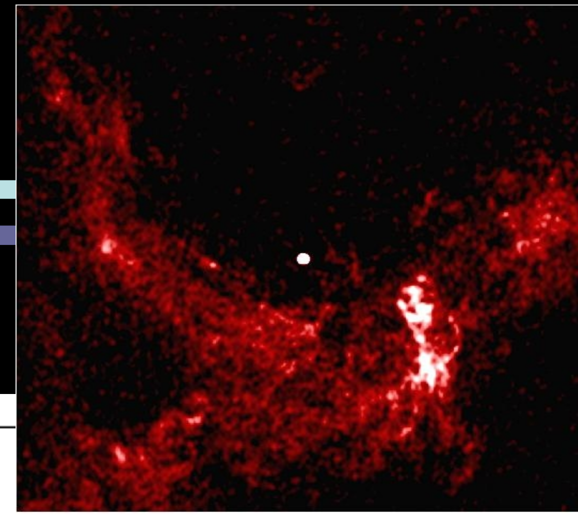
Belanger et al. 2006

22min periodicity (X-ray)
1 in 3 million chance of being random
--> a > 0.22





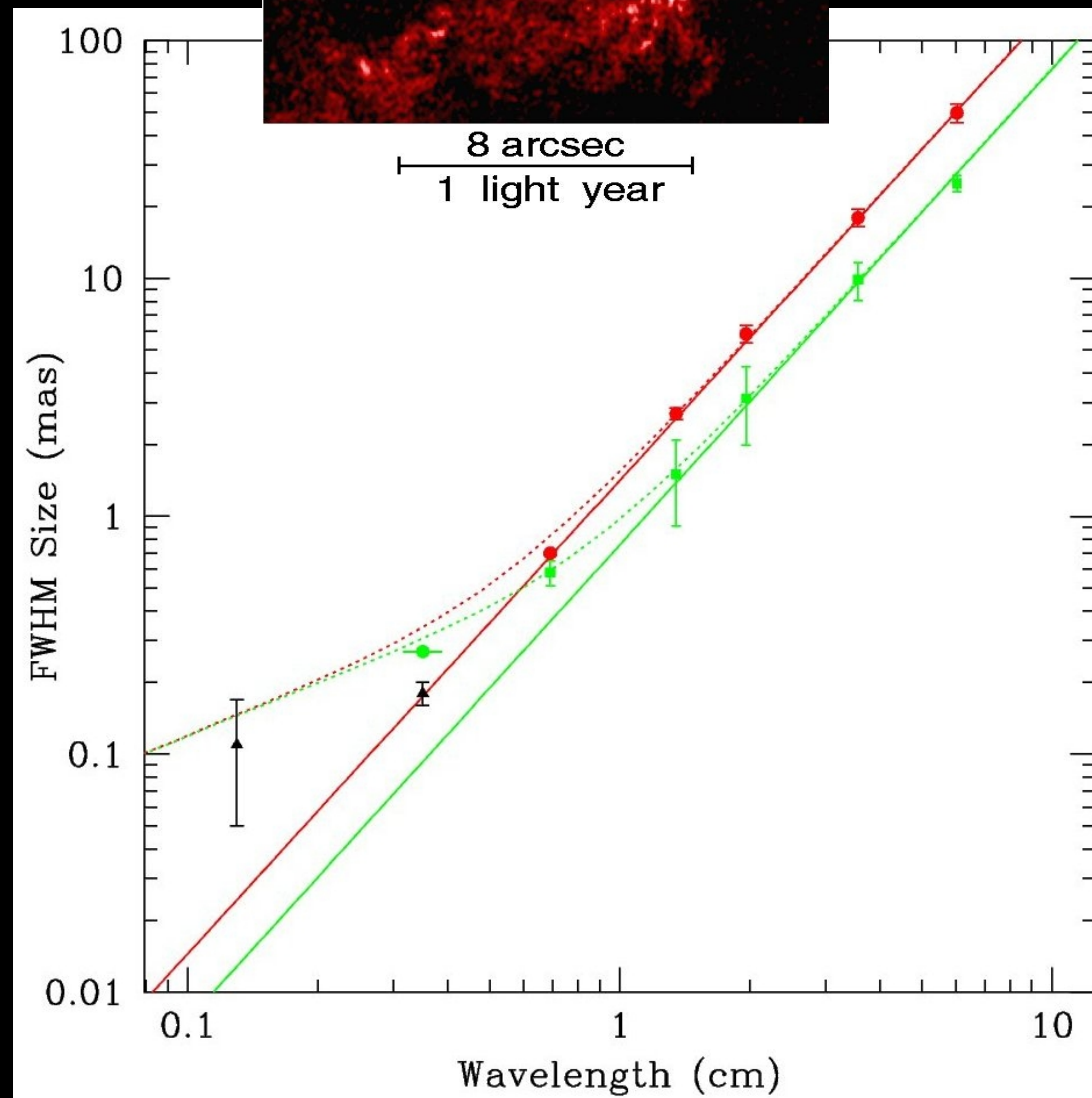
Sgr A* in the Radio



Shen et al. **Nature** (2005)
d < 25 M ~ 2 AU

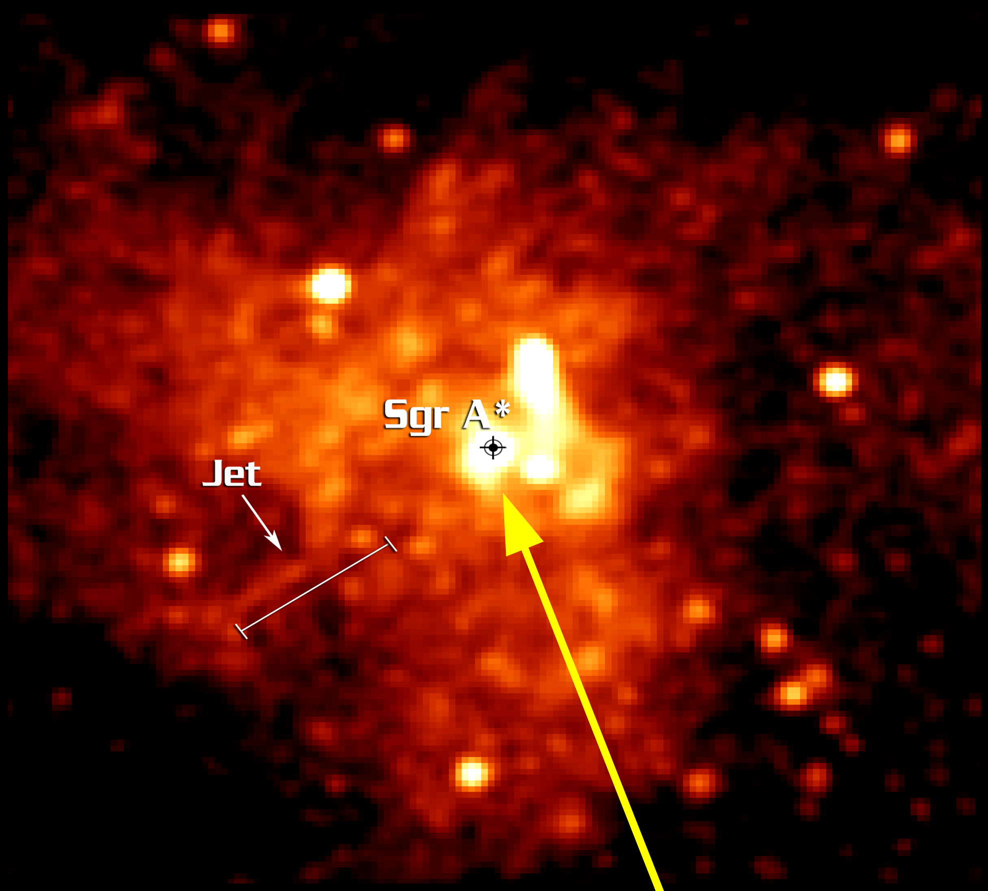
- Shrinking with increasing frequency
- Power also increases with frequency to ~1mm
- Suggests disk may be becoming optically thin with freq.
- At limit of VLBI radio, working on mm VLBI

Want to predict what they'll see!



X-Ray Observations

8.4 arcmin



NASA/CXC/MIT/ F.K. Baganoff et al.

X-Ray and Bondi

Modeling it as kT~1.3keV hot, optically-thin emission:

$$n_e = 30\,cm^{-3}$$

$$c_s^2 = \gamma\, k\, T / \mu\, m = 550\, km/s = v_{wind}$$

$$R_B = 2\, G\, M_{SgrA}/c_s^2 = 0.1\mathrm{pc} = 2.7\, arcsec.$$

$$\rightarrow \quad R_B = 2\, R_{X-rays}$$

The Luminosity Problem

$$L_{SgrA} = 10^{36} erg/s$$

$$L_{Edd} = 4\pi c G M \mu_e/\sigma_T = 1.51\times10^{38}(M/M_{sun})erg/s$$

$$L_{Edd}(M_{Sgr}) = 5.44\times10^{44} erg/s$$

$$\boxed{\rightarrow \quad L_{Sgr}=10^{-8} L_{Edd}}$$

The Luminosity Problem

$$\dot{M}_{X-rays} = 4\pi R_B^2 \rho c_s = 4\times 10^{-5} M_{sun}/yr$$

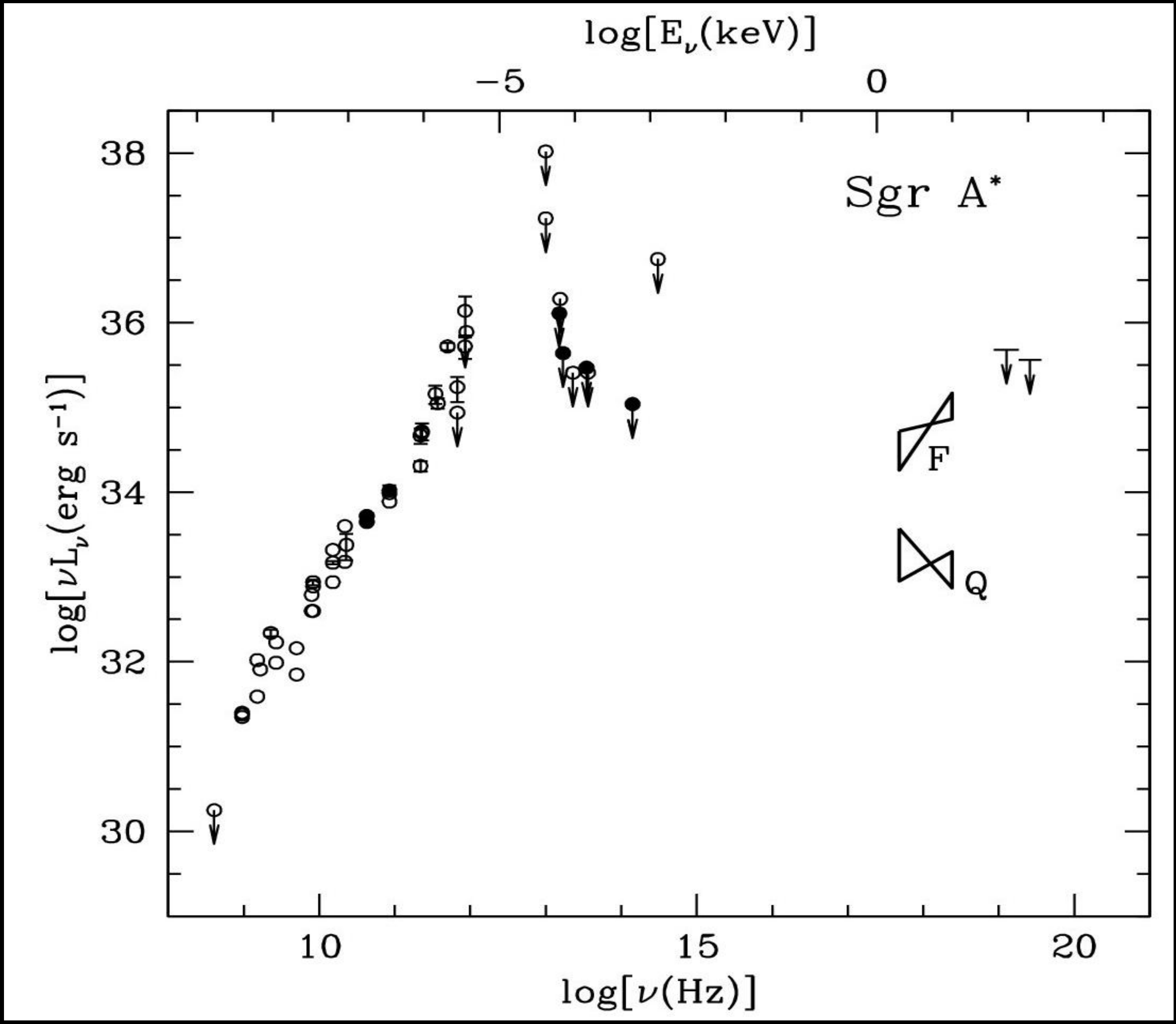
$$\dot{M}_{inner} = 10^{-3}\dot{M}_{X-rays} \rightarrow \eta = 10^{-3}\eta_{thin}$$

$$\boxed{\dot{M}_{X-rays} = 4\pi R_B^2 \rho c_s = 4\times 10^{-5} M_{sun}/yr}$$

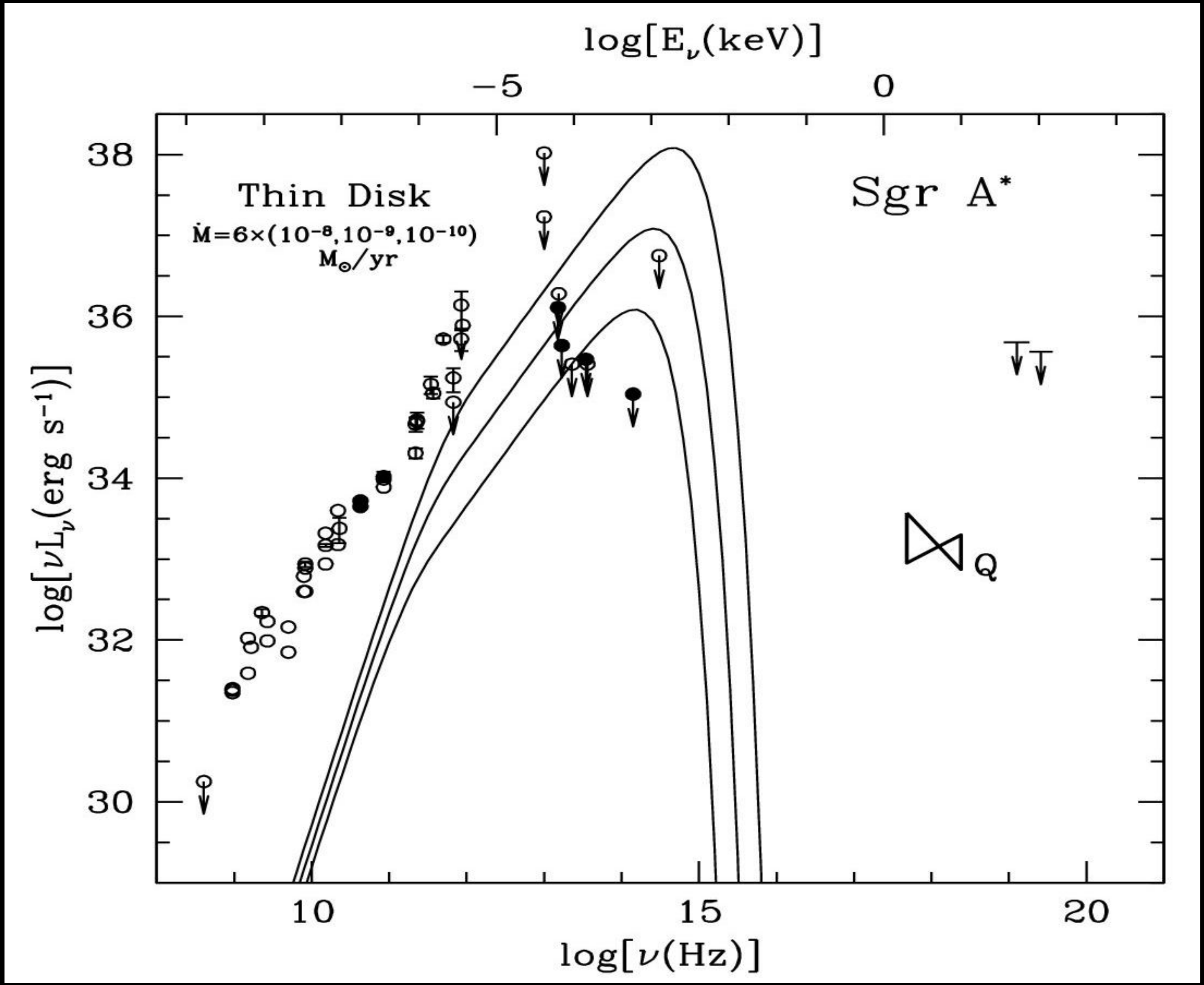
Radio Linear Polarization constraints:

$$\boxed{\dot{M}_{inner} = 10^{-3}\dot{M}_{X-rays} \rightarrow \eta = 10^{-3}\eta_{thin}}$$

Composite Spectrum

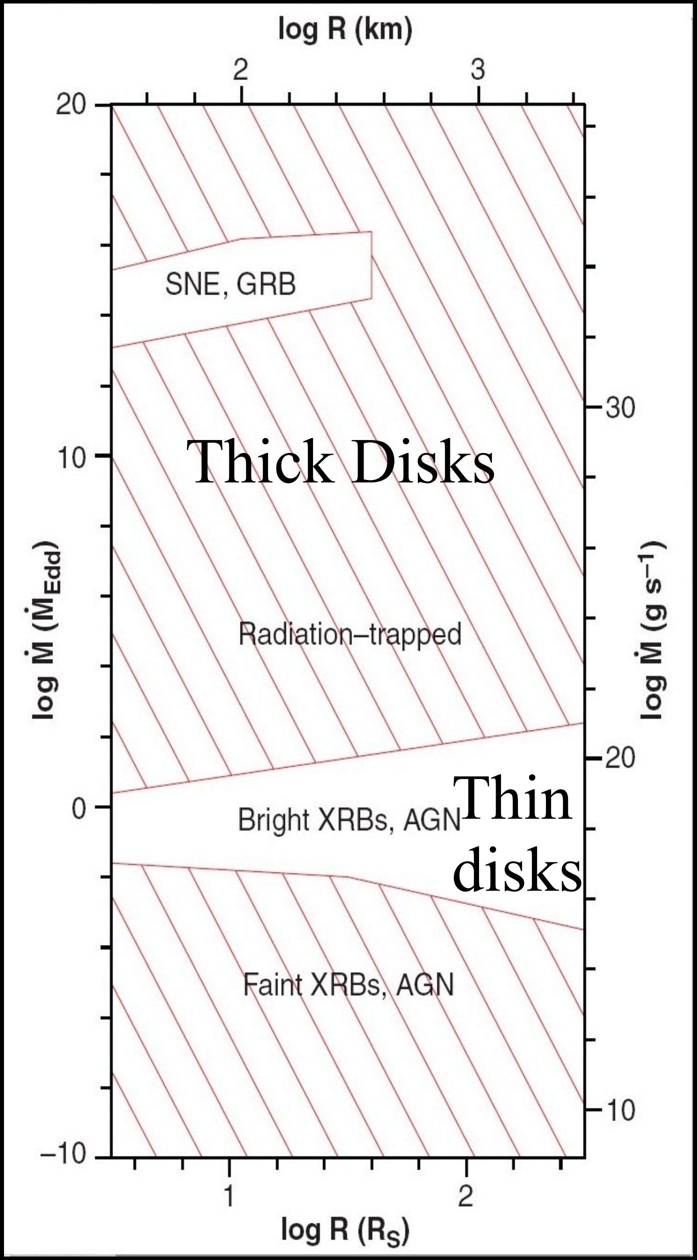


Thermal Thin Disk Spectrum



Narayan astro-ph/0201260

Thick Disks: RIAFs (Radiatively Inefficient Accretion Flows)



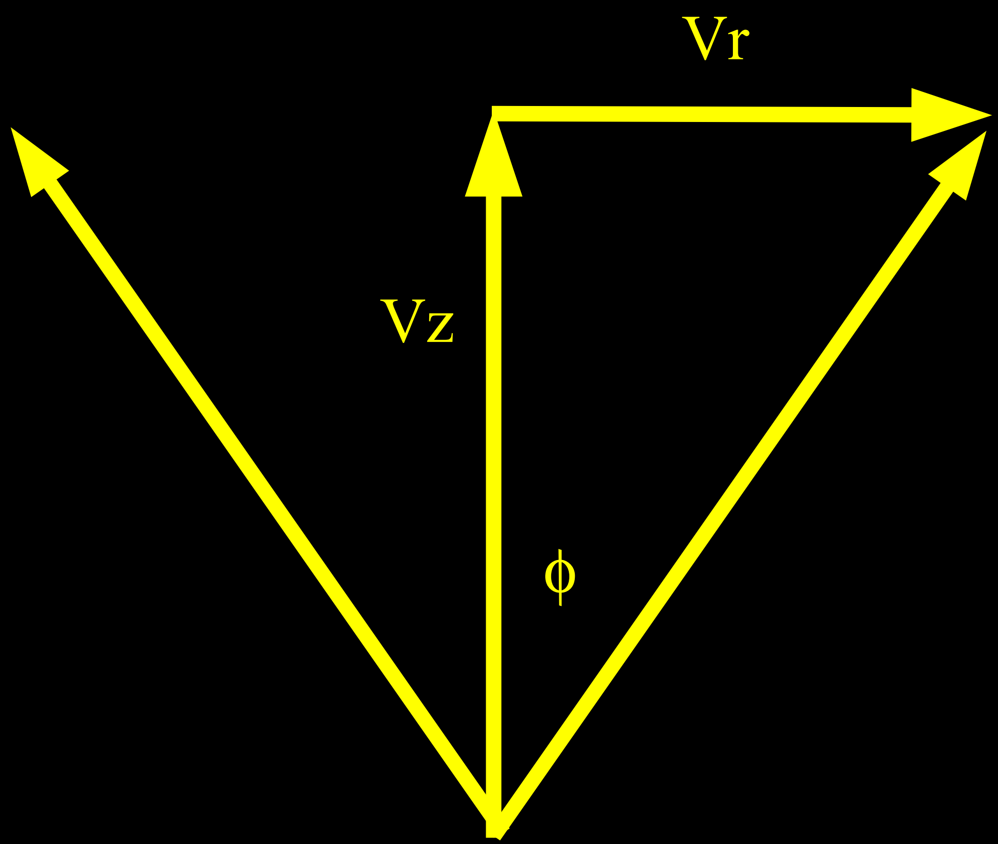
- Narayan & Yi (1994-5), Yuan et al. (2003-4)
- Blandford & Begelman (1999)
- Quataert & Gruzinov (2000)
  - 2-T flows, advection stabilizes
  - Thick disks, ~spherical
  - Energy transported outward via wind
  - Convection helps transport ang. mom. inward
  - Many parameters!!
  - Not dynamical!!

$$Q_{diss} > Q_{rad.}$$

$$\rho \;\alpha\; r^{-3/2+s}$$

$$\dot{M}_{in} = \dot{M}_{out}\left(R_{in}/R_{out}\right)^{s}$$

Jet Models



- Falcke, Melia et al (1995-now)
- Blandford & Konigl (1979)
- Free expanding, rel. jet
- Conical confinement due to rel. vel.
- Jet fed at constant rate
- Not dynamical!!

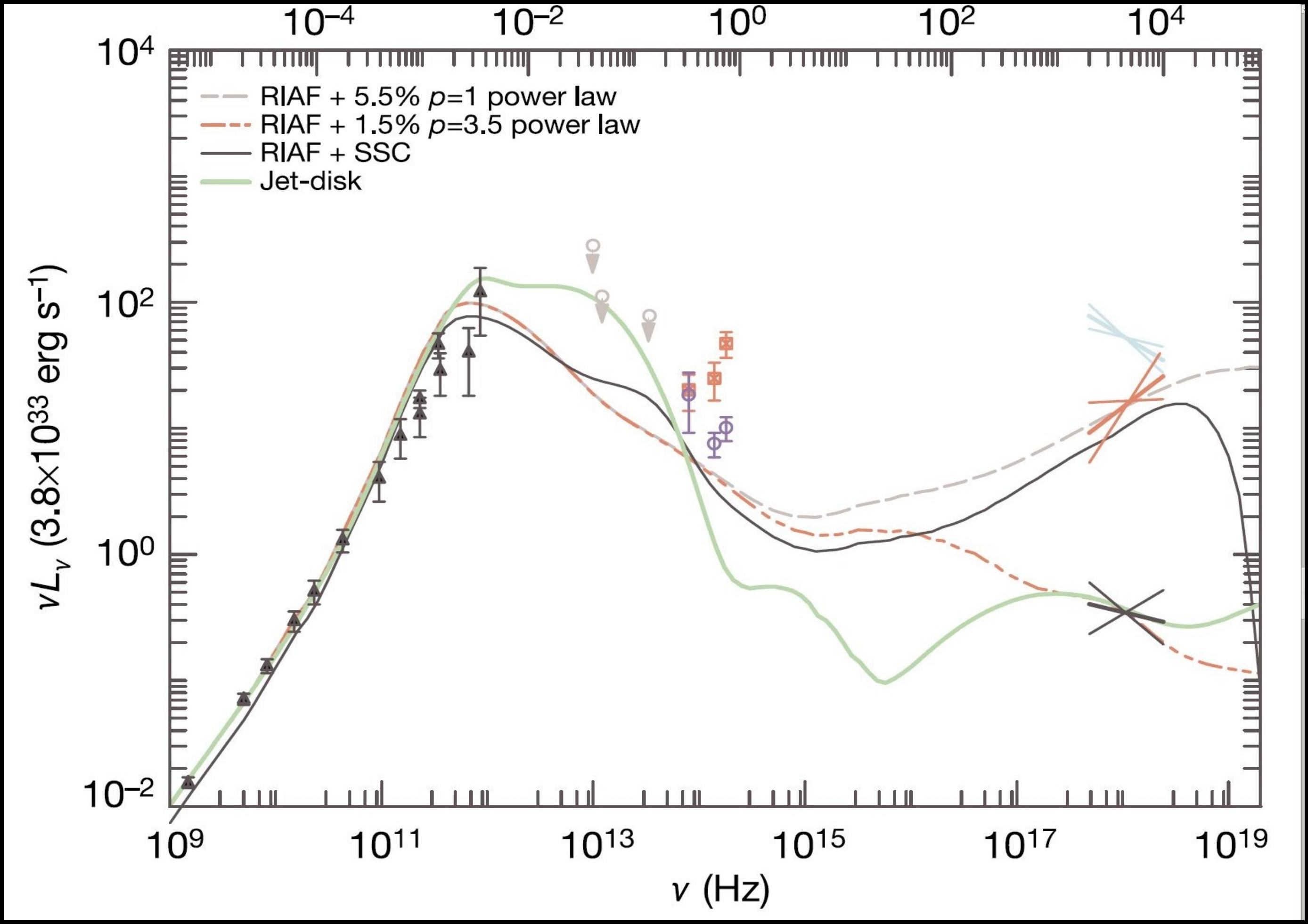
$$v_z = \gamma \beta c \ , \quad v_r = \gamma_s \beta_s c$$

$$\phi = 1/M = v_r / v_z (small)$$

$$\dot{M}_{jet} = \rho v A = m_p n(r) \gamma \beta c \pi r^2 \rightarrow n(r) \alpha 1/r^2$$

$$\dot{E}_{B,\,jet} = \rho_B v A = B^2(r) \gamma \beta c \pi r^2 \rightarrow B(r) \alpha 1/r$$

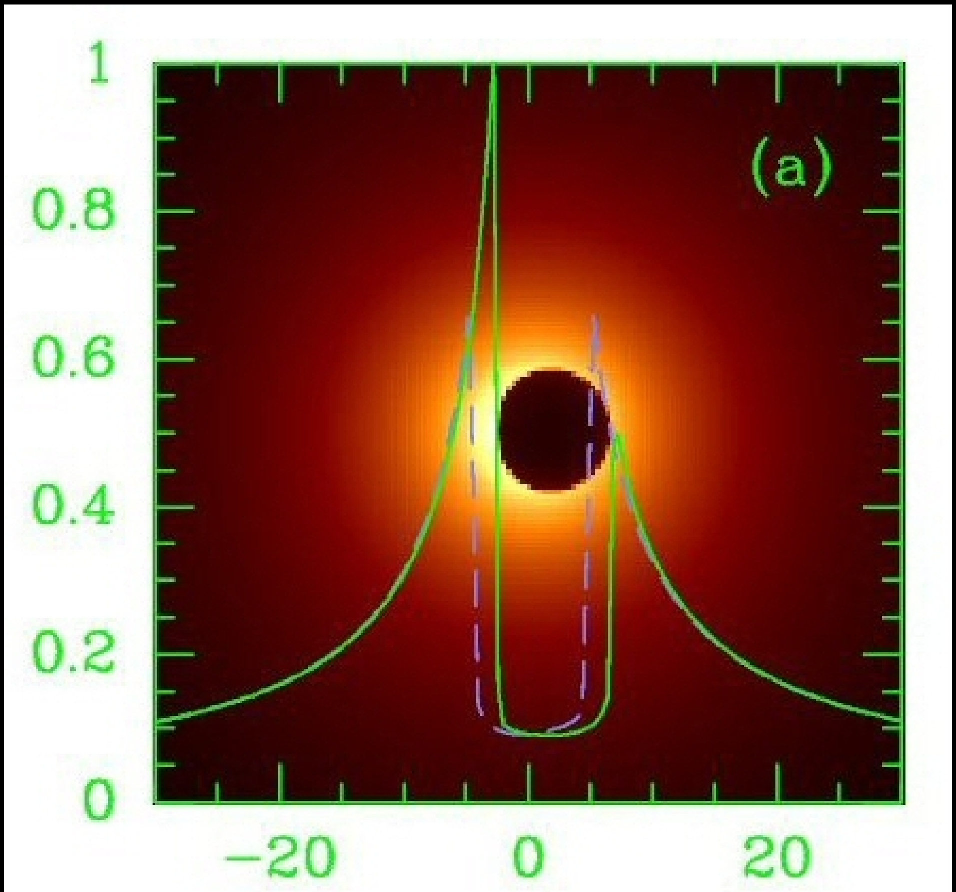
Composite Spectrum (comparison)



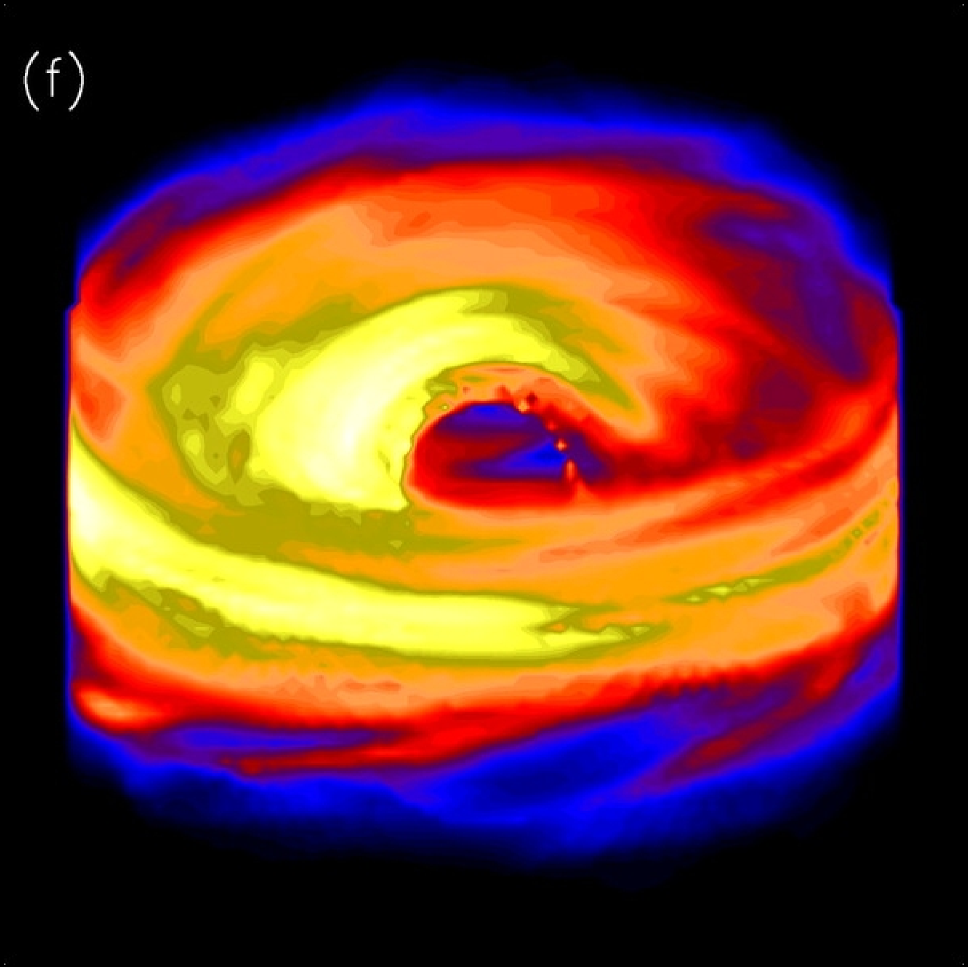
- RIAF's have problem with var. of brem. since $R_{brem} \sim 10^5 R_s$
- Instead, add PL n_e gives hard IC/SSC photons
- Solves Radio under-lum.
- Modern RIAF's have many parameters, need better constraints: simult. wide-freq. survey, submm VLBI

- Jets lack a launching mechanism
- Reliant on a disk model of some type
- Can it predict X-ray flare state?
- SgrA* may have been more active in the past...?

Example RT Calculations

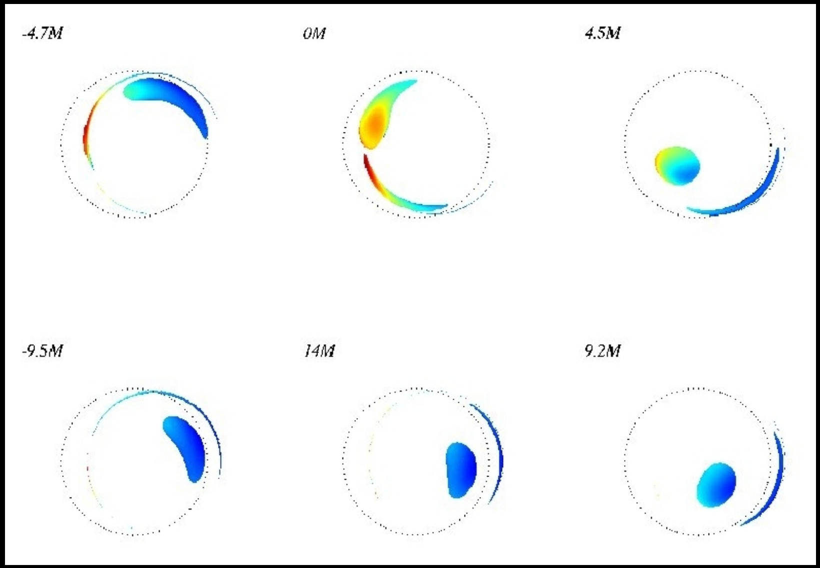


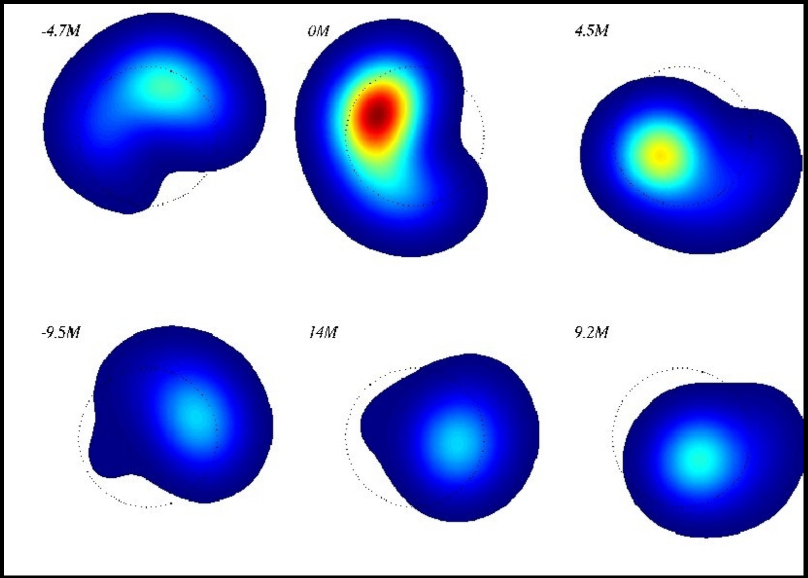
Falcke, Melia, Agol (2000)



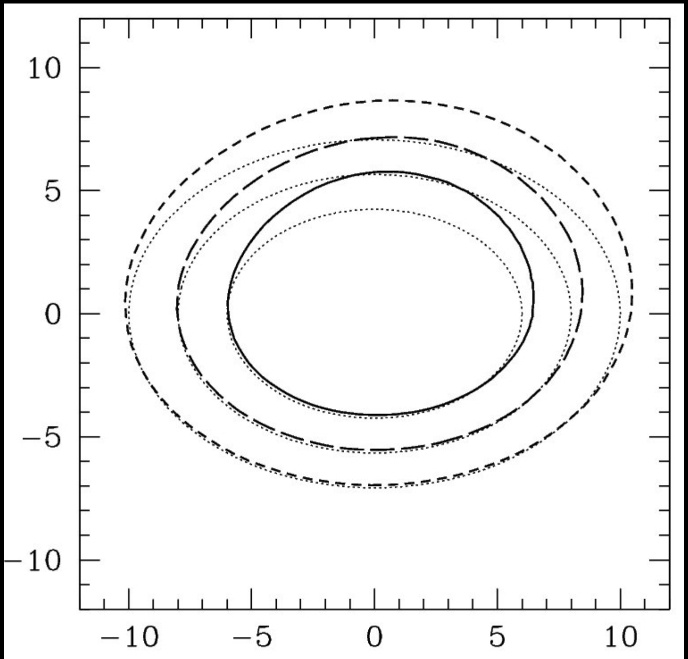
Schnittman, Krolik & Hawley (2006)

Other RT Calculations

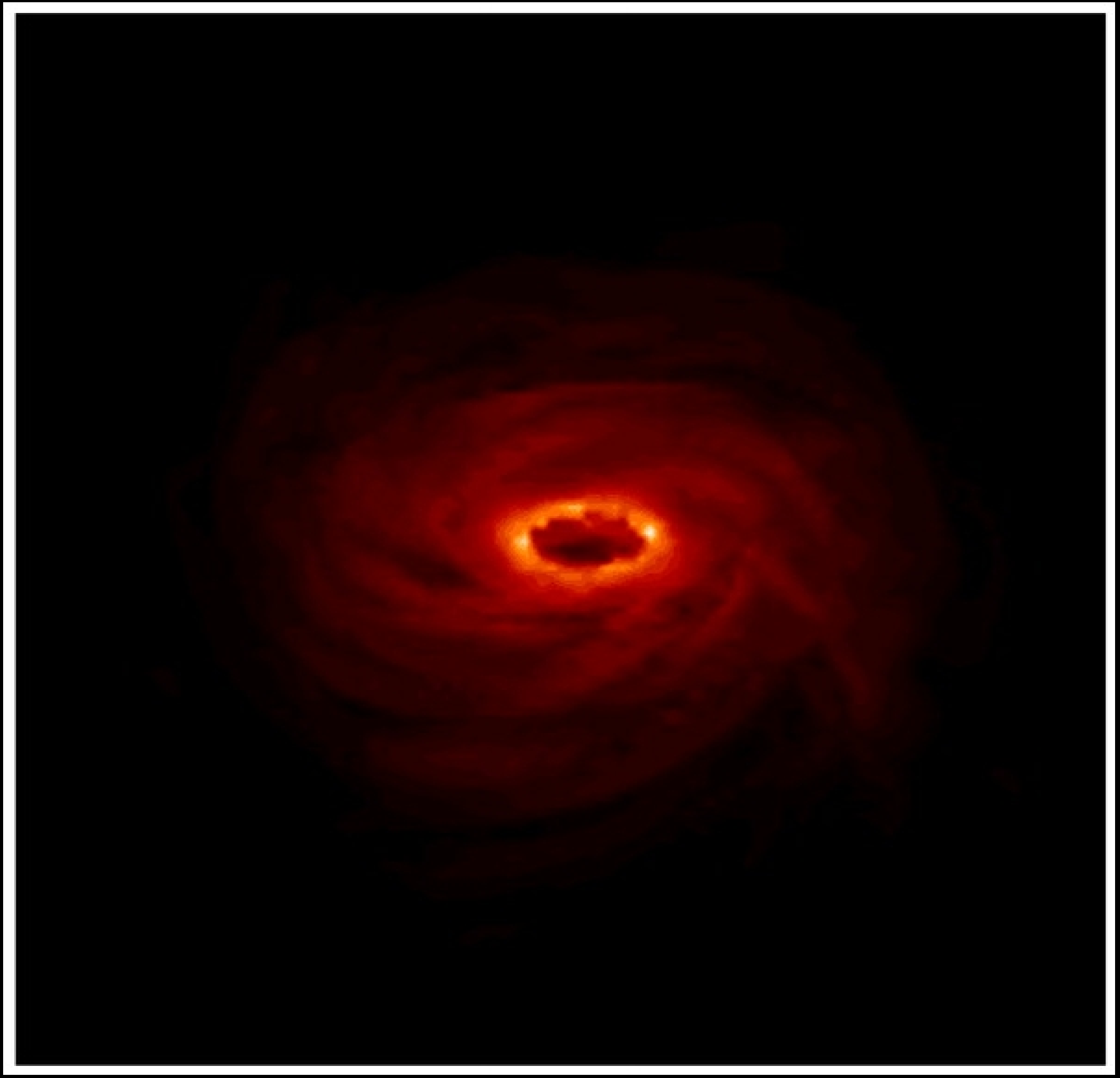




Broderick and Loeb (2005)

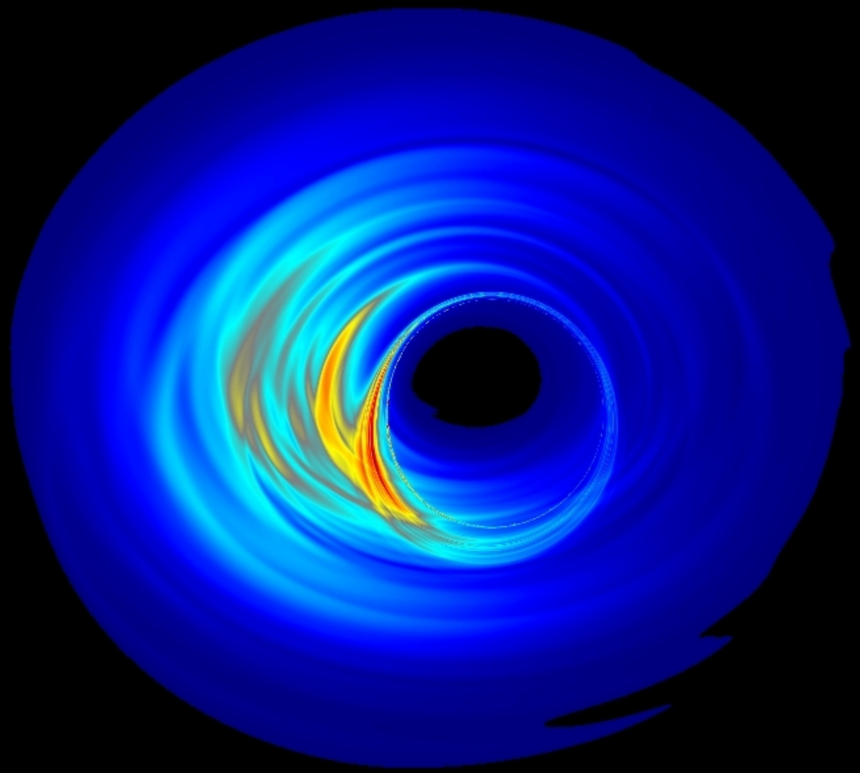


RIAF Simulations



- Goldston, Quataert, Igumenshchev (2005)
- 3D RIAF (Newt. MHD) Simulation
- $T_e = a\, T_{tot}$
- n_e ~ Maxwellian + PLT
- Non-relativistic sim. & radiation

Default Model



$$\boxed{\nu_{obs}, \dot{M}, a, \theta_{inc}}$$

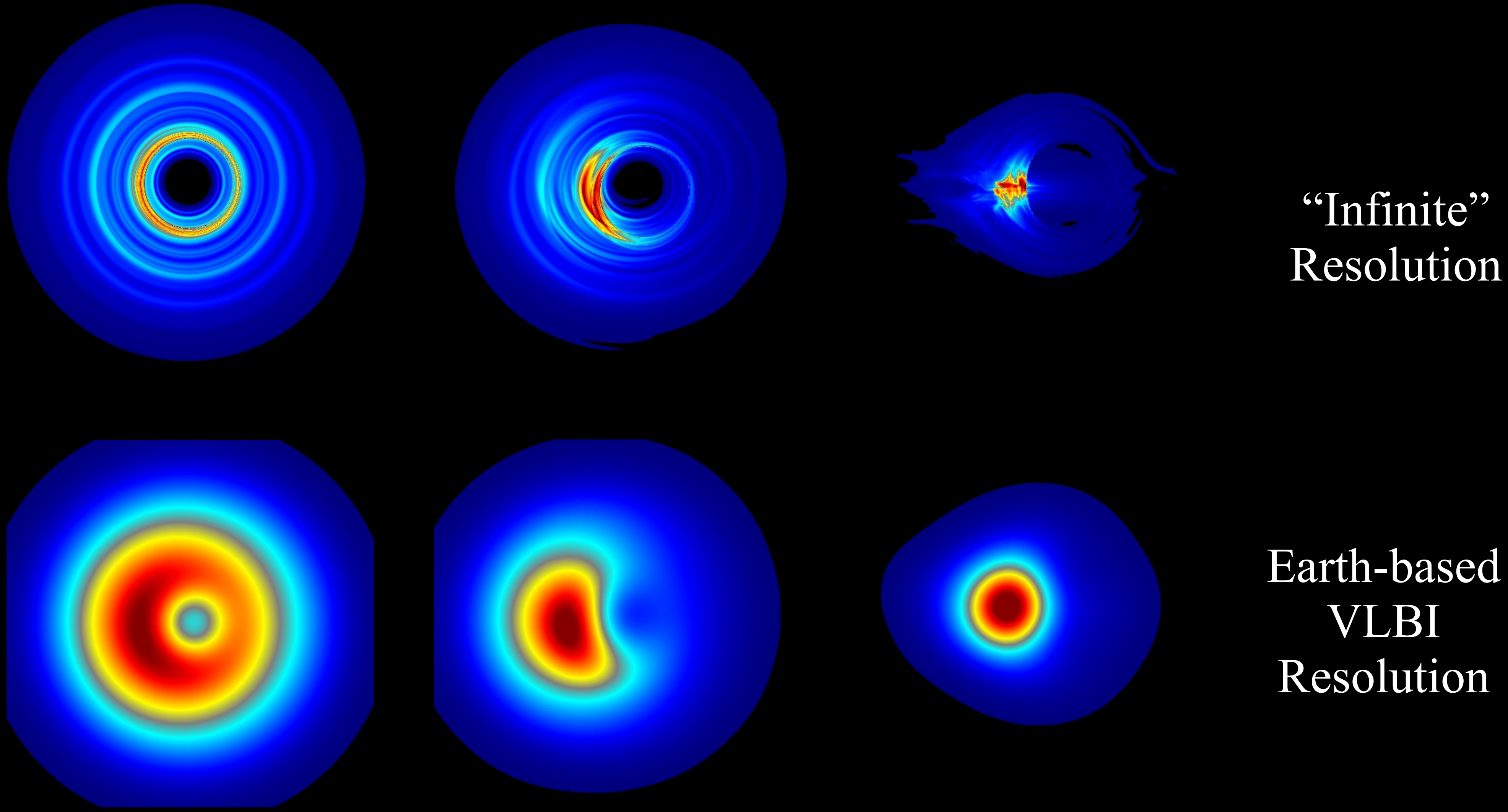
$$\nu_{obs} = 3\text{x}10^{11}\, Hz\,(1\text{mm})$$

$$\dot{M} = 5\text{x}10^{-9}\, M_{sun}\, yr^{-1}$$

$$a = 0.94$$

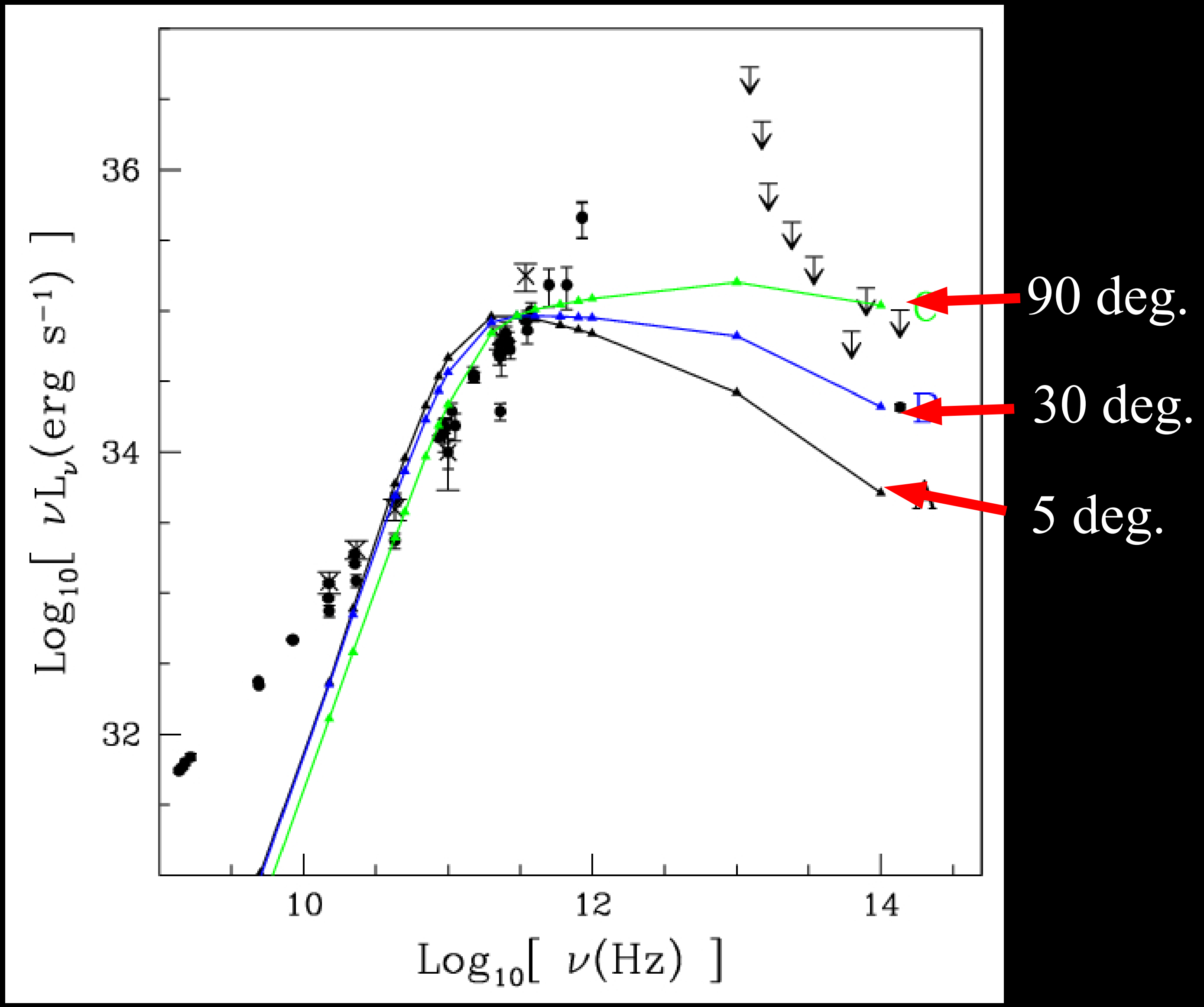
$$\theta_{inc} = 30\, degrees$$

Inclination Survey



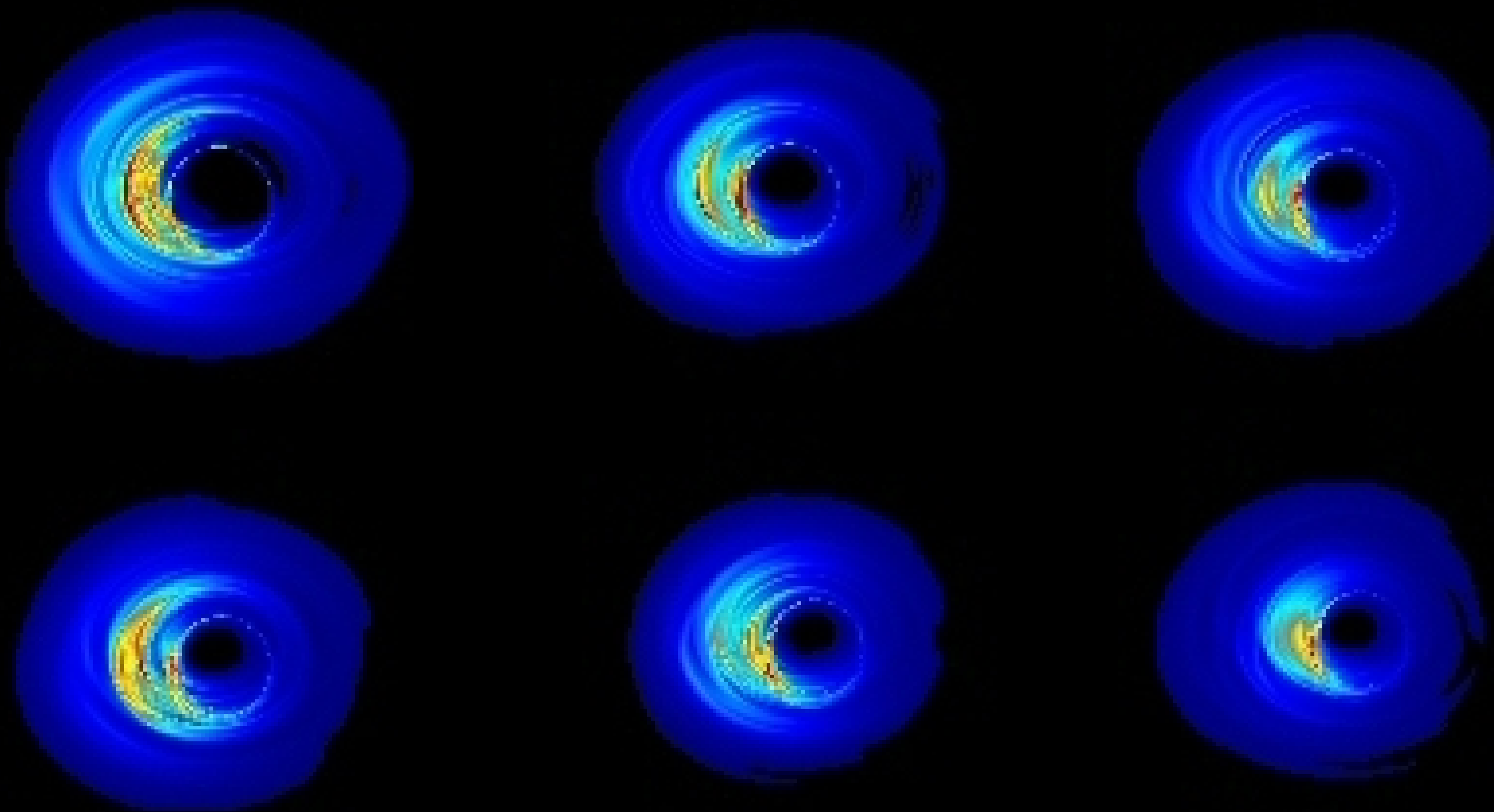
$\theta_{inc} = 5$ 30 $90\ degrees$

Inclination Survey



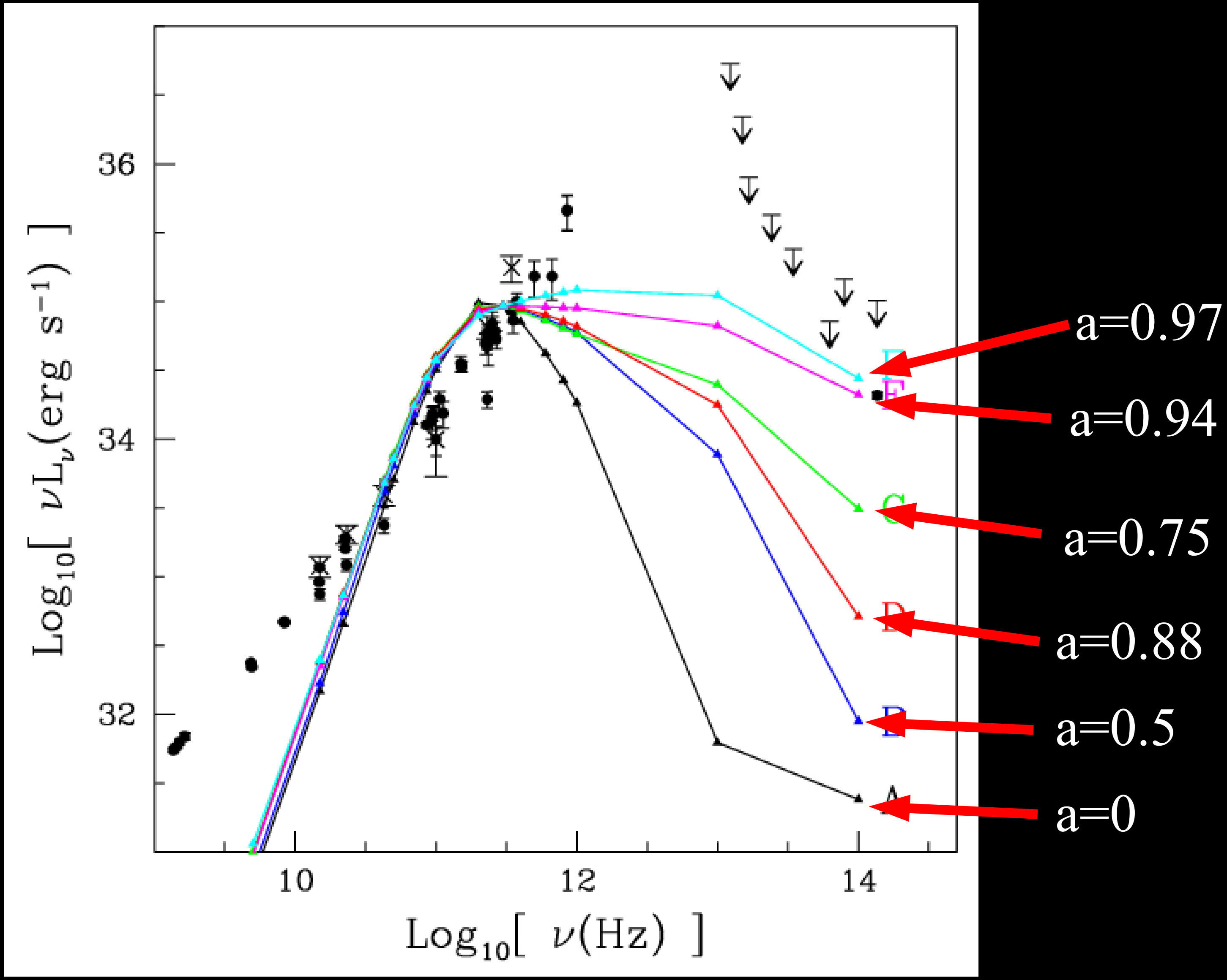
Spin Survey

0 0.5 0.75

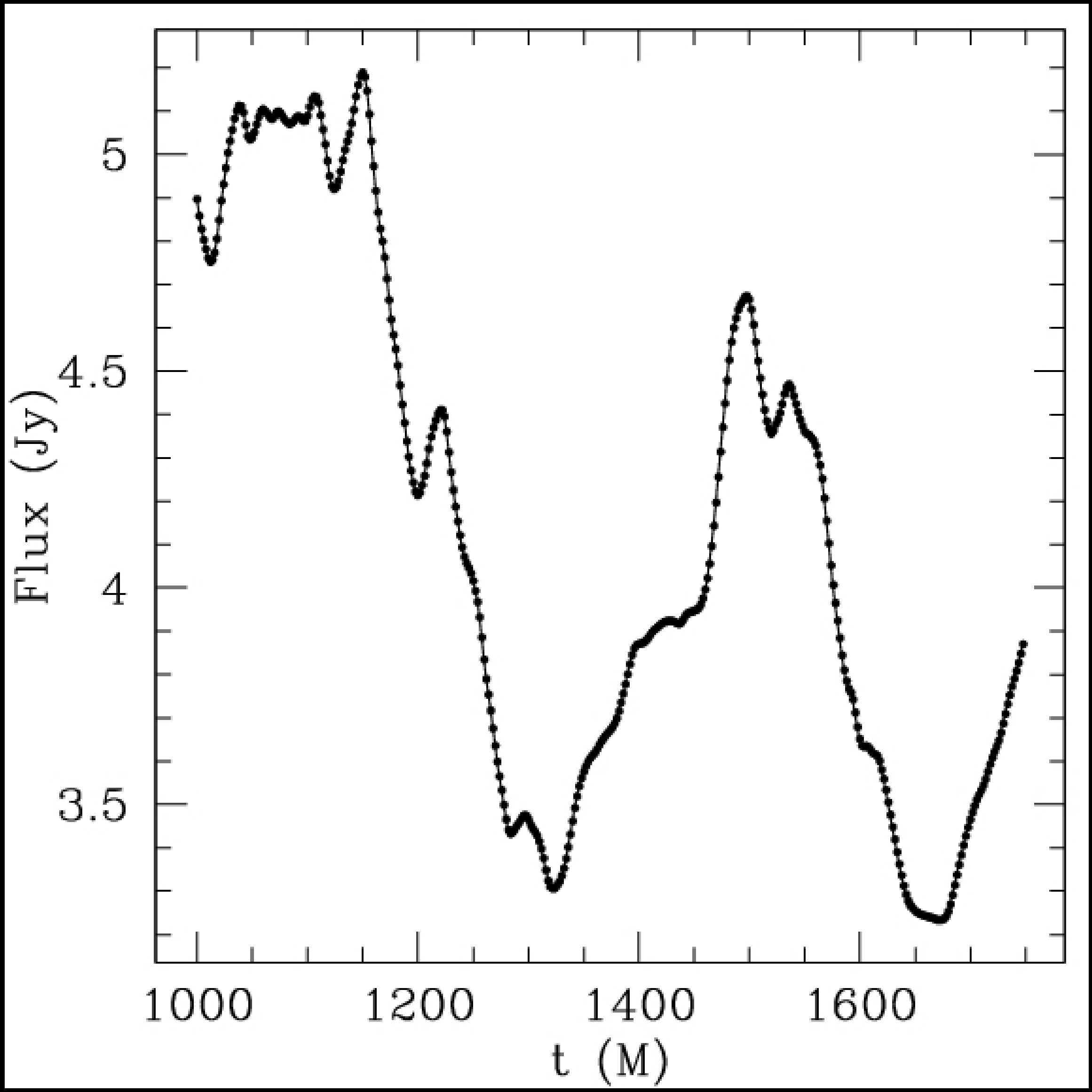


0.88 0.94 0.97

Spin Survey

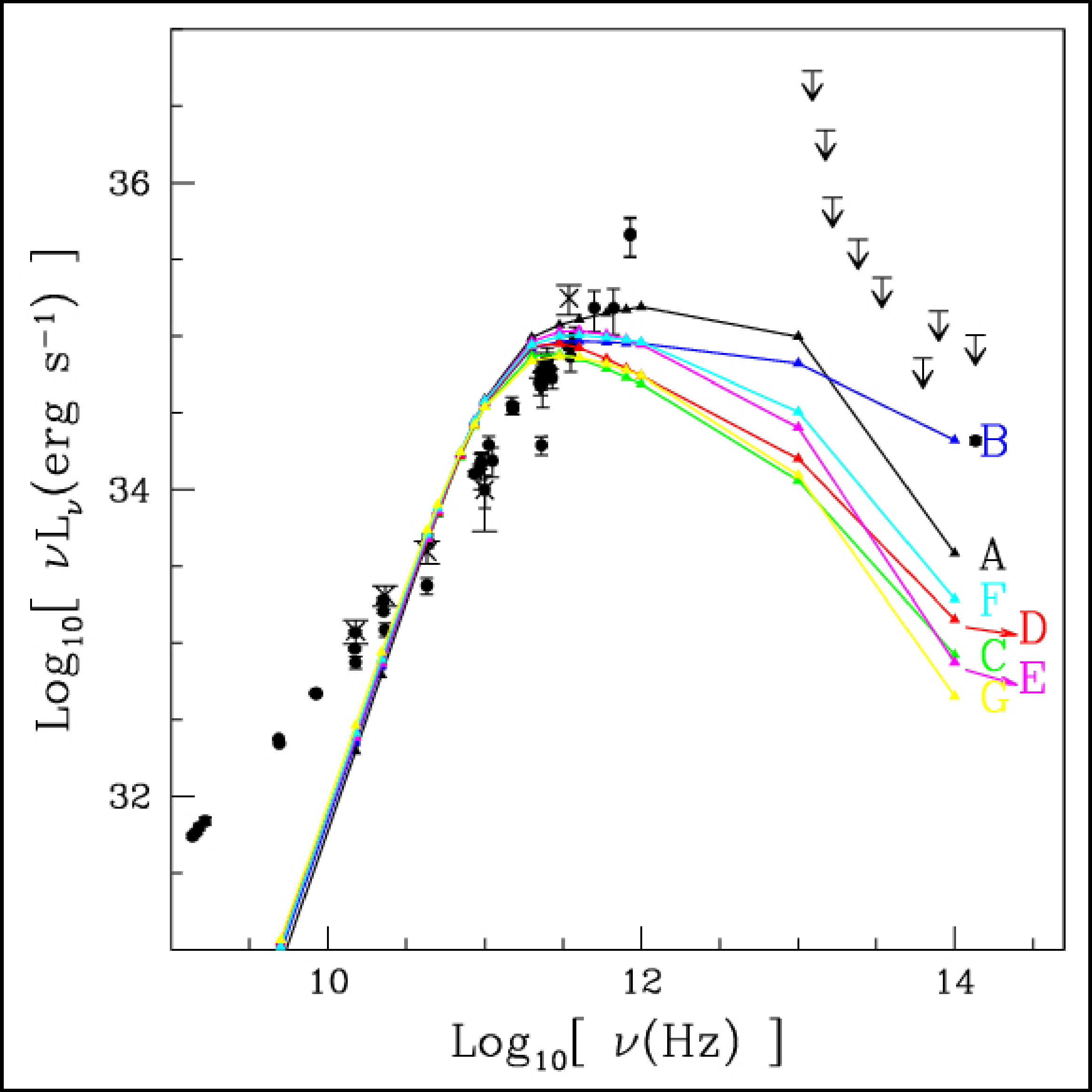

36
34
32
10
12
14
Log10[νLν(erg s−1)]
Log10[ν(Hz)]
a=0.97
a=0.94
a=0.75
a=0.88
a=0.5
a=0

Time Variation



(t = 1150M, 1250M, 1326M, 1434M, 1500M, 1666M)

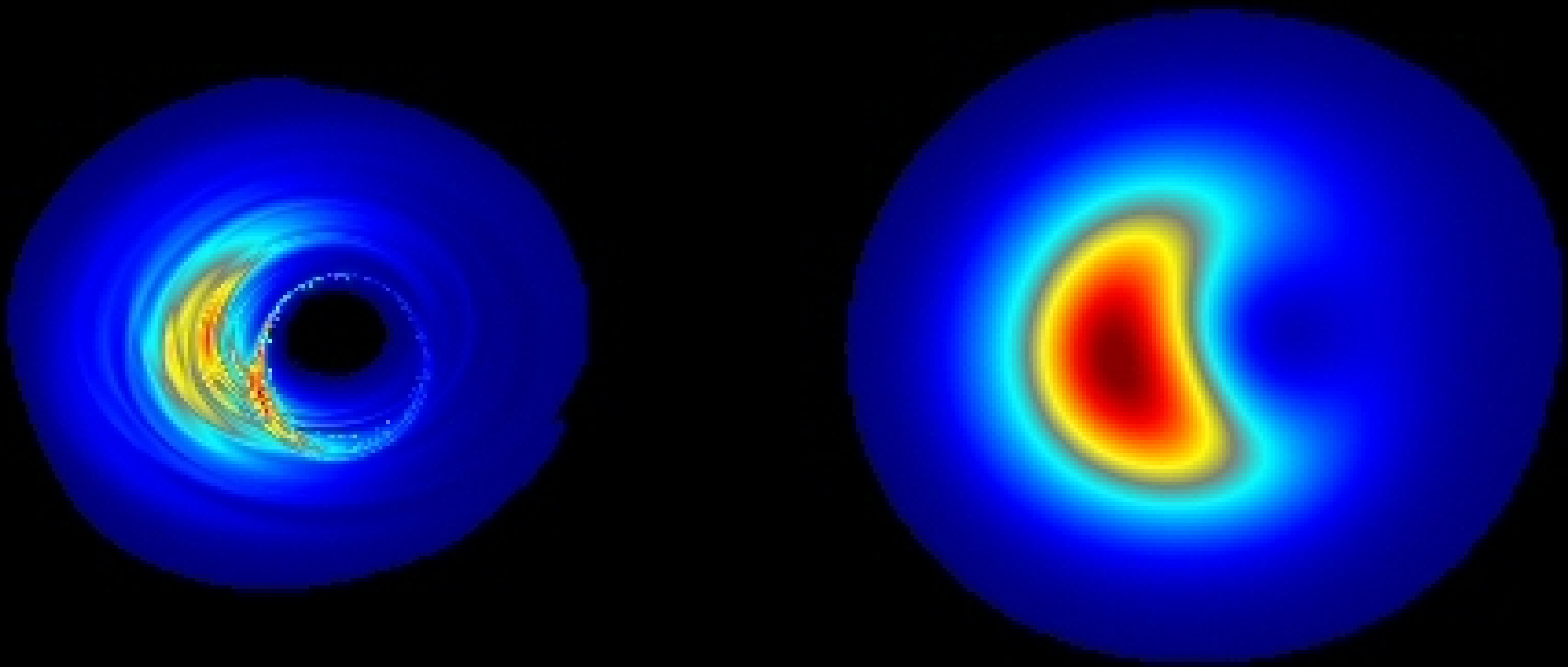
Time Variation



t = 1150M, 1250M, 1326M, 1434M, 1500M, 1666M

Time Variation

t = 1000M - 1700M



Summary

- Observable shadow for $\theta_{inc} < 30\,deg.$
- Spectral dependence on all degrees of freedom
- Greatest variability seen between disks of different spin
- Greatest variability seen at larger frequencies (ala relativistic beaming near horizon)
- Spatial/temporal variability important --- need dynamic models
- Amenable for identifying characteristics of SgrA*'s spacetime
- $\dot{M}_{num}$ $\dot{M}_{obs}$

Future Work

- Interpolate simulation data in TIME & space
- Temporal variability --> Need to time average images/spectra
- Calculate polarized emission.... (in the works: P. K. Leung)
- Add non-thermal distribution of electrons to model
    - Requires evolution of electron energy eq. in simulations
- Efficient image/flux convergence:
    - Berger & Collela "Pixel Refinement"
- Compton scattering
    - Most likely needs Monte Carlo (C. Gammie)
- Use 3D simulation data (HARM3D in the works...)

EXTRA SLIDES

Synchrotron Calculation: Exact

$$j_\nu(\vartheta) = \int_1^\infty \mathrm{d}\gamma \frac{1}{2} n_\mathrm{e}(\gamma) \int_{-1}^{1} \mathrm{d}\mu\, \eta_\nu(\vartheta, \mu, \gamma)$$

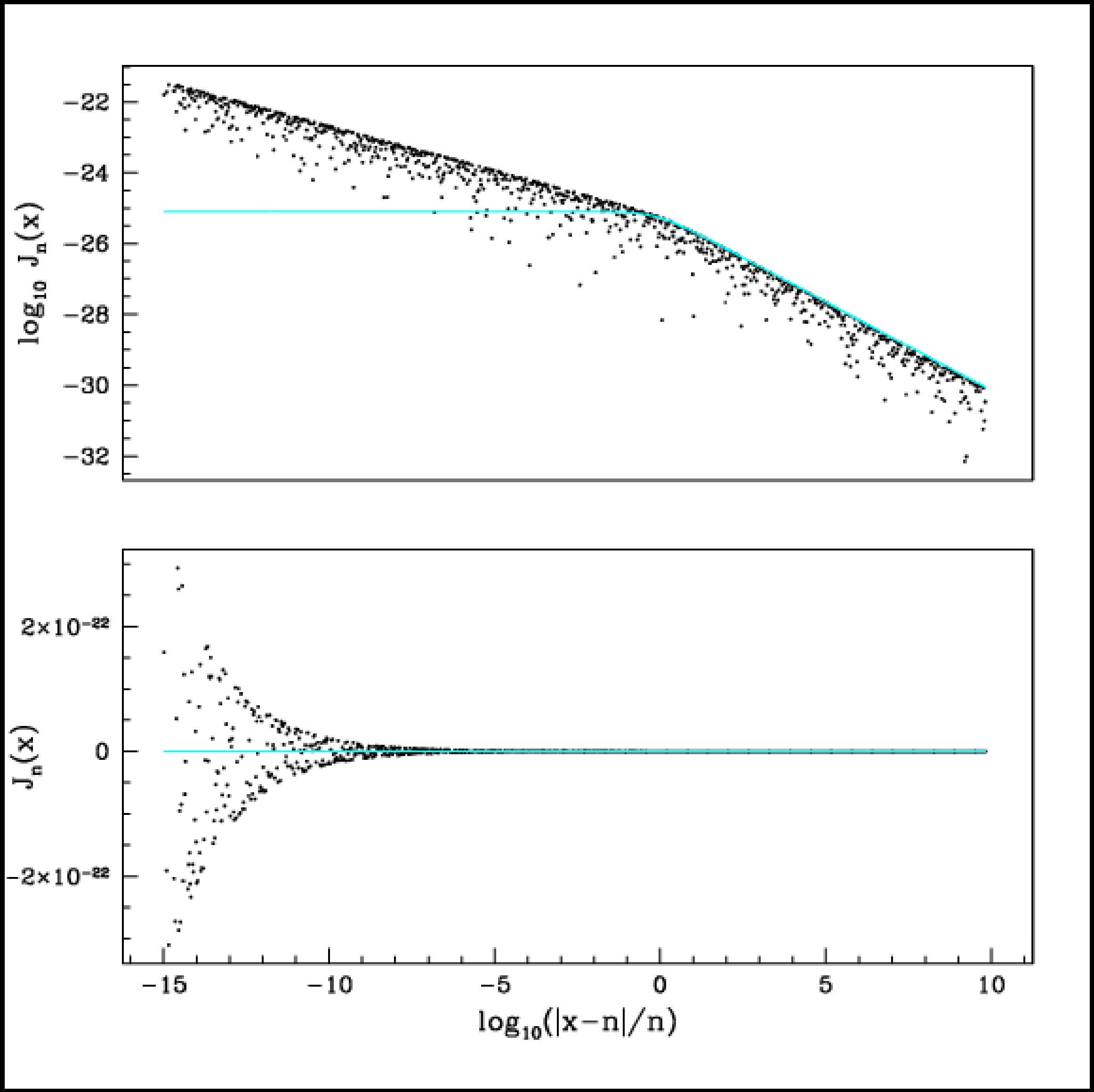
$$\eta_\nu \equiv \frac{\mathrm{d}W}{\mathrm{d}\nu\,\mathrm{d}\Omega\,\mathrm{d}t}(\vartheta, \xi, \gamma) = \frac{2\pi e^2 \nu^2}{c} \times \sum_{n=1}^{\infty} \delta(y_n) \left[\left(\frac{\cos\vartheta - \beta\cos\xi}{\sin\vartheta} \right)^2 J_n^2(z) + \beta^2 \sin^2 \xi J_n'^2(z) \right]$$

$$z \equiv \frac{\nu\gamma\beta \sin\vartheta \sin\xi}{\nu_\mathrm{c}}$$

$$y_n \equiv \frac{n\nu_\mathrm{c}}{\gamma} - \nu(1 - \beta\cos\xi\cos\vartheta)$$

$$\nu_\mathrm{c} \equiv eB/2\pi m_\mathrm{e} c$$

Synchrotron Calculation: Exact



$$n = 10^{50}$$

Chishtie et al. (2005)

- Asymptotic expansions to calculate Jn(x) for pulsar grav. waves
- Found matching conditions empirically
- n = 1e50 possible

Synchrotron Calculation: Approx.

Wardzinski & Zdziarski (2000):

$$j_\nu(\vartheta) = \frac{2^{1/2}\pi e^2 n_\mathrm{e} \nu}{3cK_2(1/\Theta)} \exp\left[-\left(\frac{9v}{2\sin\vartheta}\right)^{1/3}\right]$$

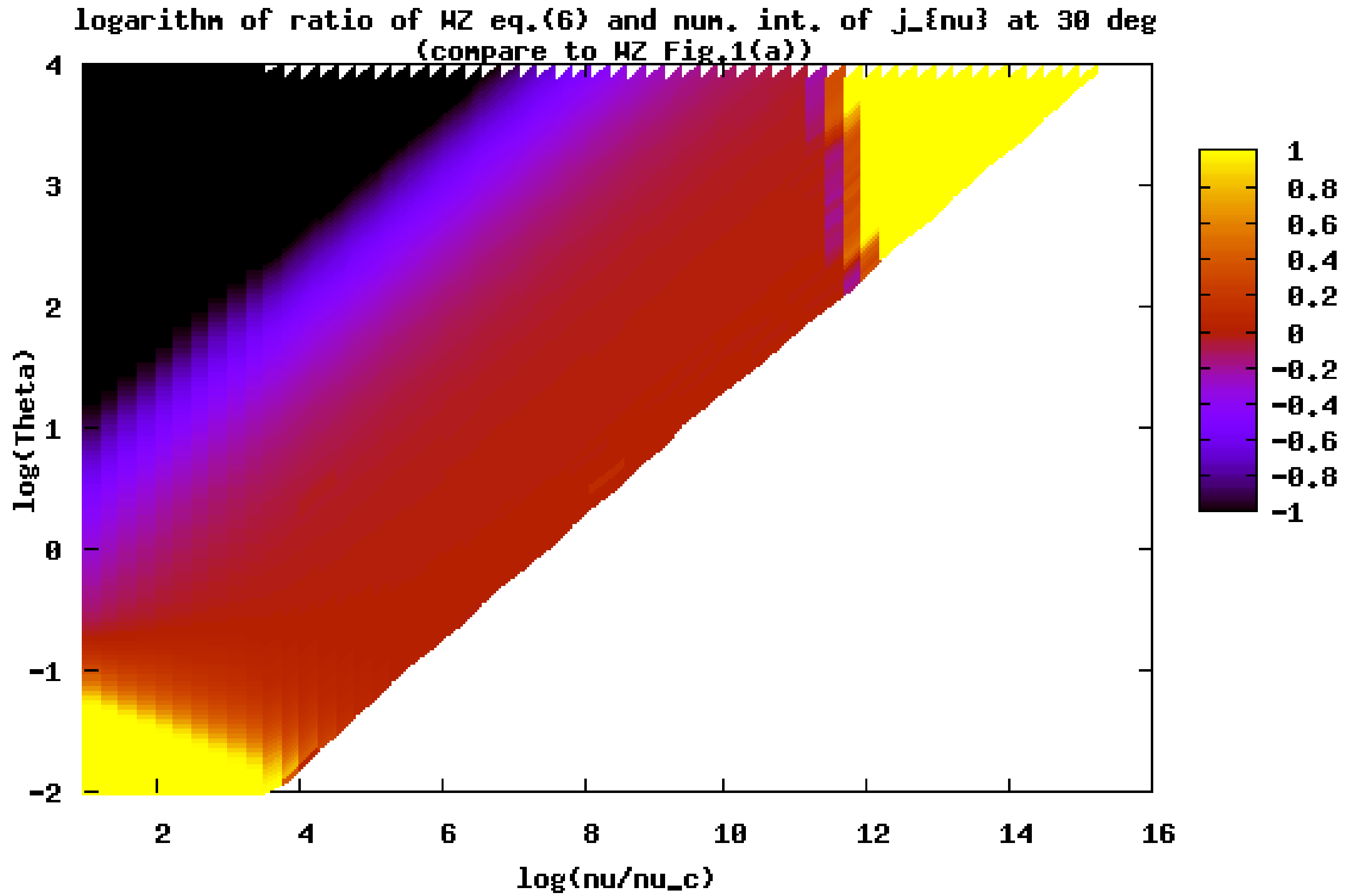
$$\Theta \equiv kT/m_\mathrm{e}c^2 \qquad v \equiv \nu/\nu_\mathrm{c}\Theta^2$$

- Also use an approximate equation by Baring (1988)

Synchrotron Calculation: Approx.

Wardzinski & Zdziarski (2000)

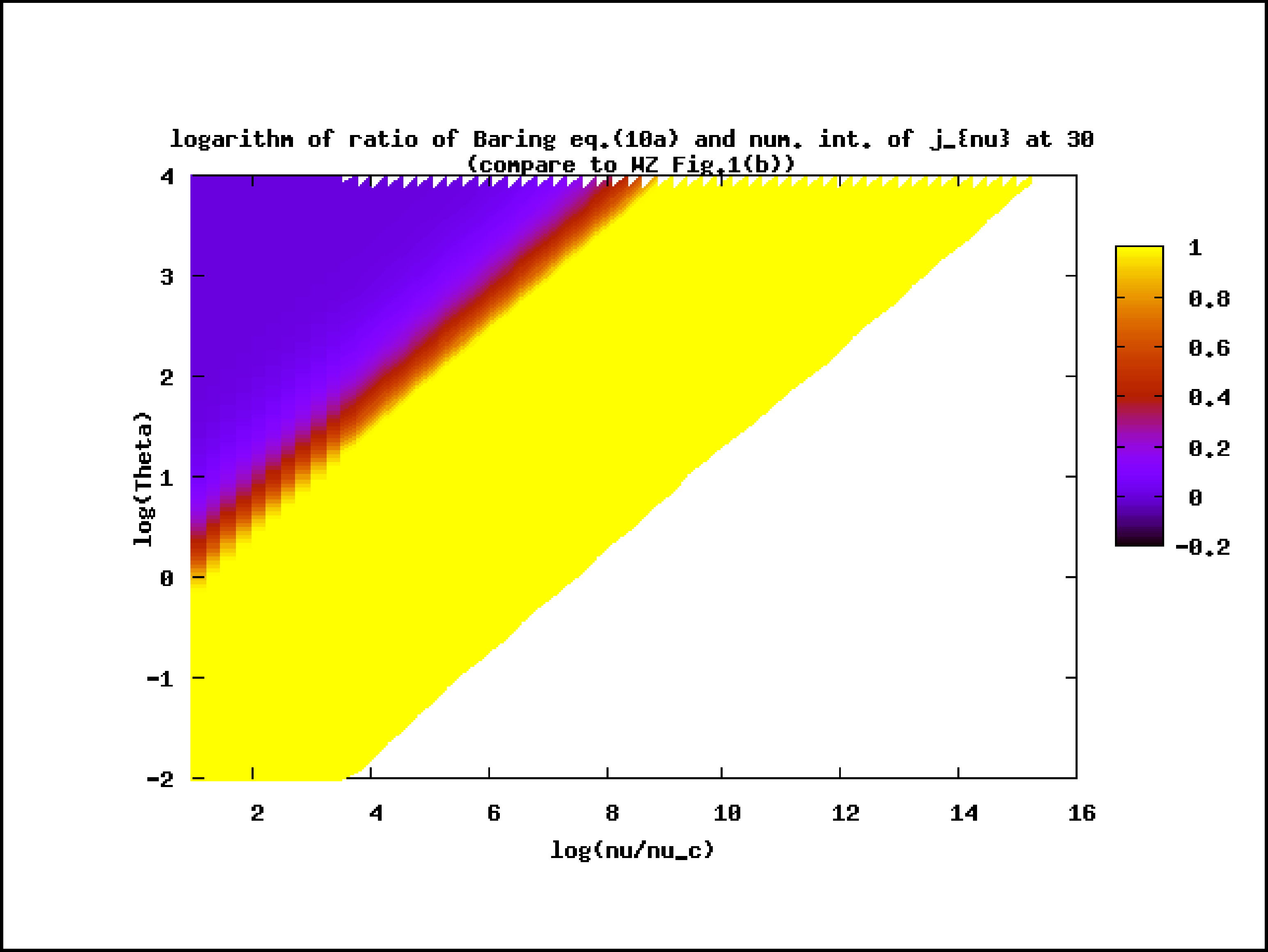
Baring (1988)



Synchrotron Calculation: Approx.

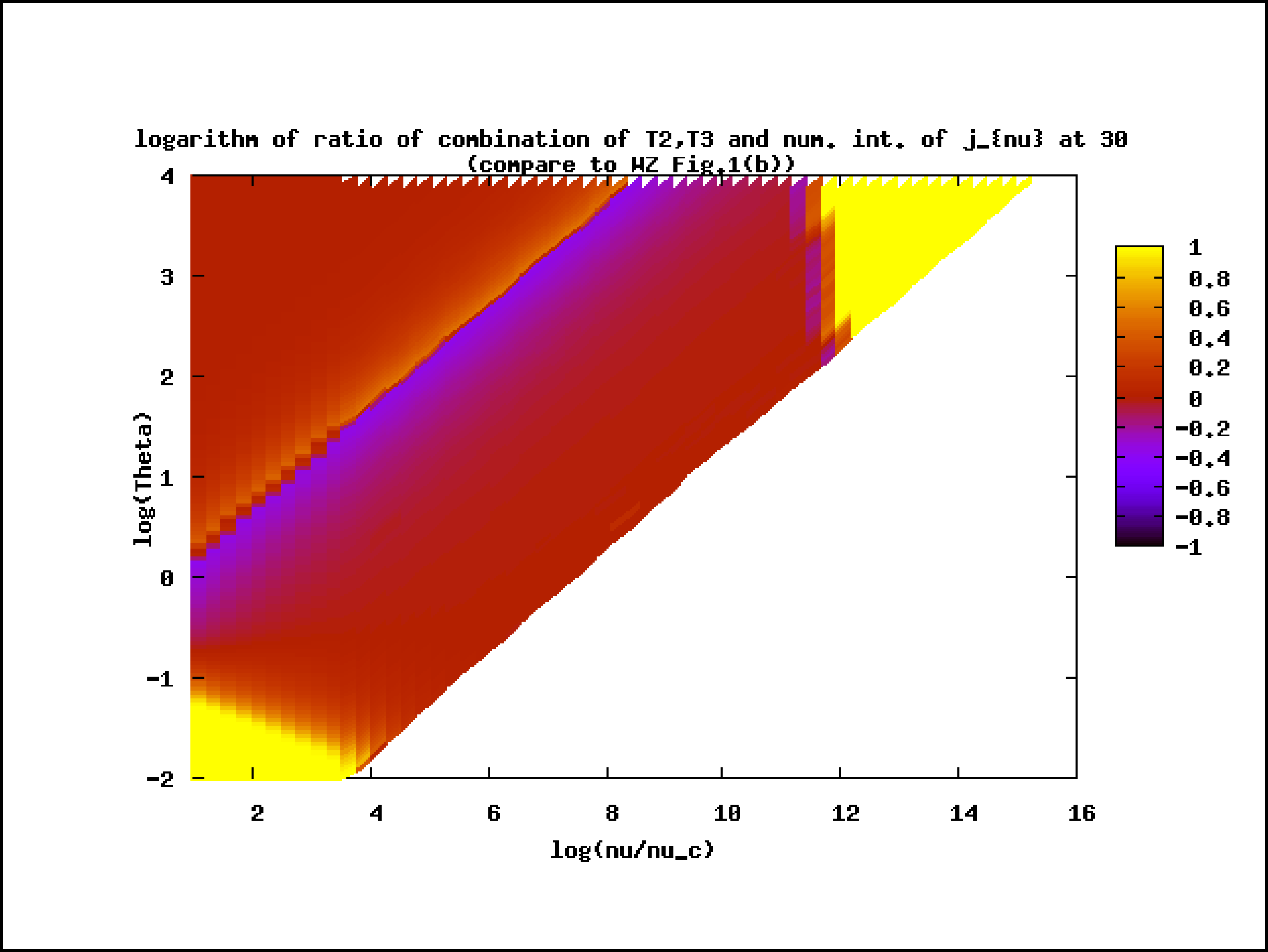
Wardzinski & Zdziarski (2000)

Baring (1988)



Synchrotron Calculation: Approx.

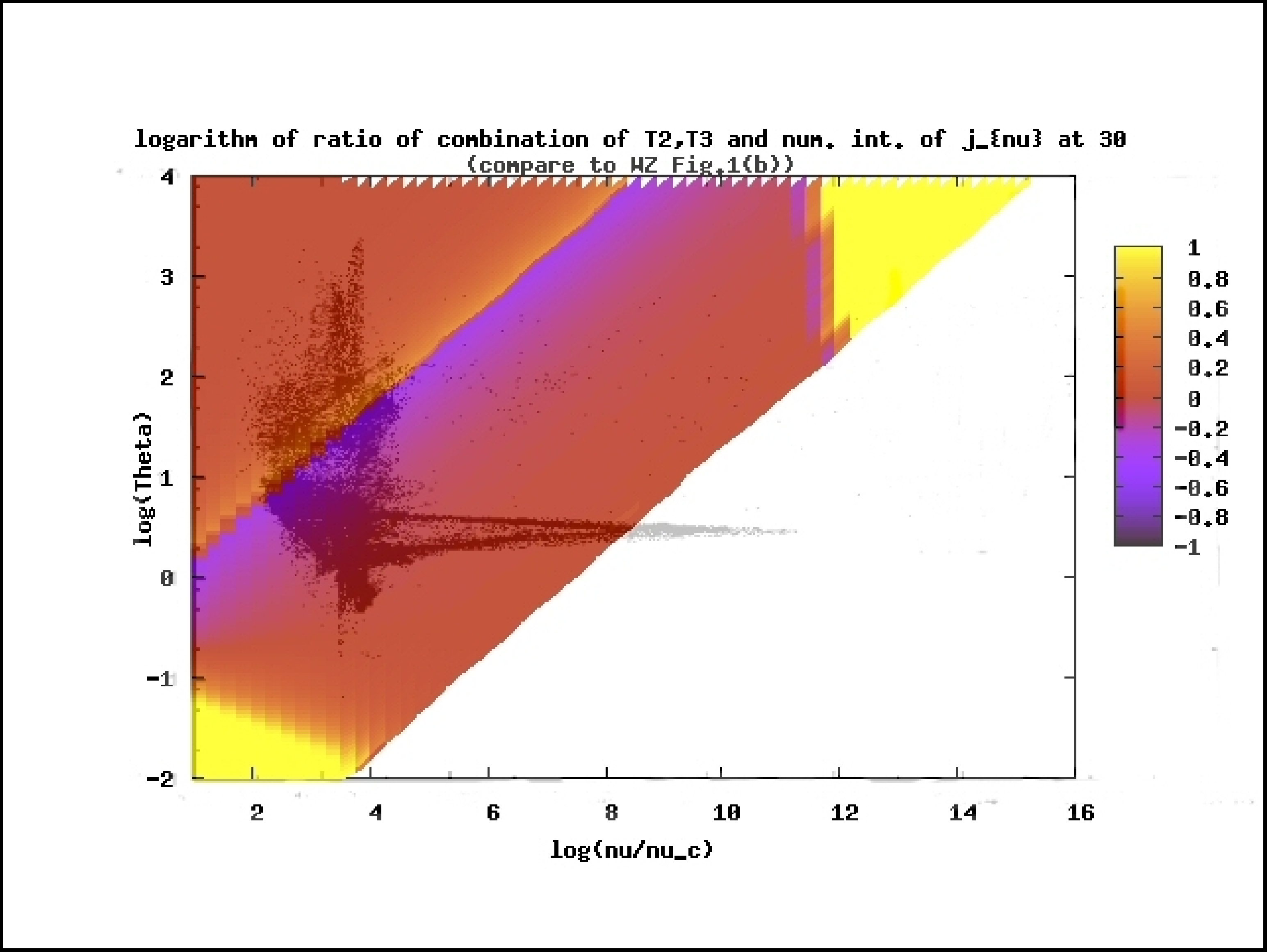
Wardzinski & Zdziarski (2000) Baring (1988)



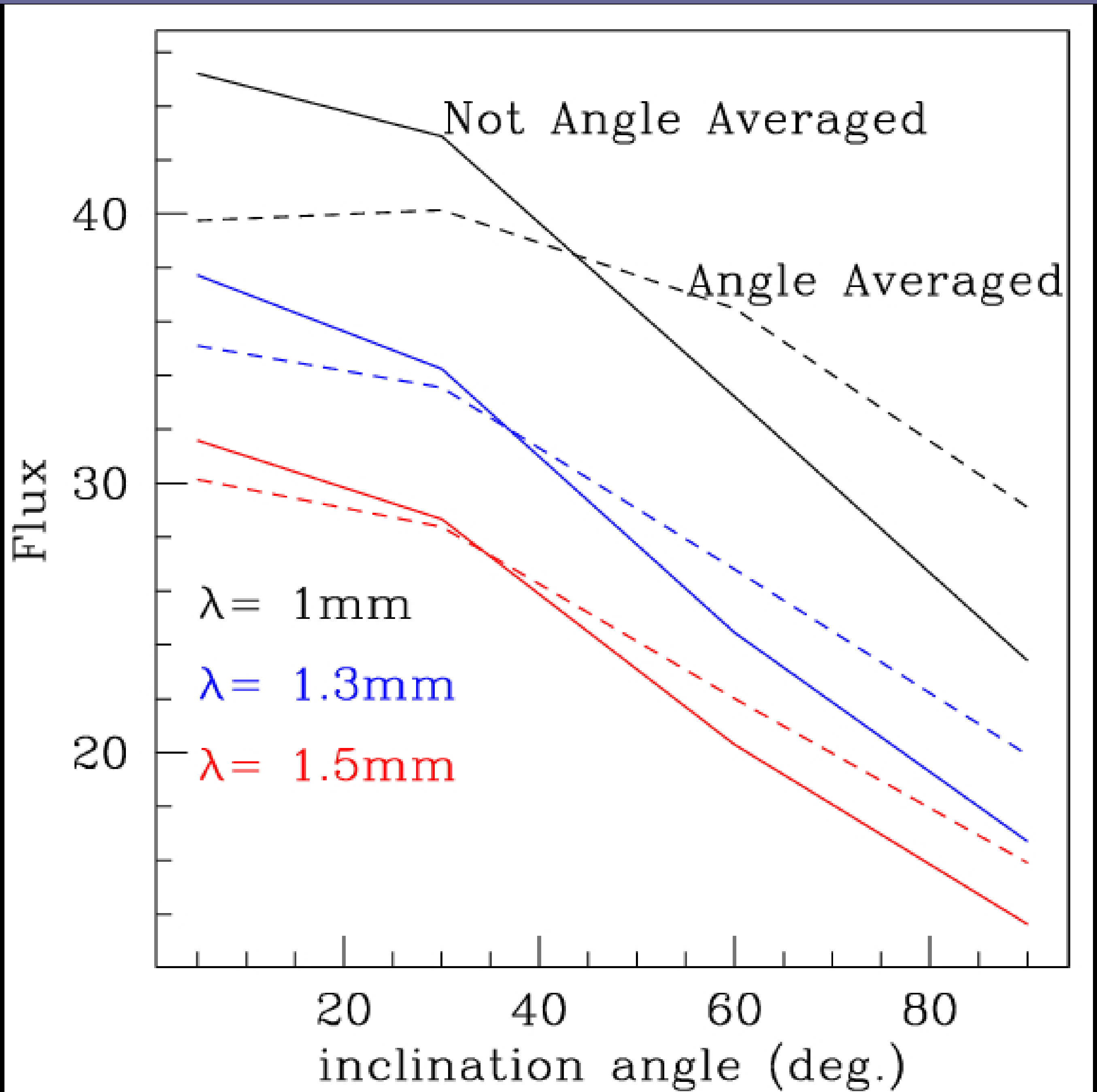
Synchrotron Calculation: Approx.

Wardzinski & Zdziarski (2000)

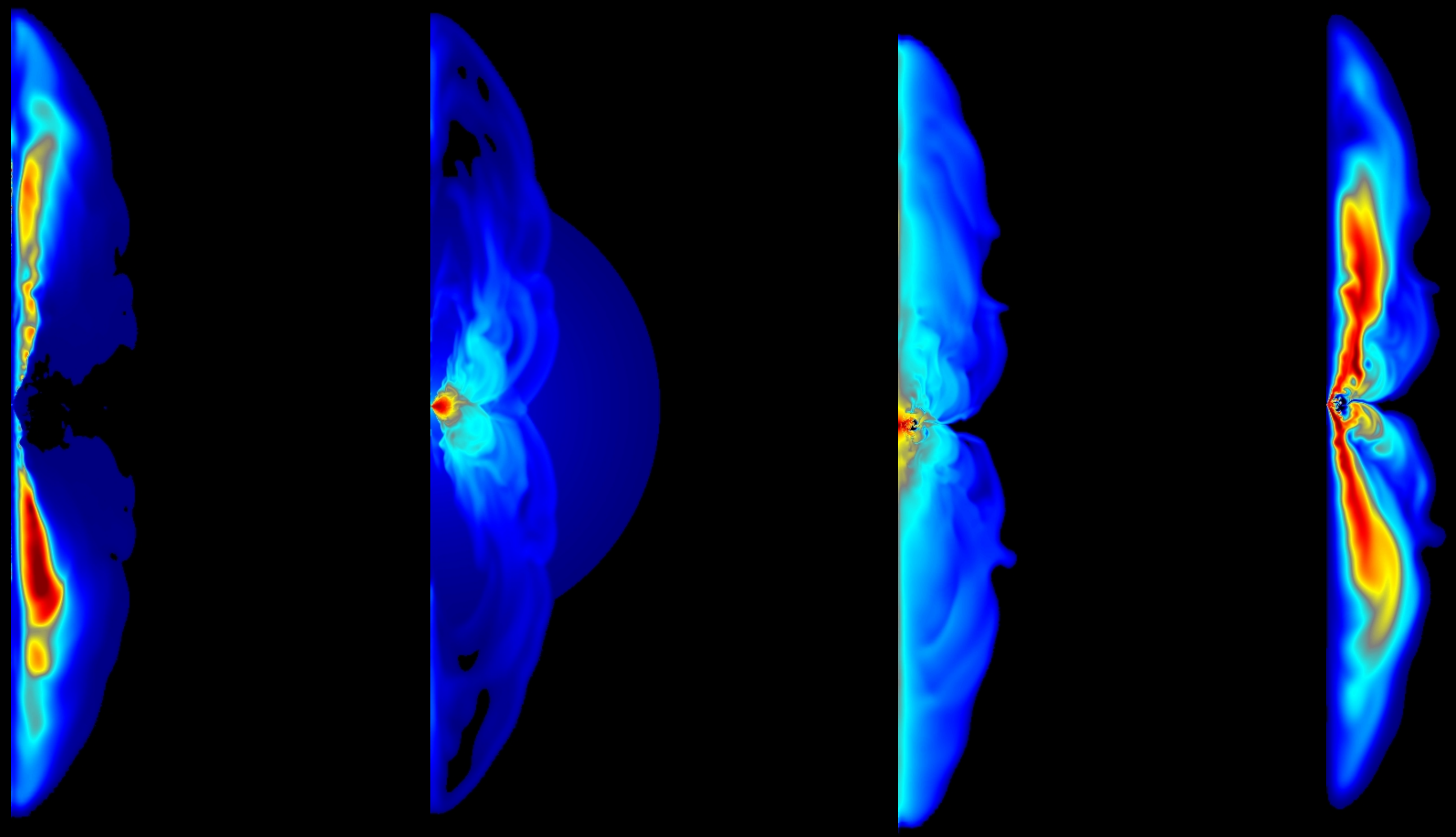
Baring (1988)



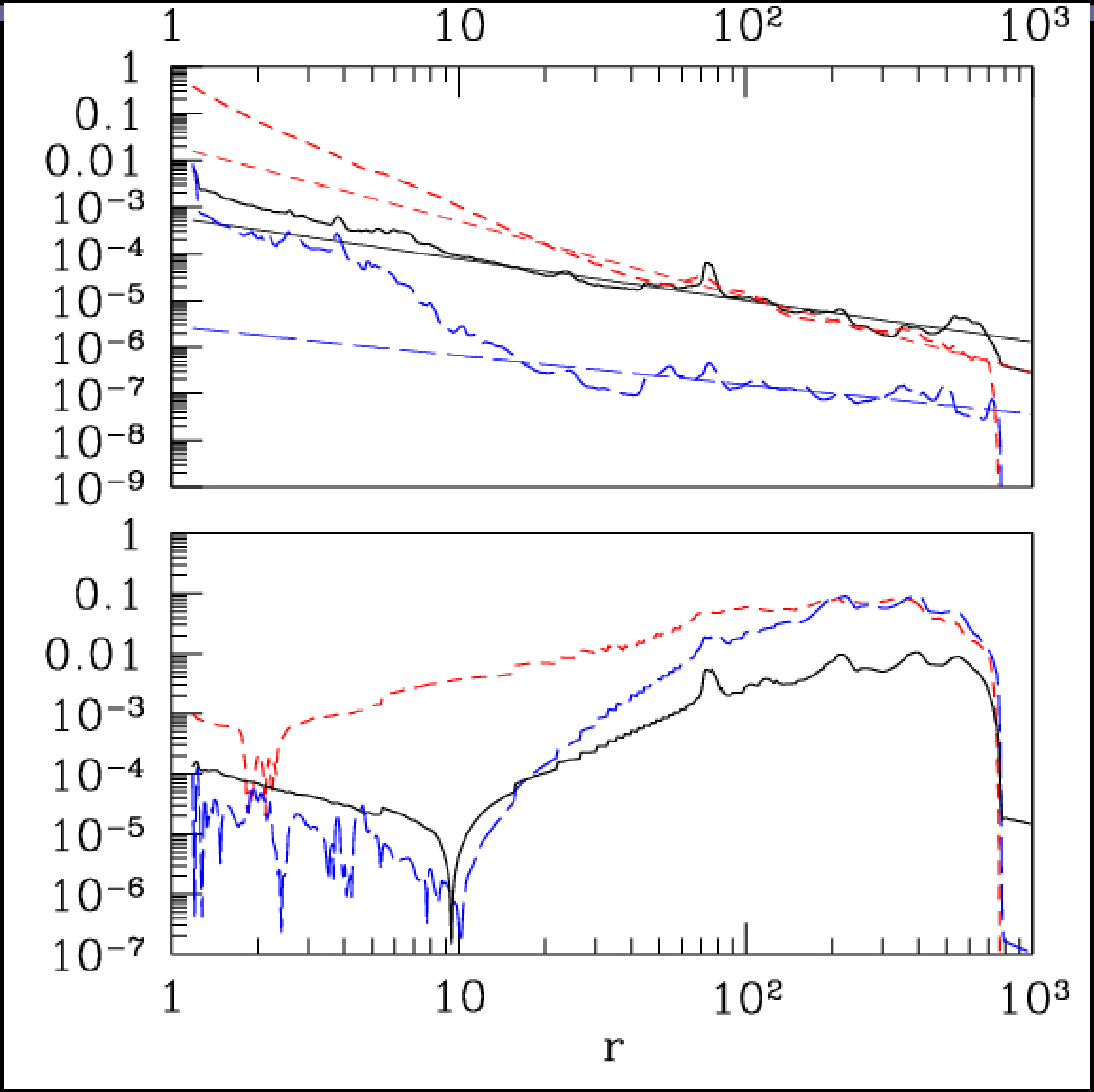
Synchrotron Calculation: Angle Avg.?



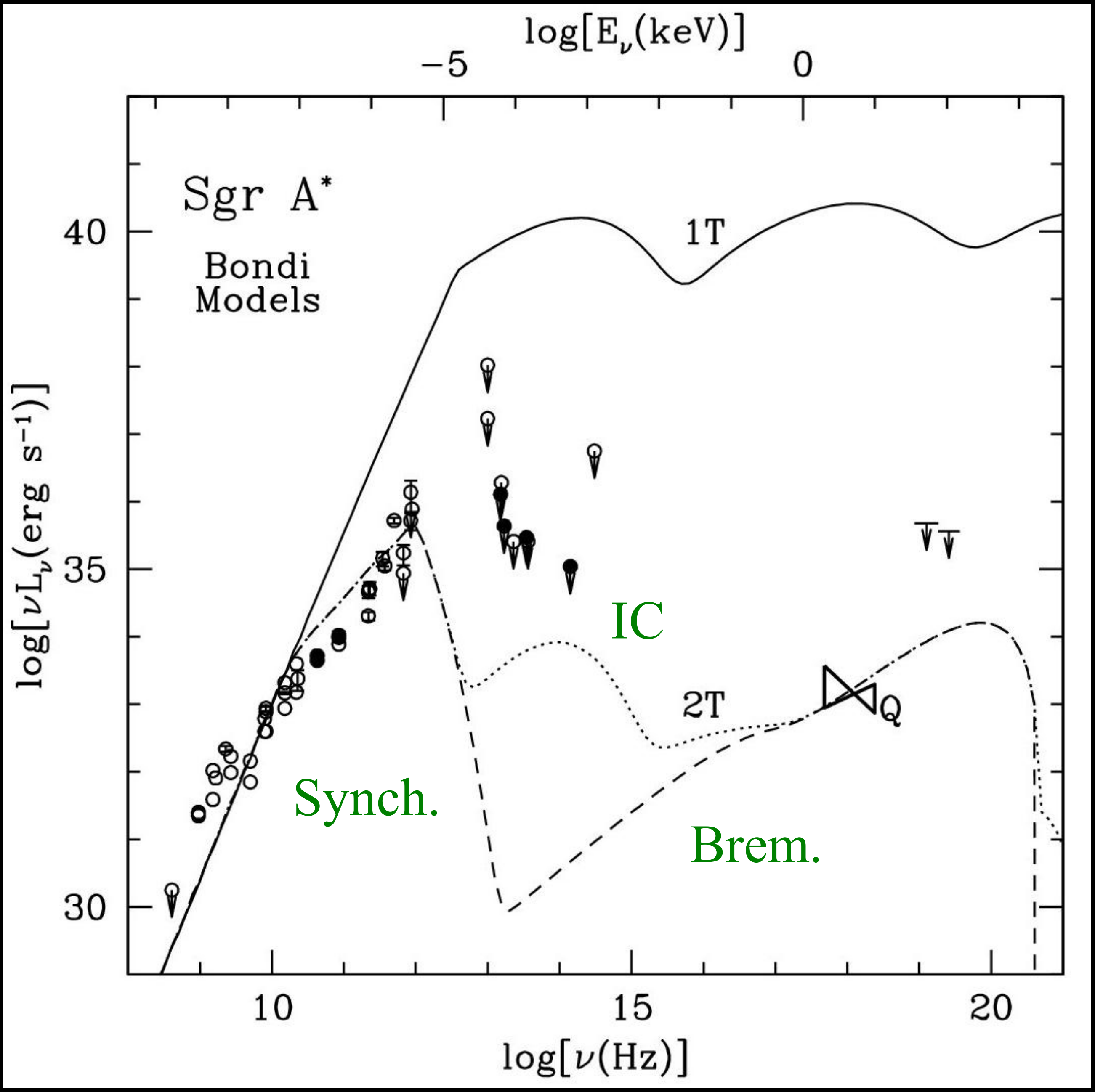
Disk Outflows



Disk Outflows



Realistic Bondi Accretion Spectrum



- Spherical accretion
- 2T : Te << Tp
- Including Synch., Bremsstrahlung (+IC)
- At low ρ, e's & ions decouple since

  $t_{Coulomb} > t_{infall}$
- Shapiro, Lightman, & Eardley (1976)

Narayan astro-ph/0201260