Physics A300: Classical Mechanics I

Problem Set 1

Assigned 2002 August 28
Due 2002 September 4 (accepted thru September 6)

Show your work on all problems! Note that the answers to the first problem are in the appendix of Marion & Thornton, but they should only be used to check your work, since the answers themselves are an insufficient solution to the problem.

1 Drill Problem on Matrix Operations (M & T 1-14)

Consider the matrices

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 3 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{pmatrix}$$

Calculate:

a) The determinant $\det \mathbf{AB}$

b) $\mathbf{AC}$

c) $\mathbf{ABC}$

d) $\mathbf{AB} - \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$

2 Drill Problem on Vector Operations

2.1 M & T 1-9

Consider the vectors

$$\vec{A} = \vec{e}_1 + 2\vec{e}_2 - \vec{e}_3 \qquad \vec{B} = -2\vec{e}_1 + 3\vec{e}_2 + \vec{e}_3$$

Calculate:

a) $\vec{A} - \vec{B}$ and its magnitude $\left|\vec{A} - \vec{B}\right|$

b) The component of $\vec{B}$ along $\vec{A}$

c) The angle between $\vec{A}$ and $\vec{B}$

d) $\vec{A} \times \vec{B}$

e) $(\vec{A} - \vec{B}) \times (\vec{A} - \vec{B})$

3 Properties of Rotation Matrices

3.1 Composition

Show that if $\mathbf{R}_1$ and $\mathbf{R}_2$ are orthogonal ($\mathbf{\Lambda}^{\mathrm{T}} = \mathbf{\Lambda}^{-1}$) and unimodular ($\det \mathbf{\Lambda} = 1$), then their product $\mathbf{R} = \mathbf{R}_1\mathbf{R}_2$ also satisfies both of these properties.

3.2 Rotations About Coördinate Axes

a) Write the matrices $\mathbf{R}_1(\theta_1)$, $\mathbf{R}_2(\theta_2)$, and $\mathbf{R}_3(\theta_3)$ describing rotations of θ_1, θ_2 and θ_3 about the x_1-, x_2-, and x_3-axes, respectively.

b) Show explicitly that each of them is an orthogonal matrix (its transpose is equal to its inverse) and unimodular (its determinant equals one).

4 A Meaningful Vector Identity (M & T 1-24)

Let $\vec{A}$ be an arbitrary vector, and let $\vec{e}$ be a unit vector (i.e., a vector such that $\vec{e} \cdot \vec{e} = 1$) in some fixed direction.

a) Show that

$$\vec{A} = \vec{e}(\vec{A} \cdot \vec{e}) + \vec{e} \times (\vec{A} \times \vec{e})$$

b) What is the geometrical significance of each of the two terms in the expansion?

Note: you may use the results of any of the previous book problems (1-1 to 1-23) in the demonstration.