Physics A300: Classical Mechanics I

Problem Set 3

Assigned 2002 September 16
Due 2002 September 23

Show your work on all problems!

1 Volume Integral (M & T 1-37)

Find the value of the integral

$$\oiint_{\mathcal{S}} \vec{A} \cdot d^2\vec{a}$$

where

$$\vec{A} = (x^2 + y^2 + z^2)(x\vec{e}_x + y\vec{e}_y + z\vec{e}_z)$$

and the surface $\mathcal{S}$ is a sphere of radius R centered on the origin. Do the integral

a) directly, and also

b) by using Gauss's theorem

2 Curls and Gradients

2.1 Curl of a Gradient

Let φ be any scalar field and consider the vector field

$$\vec{\nabla} \times (\vec{\nabla}\varphi)$$

Write the expression for the ith component of this vector field using the Levi-Civita symbol ε_{ijk} and, use a standard property of partial derivatives to simplify the expression. Write your final result in vector notation.

2.2 Integral of a Gradient Around a Closed Loop

Consider a curve $\mathcal{C}_1$ which begins at point P and ends at point Q and another curve $\mathcal{C}_2$ which begins at Q and ends at P. Let $\mathcal{C}$ be the closed curve which goes from P to Q along $\mathcal{C}_1$, then comes back from Q to P along $\mathcal{C}_2$. Calculate the integral

$$\oint_{\mathcal{C}} \vec{\nabla}\varphi \cdot d\vec{\ell}$$

of an arbitrary scalar field φ along the closed curve $\mathcal{C}$ by two methods:

a) Break up the integral along C into the piece along C_1 and the piece along C_2, evaluate each separately, and combine them:

$$\oint_{\mathcal{C}} \vec{\nabla}\varphi \cdot d\vec{\ell} = \int_{\mathcal{C}_1} \vec{\nabla}\varphi \cdot d\vec{\ell} + \int_{\mathcal{C}_2} \vec{\nabla}\varphi \cdot d\vec{\ell}$$

(You should be able to simplify this expression.)

b) Let $\mathcal{S}$ be any surface whose boundary $\partial\mathcal{S}$ is $\mathcal{C}$, use Stokes's theorem to rewrite the line integral along $\mathcal{C}$ as a surface integral over $\mathcal{S}$, and evaluate that integral.

Verify that the two methods give the same answer.

3 Drill Problem on Dimensional Analysis

3.1 Dimensionally Meaningful Expressions

Which of the following expressions or relations are sensible from a dimensional point of view? For the ones which don't, state the reason why not.

a) $5\,\text{m} + 100\,\text{in}$

b) $40\,\text{cm} + 100\,\text{kg}$

c) $x < 5$ where x is a length

d) $F = mx^2$ where F is a force, m is a mass, and x is a length

e) $\ddot{x} = g \sin t$ where x is a coördinate distance, $g = 9.8\,\text{m/s}^2$, and t is a time

f) $mv^2 - 5G\frac{Mm}{r}$ where m and M are masses, r is a length, and $G = 6.67 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$

3.2 Conversion of Units

Convert the following expressions into the units requested

a) $\frac{15\,\text{cm}+45\,\text{m}}{3\times10^8\,\text{m/s}}$ expressed in nanoseconds ($1\,\text{s} = 10^9\,\text{ns}$) (Your answer should be exact)

b) $1.25\,\text{in/yr}$ expressed in centimeters per second. (Your answer should be written to three significant figures.)

4 (M & T 2-17)

A softball player hits the ball at a height of $0.7\,\text{m}$ above home plate. The ball leaves the bat travelling in a direction which makes an angle $35°$ with the horizontal, and sails towards a fence $2\,\text{m}$ high and $60\,\text{m}$ away in centerfield. What must the initial speed of the softball be to clear the centerfield fence? Ignore air resistance, and take the acceleration of gravity to be $9.8\,\text{m/s}^2$.