

Physics A300: Classical Mechanics I

Problem Set 5

Assigned 2002 October 4

Due 2002 October 16

Show your work on all problems!

Note that solutions to this problem set will be distributed Wednesday, October 16, so no late homework can be accepted for this problem set.

1 Elastic Collision

1.1 Conservation of Energy and Momentum

Consider the following elastic collision: a particle of mass m moves at speed v towards another particle of mass m at rest. After the collision, the first particle has been deflected 45° from its initial trajectory. Use conservation of momentum and energy to calculate

- The speed of the first particle after the collision
- The speed of the second particle after the collision
- The direction in which the second particle is moving after the collision



Note: you do not need to model the details of the forces involved in the collision; just use the fact that the total momentum and energy of the system is the same before and after.

1.2 Galilean Transformation

Analyze problem 1.1 in the reference frame of an observer moving at a speed $v/2$ in the same direction as the first particle's original motion:

- Perform a Galilean transformation of the initial velocities of both particles into the new reference frame; sketch the pre-collision situation in this reference frame indicating the speed and direction of each particle's motion.

- b) Starting with the final velocities for the two particles which you found in problem 1.1, perform a Galilean transformation of these final velocities into the new reference frame; sketch the post-collision situation in this reference frame indicating the speed and direction of each particle's motion.
- c) Verify that energy and momentum are conserved in the new reference frame as well.

2 Perturbation Expansion

Consider a projectile fired straight up from the surface of the Earth with speed v_0 . The magnitude of the acceleration due to gravity is g at the surface of the earth, but decreases with altitude, remaining inversely proportional to the square of the distance to the *center* of the Earth. Assume the Earth is a sphere of radius R and neglect air resistance and the rotation of the Earth.

- a) Write down a dimensionless combination of the three parameters in the problem (v_0 , g , and R) which is inversely proportional to R .
- b) Calculate perturbatively the time the projectile spends in the air (between launch from and landing at the surface of the Earth) to first order in your dimensionless parameter (i.e., “to first order in $1/R$ ”).

3 Conservative Forces

Which of the following forces can be written in the form $\vec{F} = -\vec{\nabla}U$, and why?

- a) $\vec{F} = k_1(y\vec{e}_x - x\vec{e}_y)$
- b) $\vec{F} = k_2 \frac{x\vec{e}_x + y\vec{e}_y + z\vec{e}_z}{(x^2 + y^2 + z^2)^{3/2}}$
- c) $\vec{F} = k_3[3x^2y\vec{e}_x + (x^3 + y^3)\vec{e}_y]$

4 Time-averaged Quantities for a Simple Harmonic Oscillator

Consider a one-dimensional simple harmonic oscillator of mass m , and natural frequency ω_0 , oscillating with a position amplitude A . Calculate the time-average of the following quantities over a complete period:

- a) the position
- b) the velocity
- c) the kinetic energy
- d) the potential energy
- e) the total energy