Physics A301: Classical Mechanics II

Problem Set 2

Assigned 2003 January 22
Due 2003 January 29

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems.

1 Shortest Distance in Three Dimensions

Consider paths connecting two points (x_i, y_i, z_i) and (x_f, y_f, z_f) in three-dimensional space. In particular, limit attention to paths which are single-valued in z so they can be described by functions $x(z)$ and $y(z)$.

a) What are $x(z_i)$, $y(z_i)$, $x(z_f)$, $y(z_f)$ (the boundary conditions on $x(z)$ and $y(z)$)?

b) What is the infinitesimal distance $d\ell$ between (x_0, y_0, z_0) and $(x_0 + dx, y_0 + dy, z_0 + dz)$?

c) If (x_0, y_0, z_0) and $(x_0 + dx, y_0 + dy, z_0 + dz)$ both lie on the path described by functions $x(z)$ and $y(z)$, write dx and dy in terms of dz and those functions.

d) Write the infinitesimal distance $d\ell$ between the points on the path for which $z = z_0$ and $z = z_0 + dz$.

e) What is the rate $d\ell/dz$ at which the distance along the path given by $x(z)$ and $y(z)$ increases with z?

f) Write, as an integral over the coördinate z which parametrizes the path $(x(z), y(z), z)$, the total length $L[x, y]$ of that path from z_i to z_f. This should be expressed in terms of the functions $x(z)$ and $y(z)$ and their derivatives $x'(z)$ and $y'(z)$.

g) Using this result, identify the integrand $\mathcal{L}(x, y, x', y', z)$ in the functional

$$L[x, y] = \int_{z_i}^{z_f} \mathcal{L}(x(z), y(z), x'(z), y'(z), z)dz$$

h) Calculate the partial derivatives $\frac{\partial \mathcal{L}}{\partial x}$, $\frac{\partial \mathcal{L}}{\partial y}$, $\frac{\partial \mathcal{L}}{\partial x'}$, and $\frac{\partial \mathcal{L}}{\partial y'}$.

i) Using Euler's equations, find the differential equations satisfied by $x(z)$ and $y(z)$.

j) Find the particular solution to these differential equations subject to the boundary conditions in part a)

2 The Double Pendulum

Consider the system illustrated in the figure below: a mass m hangs from a fixed suspension point by a rod of length ℓ, and a second mass, also of mass m, hangs from the first mass by another rod, also of length ℓ.

a) Define a coördinate system in which the origin is at the suspension point of the first pendulum, the x direction is to the right, and the y direction is up. Let the position the first mass be given by Cartesian coördinates $(x_1(t), y_1(t))$ and the second by Cartesian coördinates $(x_2(t), y_2(t))$. Write the kinetic energies T_1 and T_2 and potential energies U_1 and U_2 in terms of the Cartesian coördinates and velocities.

b) If the first rod makes an angle $\theta_1(t)$ with the vertical and the second rod makes an angle $\theta_2(t)$ with the vertical, write

   i) $x_1(t)$ and $y_1(t)$ in terms of ℓ and $\theta_1(t)$;

   ii) $x_2(t)$ and $y_2(t)$ in terms of ℓ, $\theta_2(t)$, $x_1(t)$, and $y_1(t)$;

   iii) $x_2(t)$ and $y_2(t)$ in terms of ℓ, $\theta_2(t)$, and $\theta_1(t)$.

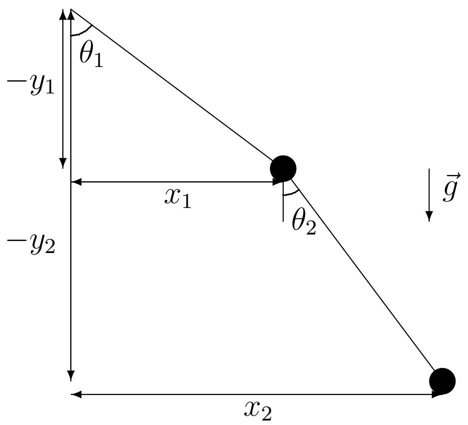
   iv) $\dot{x}_1(t)$, $\dot{y}_1(t)$, $\dot{x}_2(t)$, and $\dot{y}_2(t)$ in terms of ℓ, $\theta_1(t)$, $\theta_2(t)$, $\dot{\theta}_1(t)$, and $\dot{\theta}_2(t)$,

c) Write the kinetic energies T_1 and T_2 and potential energies U_1 and U_2 in terms of the parameters ℓ and m, the angles $\theta_1(t)$ and $\theta_2(t)$, and their derivatives $\dot{\theta}_1(t)$ and $\dot{\theta}_2(t)$.

d) Write the Lagrangian $L(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, t)$ describing the system, with θ_1 and θ_2 taken as the generalized coördinates.

e) Use the Euler-Lagrange equations to find the equations of motion satisfied by $\theta_1(t)$ and $\theta_2(t)$.

You should not assume either angle is small at any point in this problem.



3 Accelerated Pendulum

Marion & Thornton Problem 7-2.