Physics A300: Classical Mechanics I

Problem Set 2

Assigned 2002 September 4
Due 2002 September 11

Show your work on all problems! Note that the answers to the second problem are in the appendix of Symon, but they should only be used to check your work, since the answers alone are an insufficient solution to the problem.

1 Symon Chapter Two Problem 2

2 Symon Chapter Two Problem 21

3 Projectiles with Air Resistance

Consider a projectile fired into the air with initial speed v_0, in a constant gravitational field of strength g and subject to air resistance linear in the velocity with damping constant b, so that it experiences a force $F = -mg - b\dot{x}$.

a) Write the equation of motion in the form of an expression for $\ddot{x}(t)$, and its initial conditions in the form of expressions for $x(0)$ and $\dot{x}(0)$. (Assume the positive x direction is up and the projectile is launched from the origin at $t = 0$.)

b) Show that

   i) the only dimensionless combination of b, m, g, and v_0 which is linear in b is $\beta = \frac{bv_0}{mg}$

   ii) the only combination of m, g, and v_0 with units of length is $\frac{v_0^2}{g}$

   iii) the only combination of m, g, and v_0 with units of time is $\frac{v_0}{g}$

   The easiest way to do this is to consider the units of, e.g., $bm^p g^q v_0^r$ and find and solve the equation to find the values of p, q, and r needed to make the number of powers of mass, length and time vanish.

c) Defining the dimensionless quantities $\xi = \frac{x}{v_0^2/g}$ and $\tau = \frac{t}{v_0/g}$, find expressions for $\frac{dx}{dt}$ in terms of $\frac{d\xi}{d\tau}$ and $\frac{d^2x}{dt^2}$ in terms of $\frac{d^2\xi}{d\tau^2}$.

d) Using the results of part c) and the definition of ξ, rewrite the equation of motion and initial conditions from part a) in terms of $\xi(\tau)$, $\xi'(\tau)$ and $\xi''(\tau)$. You should be able to cancel out all dimensionful parameters and be left with an equation involving only the dimensionless quantities ξ, τ, and β [and the derivatives $\xi'(\tau)$ and $\xi''(\tau)$].

e) Since the particular trajectory which solves the equations of motion depends on the parameter β, we can write the solution for small values of β (small amounts of damping) as

$$\xi(\tau;\beta) = \xi_0(\tau) + \beta\xi_1(\tau) + \mathcal{O}(\beta^2) \tag{3.1}$$

where $\xi_0(\tau)$ and $\xi_1(\tau)$ are the same functions regardless of the value of β

Substitute (3.1) into your differential equation and initial conditions. Collect the terms independent of β, and the terms linear in β; any additional terms proportional to β^2 or higher powers of β can be absorbed into the notation $\mathcal{O}(\beta^2)$, which says we're not keeping track of quadratic and higher terms. In other words, rewrite the equations of motion and initial conditions each in the form

$$E_0 + \beta E_1 + \mathcal{O}(\beta^2) = 0 \tag{3.2}$$

where the expressions E_0 and E_1 do not contain β.

f) The equations (3.2) have to hold for a whole range of β values, so the coëfficients E_0 and E_1 must vanish separately. Use this to write

   i) A differential equation and initial conditions on $\xi_0(\tau)$, arising from the β-independent parts of your solution to part e)

   ii) A differential equation and initial conditions on $\xi_1(\tau)$ and $\xi_0(\tau)$ together, arising from the parts of your solution to part e) which were linear in β.