Physics A300: Classical Mechanics I

Problem Set 5

Assigned 2002 October 9
Due 2002 October 16

Show your work on all problems!

1 Fourier Analysis

Consider a square wave of period T and amplitude x_0, which is given inside the interval $0 < t < T$ by

$$F(t) = \begin{cases} F_0 & \text{when } 0 < t < T/2 \\ -F_0 & \text{when } T/2 < t < T \end{cases}$$

and which is defined outside that interval by its periodicity: $F(t+T) = F(t)$.

a) Use the methods described in section 2.11 of Symon to find the coëfficients A_n and B_n in the Fourier series

$$F(t) = \frac{1}{2} A_0 + + \sum_{n=1}^{\infty} \left(A_n \cos \frac{2\pi n t}{T} + B_n \sin \frac{2\pi n t}{T} \right)$$

Express your answers in a form involving no sines or cosines. (To do this, you'll need to write different expressions for odd and even n.)

b) Use the coëfficients $\{A_n\}$ and $\{B_n\}$ to write the force in the form

$$F(t) = \sum_{n=0}^{\infty} A_{0\,n} \cos(\omega_n t + \theta_{0\,n}) \ .$$

What are $A_{0\,n}$, ω_n, and $\theta_{0\,n}$?

c) If the square wave is applied to a harmonic oscillator of natural frequency ω_0 and damping parameter γ, i.e.:

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = F(t) \ ,$$

use the principle of superposition to find the values of $A_{s\,n}$, and β_n in the expansion

$$x(t) = \sum_{n=0}^{\infty} A_{s\,n} \sin(\omega_n t + \theta_{0\,n} + \beta_n)$$

of the steady-state solution.

2 Vector Practice

When doing the following problems from Symon, be sure to put an arrow over each vector (except for unit vectors, which get a hat). Symon uses boldface, but that's easy to lose track of, so for this course we'll insist on the arrow notation. So for example, Symon writes his equation (3.10) as

$$\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$$

while we will write

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

2.1 Explicit Calculation

Consider the vectors

$$\vec{A} = -\hat{x} + \hat{y} + 2\hat{z} \qquad \vec{B} = \hat{x} - 2\hat{y} + 3\hat{z}$$

Calculate:

a) $\vec{A} - \vec{B}$ and its magnitude $\left|\vec{A} - \vec{B}\right|$

b) $\vec{B} \cdot \vec{A}$

c) The angle between $\vec{A}$ and $\vec{B}$

d) $\vec{A} \times \vec{B}$

e) $(\vec{A} - \vec{B}) \times (\vec{A} - \vec{B})$

2.2 Symon Chapter Three Problem 1

You may skip part (e) or do it for extra credit.

2.3 Symon Chapter Three Problem 3