Physics A300: Classical Mechanics I

Problem Set 9

Assigned 2002 November 18
Due 2002 November 25

1 Properties of a Mass Distribution

Consider four identical particles, each of mass m, each moving in counter-clockwise around a circle of radius a in the x-y plane, centered at the origin, at constant angular velocity $\Omega > 0$, with their positions evenly spaced around the circle.

a) Sketch this situation, and label the particles 1 through 4.

b) Write the position vectors $\vec{r}_1(t)$, $\vec{r}_2(t)$, $\vec{r}_3(t)$, and $\vec{r}_4(t)$ if particle 1 crosses the positive x axis at $t = 0$. (Assume the orbital plane is $z = 0$.)

c) Calculate the velocities $\dot{\vec{r}}_1(t)$, $\dot{\vec{r}}_2(t)$, $\dot{\vec{r}}_3(t)$, and $\dot{\vec{r}}_4(t)$.

d) Calculate *explicitly*

   i) the total mass M;

   ii) the total momentum $\vec{P}$;

   iii) the position vector $\vec{R}$ of the center of mass;

   iv) the total angular momentum $\vec{L}$;

   v) the total kinetic energy T

   You don't need to calculate any components of vectors which vanish as a result of the motion being confined to a plane, but you should calculate all other components of the relative vectors, even if they turn out to be zero due to symmetry.

2 Decomposition of Angular Momentum

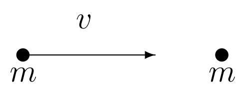
a) Substitute Symon's (4.19), the definition of angular momentum of particle k about a point $\mathcal{Q}$, into Symon's (4.23), the total angular momentum of a system about $\mathcal{Q}$. Expand the cross product $(\vec{r}_k - \vec{r}_{\mathcal{Q}}) \times (\dot{\vec{r}}_k - \dot{\vec{r}}_{\mathcal{Q}})$ appearing inside the sum in the resulting expression for $\vec{L}_{\mathcal{Q}}$ and use the definitions of the total mass M, center of mass $\vec{R}$, and total momentum $\vec{P}$ of the system to simplify your expression for $\vec{L}_{\mathcal{Q}}$ so that only one of the four terms still explicitly contains the sum over k, and the rest only contain M, $\vec{R}$, $\vec{P}$, $\vec{r}_{\mathcal{Q}}$, and $\dot{\vec{r}}_{\mathcal{Q}}$.

b) Simplify the result of part a) in the special case where the point $\mathcal{Q}$ is the origin of coördiates ($\vec{r}_{\mathcal{Q}} = \vec{0}$). Call this the total angular momentum $\vec{L}$.

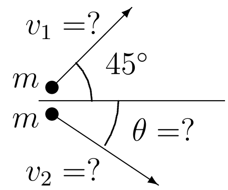
c) Simplify the result of part a) in the special case where the point $\mathcal{Q}$ is the center of mass ($\vec{r}_{\mathcal{Q}} = \vec{R}$). Call this the angular momentum $\vec{L}_{\text{com}}$ relative to the center of mass.

d) Use the results of parts b) and c) to find an expression for the total angular momentum $\vec{L}$ in terms of $\vec{R}$, $\vec{P}$, and $\vec{L}_{\text{com}}$.

3 Elastic Collision



Pre-collision



Post-collision

a) Consider a particle of mass m moving in the positive x direction at speed v towards another particle of mass m. (The figure labelled "Pre-collision" in the diagram above.) What are the total momentum $\vec{P}_I$ and total kinetic energy T_I of the system in terms of m and v?

b) Suppose the two particles have a collision, with the result that the first particle is moving in the x-y plane, at an angle of 45° above the positive x axis, with a speed v_1, and the second particle is moving in the x-y plane, at an angle of θ below the positive x axis. (The figure labelled "Post-collision" in the diagram above.) What are the total momentum $\vec{P}_F$ and total kinetic energy T_F of the system in terms of m, v_1, v_2, and θ? (Be sure to evaluate any trig functions of the known angle 45°.)

c) If the collision was elastic, and no other forces were involved aside from the short-range forces between the colliding particles, use conservation of momentum and energy to write three equations in the three unknowns v_1, v_2, and θ and solve for these in terms of m and v.

d) Consider the "Pre-collision" picture in a reference frame moving in the positive x direction at speed $v/2$, so that the two particles have velocities

$$\vec{v}'_{1I} = \vec{v}_{1I} - \hat{x}\,\frac{v}{2} \tag{3.1a}$$

$$\vec{v}'_{2I} = \vec{v}_{2I} - \hat{x}\,\frac{v}{2} \tag{3.1b}$$

i) Sketch the "Pre-collision" picture in this reference frame by working out the speeds and directions corresponding to the velocity vectors (3.1).

ii) Find the total momentum $\vec{P}'_I$ and total kinetic energy T'_I in this reference frame.

e) Consider the "Post-collision" picture in a reference frame moving in the positive x direction at speed $v/2$, so that the two particles have velocities

$$\vec{v}'_{1F} = \vec{v}_{1F} - \hat{x}\,\frac{v}{2} \tag{3.2a}$$

$$\vec{v}'_{2F} = \vec{v}_{2F} - \hat{x}\,\frac{v}{2} \tag{3.2b}$$

i) Sketch the "Post-collision" picture in this reference frame by working out the speeds and directions corresponding to the velocity vectors (3.2).

ii) Find the total momentum $\vec{P}'_F$ and total kinetic energy T'_F in this reference frame, and verify that energy and momentum are still conserved.