Vector and Tensor Notation Crib Sheet

Physics A301*

Spring 2004

Object	Symon's Notation	Our Notation (typeset)	Our Notation (handwritten)
Vector	$\mathbf{A}$	$\vec{A}$	$\vec{A}$
Tensor	$\mathbf{T}$	$\overleftrightarrow{T}$	$\overleftrightarrow{T}$
Matrix	$\mathbf{T}$	$\mathbf{T}$	$\underset{\sim}{T}$
Dyad	$\mathbf{AB}$	$\vec{A}\otimes\vec{B}$	$\vec{A}\otimes\vec{B}$

Note that we do *not* treat a tensor and the corresponding matrix as interchangeable within the same equation: the crossed-out equation in $\cancel{\overleftrightarrow{T}=\mathbf{T}}$ is not only not correct, it's meaningless, as it relates two different type of objects.

Given a coördinate system, there is a one-to-one correspondence between a tensor and the matrix containing its components. If we're dealing with two different sets of axes, the *same* tensor is represented by two different matrices. Explicitly,

$$\begin{aligned}\overleftrightarrow{T} =& T_{xx}(\hat{x}\otimes\hat{x}) + T_{xy}(\hat{x}\otimes\hat{y}) + T_{xz}(\hat{x}\otimes\hat{z}) + T_{yx}(\hat{y}\otimes\hat{x}) + T_{yy}(\hat{y}\otimes\hat{y}) + T_{yz}(\hat{y}\otimes\hat{z})\\ &+ T_{zx}(\hat{z}\otimes\hat{x}) + T_{zy}(\hat{z}\otimes\hat{y}) + T_{zz}(\hat{z}\otimes\hat{z})\\ =& T'_{xx}(\hat{x}'\otimes\hat{x}') + T'_{xy}(\hat{x}'\otimes\hat{y}') + T'_{xz}(\hat{x}'\otimes\hat{z}') + T'_{yx}(\hat{y}'\otimes\hat{x}') + T'_{yy}(\hat{y}'\otimes\hat{y}') + T'_{yz}(\hat{y}'\otimes\hat{z}')\\ &+ T'_{zx}(\hat{z}'\otimes\hat{x}') + T'_{zy}(\hat{z}'\otimes\hat{y}') + T'_{zz}(\hat{z}'\otimes\hat{z}')\end{aligned}$$

is represented by the matrix

$$\mathbf{T} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$$

in the basis $\{\hat{x},\hat{y},\hat{z}\}$ and the matrix

$$\mathbf{T}' = \begin{pmatrix} T'_{xx} & T'_{xy} & T'_{xz} \\ T'_{yx} & T'_{yy} & T'_{yz} \\ T'_{zx} & T'_{zy} & T'_{zz} \end{pmatrix}$$

in the basis $\{\hat{x}',\hat{y}',\hat{z}'\}$.

*Copyright 2004, John T. Whelan, and all that