Rigid Body Motion
(Symon Chapter Eleven)

Physics A301*

Spring 2004

Contents

1 Euler's Equations

Now consider how a rigid body actually moves → Chapter 11.

Total angular momentum $\vec{L} = \overleftrightarrow{I} \cdot \vec{\omega}$ for the body is because

$$\frac{d\vec{L}}{dt} = \vec{N} \tag{1.1}$$

(where $\vec{N}$ is the total external torque) just as

$$\frac{d\vec{P}}{dt} = \vec{F} \tag{1.2}$$

(where $\vec{F}$ is the total external force).

In an inertial coördinate system the body will in general change its orientation and the components of $\overleftrightarrow{I}$ will change.

To simplify the equation of motion for $\vec{\omega}$, analyze in a rotating basis co-moving with the body; then choose the axes to point along the principal axes of inertia sothat $\vec{e}'_i = \hat{u}_i$. Then

$$I'_{xx} = I_1 \tag{1.3a}$$
$$I'_{yy} = I_1 \tag{1.3b}$$
$$I'_{zz} = I_1 \tag{1.3c}$$

and the off-diagonal components vanish. This means

$$\vec{L} = \overleftrightarrow{I} \cdot \vec{\omega} = \sum_{i=1}^{3} L'_i \vec{e}'_i = \sum_{i=1}^{3}\sum_{j=1}^{3} I'_{ij}\omega'_j \vec{e}'_i = \sum_{i=1}^{3} I_i \omega'_i \vec{e}'_i \tag{1.4}$$

Now since the basis $\{\vec{e}'_i\}$ is rotating,

$$\vec{N} = \frac{d\vec{L}}{dt} = \frac{d'\vec{L}}{dt} + \vec{\omega} \times \vec{L} \tag{1.5}$$

where

$$\frac{d'\vec{L}}{dt} = \sum_{i=1}^{3} \frac{dL'_i}{dt} \vec{e}'_i \tag{1.6}$$

is the usual vector made up of the time derivatives of the components of a vector in the rotating coördinate system.

Note that as long as the primed basis vectors are co-rotating with the rigid body,

$$\frac{d'\overleftrightarrow{I}}{dt} = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{dI'_{ij}}{dt} \vec{e}'_i \vec{e}'_j \tag{1.7}$$

which means that

$$\vec{N} = \overleftrightarrow{I} \cdot \frac{d'\vec{\omega}}{dt} + \vec{\omega} \times \overleftrightarrow{I} \cdot \vec{\omega} \tag{1.8}$$

Looking at the x' component in detail, we have

$$N'_x = I_1 \frac{\omega'_x}{dt} + (\omega'_y L'_z - \omega'_z L'_y) = I_1 \frac{\omega'_x}{dt} + (I_3 \omega'_y \omega'_z - I_2 \omega'_z \omega'_y) \tag{1.9}$$

Things work out similarly for the y' and z' components, and we have

$$N'_x = I_1 \frac{\omega'_x}{dt} + (I_3 - I_2)\omega'_y \omega'_z \tag{1.10a}$$

$$N'_y = I_2 \frac{\omega'_y}{dt} + (I_1 - I_3)\omega'_x \omega'_z \tag{1.10b}$$

$$N'_z = I_3 \frac{\omega'_z}{dt} + (I_2 - I_1)\omega'_x \omega'_y \tag{1.10c}$$

These are called *Euler's Equations*. Symon writes this as his equation (11.7), but seems to have forgotten he's talking about the components in the coördinate system co-rotating with the body (since he calls the components of $\vec{\omega}$ simply $\{\omega_1, \omega_2, \omega_3\}$ rather than $\{\omega'_x, \omega'_y, \omega'_z\}$.

Note that if thee is no external torque, the components of $\vec{\omega}$ in the body system can still change, if the inertia tensor is not isotropic (e.g., if $I_1 \neq I_2$).

1.1 Free Precession of a Prolate or Oblate Object

For example, consider the case where $\vec{N} = \vec{0}$ and $I_1 = I_2 \neq I_3$. Examples of this would be a spheroid (an ellipsoid with two equal axes) or a square prism.

Euler's equations become

$$\dot{\omega}'_x = \frac{I_2 - I_3}{I_1}\omega'_y \omega'_z = \frac{I_1 - I_3}{I_1}\omega'_y \omega'_z \tag{1.11a}$$

$$\dot{\omega}'_y = \frac{I_3 - I_1}{I_1}\omega'_x \omega'_z \tag{1.11b}$$

$$\dot{\omega}'_z = 0 \tag{1.11c}$$

So ω'_z is a constant and

$$\Omega = \frac{I_3 - I_1}{I_1}\omega'_z \tag{1.12}$$

is a constant frequency, in terms of which the equations for $\dot{\omega}'_x$ and $\dot{\omega}'_y$ become

$$\dot{\omega}'_x = -\Omega \omega'_y \tag{1.13a}$$

$$\dot{\omega}'_y = \Omega \omega'_x \tag{1.13b}$$

This is not the most difficult system of ordinary differential equations in the world. The general solution is

$$\omega'_x = A\cos(\Omega t + \delta) \tag{1.14a}$$

$$\omega'_y = A\sin(\Omega t + \delta) \tag{1.14b}$$

where A and δ are constants chosen to match the initial conditions.

Let's visualize what's happening in the two cases:

1.1.1 Oblate $I_3 > I_1$

Then

$$\beta = \frac{\Omega}{\omega_z'} > 0 \tag{1.15}$$

Now, since

$$\omega \cdot \omega = {\omega_x'}^2 + {\omega_y'}^2 + {\omega_z'}^2 = A^2 + {\omega_z'}^2 = \text{constant} \tag{1.16}$$

and $\frac{d\vec{L}}{dt} = \vec{0}$ the lengths of the vectors $\vec{\omega}$ and $\vec{L}$ don't change although their components in one or more bases can.

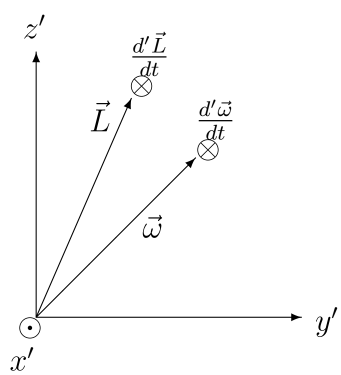
The components of $\vec{L}$ along the body axes are

$$L_x' = I_1\omega_x' = I_1 A\cos(\Omega t + \delta) \tag{1.17a}$$

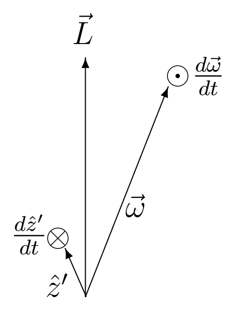
$$L_y' = I_1\omega_y' = I_1 A\sin(\Omega t + \delta) \tag{1.17b}$$

$$L_z' = I_3\omega_z' \tag{1.17c}$$

So if you look at the components in along the primed axes ("in the body frame") $\vec{L}$ and $\vec{\omega}$ appear to *precess* about a "fixed" z' axis with and angular frequency $\Omega = \frac{I_3 - I_1}{I_1}\omega_z'$. Assuming $\omega_z' > 0$ and taking a snapshot at an instant when ω_x' (and thus L_x') happens to vanish, it looks like this:



Of course, in the inertial frame, it is $\vec{L}$ that is fixed ($\frac{d\vec{L}}{dt} = \vec{N} = \vec{0}$) and $\vec{\omega}$ and $\hat{z}'$ both precess about it.



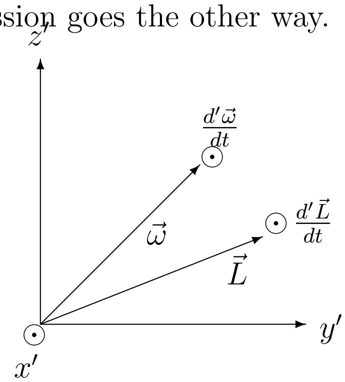
This is why there's no permanent South Pole: The Earth's rotation is not quite aligned with its body axis, so it "wobbles". The South Pole is where the direction of ω intersects the Earth, and that is precessing.

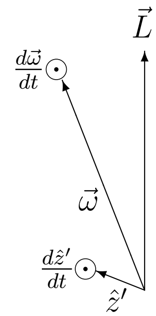
1.1.2 Prolate $I_3 < I_1$

Then

$$\frac{\Omega}{\omega'_z} < 0 \tag{1.18}$$

and the precession goes the other way.





This is why, when a football is not thrown in a tight spiral, you see the nose spin.

2 Euler Angles

Three numbers are needed to describe the orientation of a rigid bodyin space. For example, if you consider the orthonormal unit vectors $\hat{u}_1$, $\hat{u}_2$, $\hat{u}_3$ associated with the principal axes, specifying $\hat{u}_1$ takes two parameters (it's a vector, but you know $|\hat{u}_1| = 1$), then specifying $\hat{u}_2$, which lies ina planeperpendicular to $\hat{u}_1$, requires one,andthen you're done because $\hat{u}_3 = \hat{u}_1 \times \hat{u}_2$.

There are *lots* of different conventions on what those numbers are, e.g., in aeronautics one uses yaw, pitch, and roll. Our convention (i.e., Symon's, but it's a good one) is as follows. (See Figure 11.4 in Symon.) Rather than build up a mondo rotation matrix out of three rotations, focus on the two sets of axes $\hat{x}$, $\hat{y}$, $\hat{z}$ and $\hat{u}_1 = \hat{x}'$, $\hat{u}_2 = \hat{y}'$, $\hat{u}_3 = \hat{z}'$. In particular, treat the "z" axes preferentially, and look at the equatorial planes of the two systems.

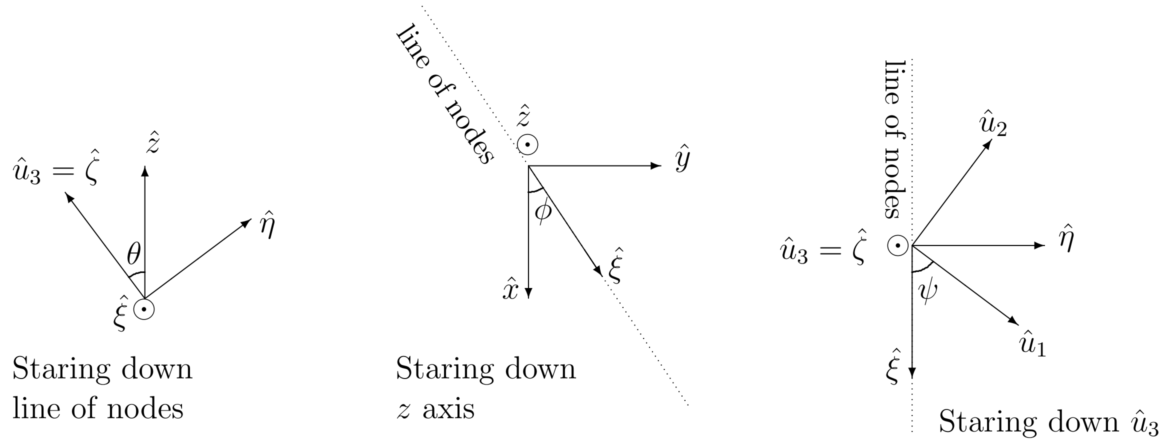
- θ is the angle between the z and z' axes, i.e., $\hat{z} \cdot \hat{u}_3 = \cos\theta$;
- ϕ completes the specification of $\hat{u}_3$;
- ψ locates $\hat{u}_1$ and $\hat{u}_2$ via a rotation about $\hat{u}_3$.

Now, we might like θ and ϕ to be the spherical coördinate angles corresponding to the direction $\hat{u}_3$, but the convention used actually makes those angles θ and $\phi - \frac{\pi}{2}$.

Convention/Definition: The two equatorial planes (perpendicular to $\hat{z}$ and perpendicular to $\hat{u}_3$, respectively) intersect in a line called the *line of nodes* (which is perpendicular to both $\hat{z}$ and $\hat{u}_3$). ϕ is the angle from the x axis to the line of nodes. It is useful to define an "intermediate" set of axes $\hat{\xi}$, $\hat{\eta}$, $\hat{\zeta}$, where $\hat{\zeta} = \hat{u}_3$, $\hat{\xi}$ points along the line of nodes, and $\hat{\eta} = \hat{\zeta} \times \hat{\xi}$.

We still have to specify the orientation of $\hat{u}_2$ and $\hat{u}_3$, and we do that by saying ψ is the angle (around the z' axis) from the line of nodes to $\hat{u}_1$.

Look at selected cross-sections…



Staring down line of nodes

Staring down z axis

Staring down $\hat{u}_3$

To rotate $\hat{x}$, $\hat{y}$, $\hat{z}$ into $\hat{u}_1$, $\hat{u}_2$, $\hat{u}_3$:

1. Rotate ϕ about $\hat{z}$
2. Rotate θ about $\hat{\xi}$
3. Rotate ψ about $\hat{\zeta} = \hat{u}_3$

Of course, what we really want is $\vec{\omega}$ in terms of $\dot{\theta}$, $\dot{\phi}$, and $\dot{\psi}$.

Symon proves that if $\hat{x}^*$, $\hat{y}^*$, $\hat{z}^*$ are rotating relative to $\hat{x}$, $\hat{y}$, $\hat{z}$ at angular velocity $\vec{\omega}_1$ and $\hat{x}'$, $\hat{y}'$, $\hat{z}'$ are rotating relative to $\hat{x}^*$, $\hat{y}^*$, $\hat{z}^*$ at angular velocity $\vec{\omega}_2$ then $\hat{x}'$, $\hat{y}'$, $\hat{z}'$ are rotating relative to $\hat{x}$, $\hat{y}$, $\hat{z}$ at angular velocity $\vec{\omega}_1 + \vec{\omega})_2$. Basically, this is a manifestation of the fact that infinitesimal rotations add like vectors.

This means

$$\vec{\omega} = \dot{\theta}\hat{\xi} + \dot{\phi}\hat{z} + \dot{\psi}\hat{u}_3 \tag{2.1}$$

Now, to attach lots of problems, we want to use a Lagrangian method, which means finding $T = \frac{1}{2}\vec{\omega} \cdot \overleftrightarrow{I} \cdot \vec{\omega}$ in terms of θ, ϕ, ψ, $\dot{\theta}$, $\dot{\phi}$, $\dot{\psi}$ so we should try to resolve

$$\vec{\omega} = \omega_1'\hat{u}_1 + \omega_2'\hat{u}_2 + \omega_3'\hat{u}_3 \tag{2.2}$$

since

$$T = \frac{1}{2}\vec{\omega} \cdot \vec{\omega} = \frac{1}{2}I_1{\omega_1'}^2 + \frac{1}{2}I_2{\omega_2'}^2 + \frac{1}{2}I_3{\omega_3'}^2 \tag{2.3}$$

So, look at the geometry in order to get $\hat{\xi}$ and $\hat{z}$ in terms of the basis vectors pointing along principal axes of inertia.

$$\hat{\xi} = \hat{u}_1 \cos\psi - \hat{u}_2 \sin\psi \tag{2.4a}$$
$$\hat{\eta} = \hat{u}_1 \sin\psi + \hat{u}_2 \cos\psi \tag{2.4b}$$
$$\hat{\zeta} = \hat{u}_3 \tag{2.4c}$$

which means

$$\hat{z} = \hat{\eta}\sin\theta + \hat{\zeta}\cos\theta = \hat{u}_1 \sin\theta\sin\psi + \hat{u}_2 \sin\theta\cos\psi + \hat{u}_3\cos\theta \tag{2.5}$$

Putting it together,

$$\vec{\omega} = (\dot{\theta}\cos\psi + \dot{\phi}\sin\theta\sin\psi)\hat{u}_1 + (-\dot{\theta}\sin\psi + \dot{\phi}\sin\theta\cos\psi)\hat{u}_2 + (\dot{\phi}\cos\theta + \dot{\psi})\hat{u}_3 \tag{2.6}$$

In general,

$$T(\theta,\phi,\psi,\dot\theta,\dot\phi,\dot\psi) = \frac{1}{2}I_1(\dot\theta\cos\psi+\dot\phi\sin\theta\sin\psi)^2+\frac{1}{2}I_2(-\dot\theta\sin\psi+\dot\phi\sin\theta\cos\psi)^2+\frac{1}{2}I_3(\dot\phi\cos\theta+\dot\psi)^2 \tag{2.7}$$

This has cross terms involving $\dot\theta\dot\phi$ and $\dot\phi\dot\psi$, but if $I_1 = I_2$, the $\dot\theta\dot\phi$ terms cancel, and it simplifies somewhat. Then

$$T = \frac{1}{2}I_1\dot\theta^2 + \frac{1}{2}I_1\sin^2\theta\dot\phi^2 + \frac{1}{2}I_3(\dot\phi\cos\theta+\dot\psi)^2 \qquad (\text{when } I_1 = I_2) \tag{2.8}$$

This is the setup for the *symmetrical top*

3 The Symmetrical Top

One of the classic rigid body problems.

Consider a "solid of rotation" which has rotational symmetry about a symmetry axis which we call $\hat u_3$. From the geometry, $\hat u_3$ is a principal axis of inertia, and $I_1 = I_2$. Also, the center of mass is on the symmetry axis. Let ℓ be the distance of the center of mass from the tip of the top. Describe the situation where one point (the aforementioned tip) is fixed, in a Lagrangian formalism.

The generalized coördinates are the Euler angles θ, ϕ, ψ. From Section 2, we know

$$T = \frac{1}{2}I_1\dot\theta^2 + \frac{1}{2}I_1\sin^2\theta\dot\phi^2 + \frac{1}{2}I_3(\dot\phi\cos\theta+\dot\psi)^2 \tag{3.1}$$

If the top is moving in a uniform gravitational field in the z direction, with the zero of potential energy defined at the height of the fixed tip, the potential energy is

$$V = \iiint \rho g z\,dx\,dy\,dz = Mg\cdot(z\text{ coörd of C.O.M.}) = Mg\ell\cos\theta \tag{3.2}$$

so

$$L = \frac{1}{2}I_1\dot\theta^2 + \frac{1}{2}I_1\sin^2\theta\dot\phi^2 + \frac{1}{2}I_3(\dot\phi\cos\theta+\dot\psi)^2 - Mg\ell\cos\theta \tag{3.3}$$

We can analyze the motion using symmetries.

$$\frac{\partial L}{\partial\phi} = 0 = \frac{\partial L}{\partial\psi} \tag{3.4}$$

so ϕ and ψ are ignorable coördinates and p_ϕ and p_ψ are conserved. Also

$$\frac{\partial L}{\partial t} = 0 \tag{3.5}$$

so H will be conserved.

The conserved conjugate momenta are

$$p_\phi = \frac{\partial L}{\partial\dot\phi} = I_1\sin^2\theta\dot\phi + I_3\cos\theta(\dot\psi+\dot\phi\cos\theta) \tag{3.6a}$$

$$p_\psi = \frac{\partial L}{\partial\dot\psi} = I_3(\dot\psi+\dot\phi\cos\theta) \tag{3.6b}$$

For reference,

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = I_1 \dot{\theta} \tag{3.6c}$$

The Hamiltonian is

$$\begin{aligned}
H &= p_\theta \dot{\theta} + p_\phi \dot{\phi} + p_\psi \dot{\psi} - L \\
&= I_1 \dot{\theta}^2 + I_1 \sin^2\theta \dot{\phi}^2 + I_3 \dot{\phi}\cos\theta(\dot{\psi} + \dot{\phi}\cos\theta) + I_3 \dot{\psi}(\dot{\psi} + \dot{\phi}\cos\theta) \\
&\quad - \frac{1}{2} I_1 \dot{\theta}^2 - \frac{1}{2} I_1 \sin^2\theta \dot{\phi}^2 - \frac{1}{2} I_3 (\dot{\phi}\cos\theta + \dot{\psi})^2 + Mg\ell\cos\theta \\
&= T + V = E
\end{aligned} \tag{3.7}$$

I.e., since the kinetic energy (3.1) is quadratic in the velocities (this time including cross terms) the Hamiltonian is equal to the total energy.

If we make the substitution

$$\dot{\psi} + \dot{\phi}\cos\theta = \frac{p_\psi}{I_3} \tag{3.8}$$

(3.6a) becomes

$$p_\phi = I_1 \sin^2\theta \dot{\phi} + p_\psi \cos\theta \tag{3.9}$$

And

$$E = \frac{1}{2} I_1 \dot{\theta}^2 + \frac{(p_\phi - p_\psi\cos\theta)^2}{2I_1\sin^2\theta} + \frac{p_\psi^2}{2I_3} + mg\ell\cos\theta \tag{3.10}$$

which can be analyzed analogous to

$$E = \frac{1}{2} m\dot{x}^2 + V(x) \tag{3.11}$$

or

$$E = \frac{1}{2} m\dot{r}^2 + V_{\text{eff}}(r) \tag{3.12}$$

4 Course Summary

I A. Gravity (Chapter 6)

B. Moving Coördinate Systems (Sec 7.1-7.4 + tides)

II C. Lagrangian Mechanics (Sec 9.1-9.10)

III D. Tensors (Sec 10.1-10.5 – Note differences in our notation)

E. Rigid Body Motion (Sec 11.1, 11.2, 11.4, [11.5])

4.1 Gravity & Non-Inertial Coördinates

See summary from 2004 February 17 (Section 6 of Chapter 7 Notes)

4.2 Lagrangian Mechanics

See lightning recap from 2004 March 25 and review from 2004 April 13 (Sections 6-7 of Chapter 9 notes)

4.3 Tensors

Tensor Notation | **Matrix Notation**

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z} \tag{4.1}$$

$$\hat{x} = \hat{e}_1, \qquad \hat{y} = \hat{e}_2, \qquad \hat{z} = \hat{e}_3 \tag{4.2}$$

$$\mathbf{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \qquad \mathbf{A}^\mathrm{T} = \begin{pmatrix} A_x & A_y & A_z \end{pmatrix} \tag{4.4}$$

so

$$\vec{A} = \sum_{i=1}^{3} A_i\hat{e}_i \tag{4.3}$$

$$\mathbf{A} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \qquad \mathbf{A}^\mathrm{T} = \begin{pmatrix} A_1 & A_2 & A_3 \end{pmatrix} \tag{4.5}$$

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z = \mathbf{A}^\mathrm{T}\mathbf{B} \tag{4.6}$$

Since this is just a number, the two are equal. Otherwise, we don't talk about equality, but rather correspondence

$$\cancel{\vec{A} = \mathbf{A}} \tag{4.7}$$

$$(\vec{A} \otimes \vec{B}) \cdot \vec{C} = \vec{A}(\vec{B} \cdot \vec{C}) \tag{4.8}$$

$$(\mathbf{A}\mathbf{B}^\mathrm{T})\mathbf{C} = \mathbf{A}(\mathbf{B}^\mathrm{T}\mathbf{C}) \tag{4.9}$$

$$\overleftrightarrow{T} = \sum_{i=1}^{3}\sum_{j=1}^{3} T_{ij}\hat{e}_i\hat{e}_j \tag{4.10}$$

note

$$T_{ij} = \hat{e}_i \cdot \overleftrightarrow{T} \cdot \hat{e}_j \tag{4.11}$$

$$\mathbf{T} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix} \tag{4.12}$$

A symmetric matrix has $\mathbf{T}^\mathrm{T} = \mathbf{T}$ i.e., $T_{ij} = T_{ji}$

A symmetric tensor has $\overleftrightarrow{T}^\mathrm{T} = \overleftrightarrow{T}$.

If $\overleftrightarrow{T}$ is symmetric, there is always an *orthonormal* basis $\{\hat{u}_1, \hat{u}_2, \hat{u}_3\}$ such that

$$\overleftrightarrow{T} = T_1\hat{u}_1 \otimes \hat{u}_1 + T_2\hat{u}_2 \otimes \hat{u}_2 + T_3\hat{u}_3 \otimes \hat{u}_3 \tag{4.13}$$

$\hat{u}_i$ is an eigenvector with eigenvalue T_i, since $\overleftrightarrow{T} \cdot \hat{u}_i = T_i\hat{u}_i$

4.4 Inertia Tensor

$$\vec{L} = \overleftrightarrow{I} \cdot \vec{\omega} \tag{4.14}$$

where

$$\overleftrightarrow{I} = \sum_{k=1}^{N} m_k(r_k^2\overleftrightarrow{1} - \vec{r}_k \otimes \vec{r}_k) \tag{4.15}$$

- Be familar with derivation

- Be able to construct $\overleftrightarrow{I}$ from distribution of point masses or

$$\overleftrightarrow{I} = \iiint (r^2 \overleftrightarrow{1} - \vec{r} \otimes \vec{r})\, \rho(\vec{r})\, d^3V \tag{4.16}$$

  Note! This is a lot like

$$\varphi(\vec{r}) = -\sum_{k=1}^{N} \frac{Gm_k}{|\vec{r} - \vec{r}_k|} \tag{4.17}$$

  or

$$\vec{g}(\vec{r}) = -\iiint Gm_k \rho(\vec{r}') \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|} d^3V' \tag{4.18}$$

  Would do well to practice this!

Also note

- Rotation of axes to represent same tensor $\overleftrightarrow{I}$ in different bases.
- Translation of origin to get different $\overleftrightarrow{I}_{\mathcal{O}'}$ vs $\overleftrightarrow{I}_{\mathcal{O}}$

  Note the identity

$$\overleftrightarrow{I}_{\mathcal{O}} = \overleftrightarrow{I}_{\mathcal{G}} + M(R^2 \overleftrightarrow{1} - \vec{R} \otimes \vec{R}) \tag{4.19}$$

  where $\vec{R}$ is the position vector of the center of mass $\mathcal{G}$ with respect to the origin $\mathcal{O}$

Note $I_{ij} = I_{ji}$ emphalways.

THE INERTIA TENSOR IS SYMMETRIC

4.5 Rigid Body Motion

4.5.1 Euler's Equations

$$\vec{N} = \frac{d\vec{L}}{dt} = \frac{d'\vec{L}}{dt} + \vec{\omega} \times \vec{L} = \frac{d'(\vec{I} \cdot \vec{\omega})}{dt} + om\vec{e}ga \times \overleftrightarrow{I} \cdot L \tag{4.20}$$

where $\vec{N}$ is the external torque. If the primed coördinate axes are chosen to move and rotate with the rigid body, and line up with the body axes, then $I'_{ij} = \delta_{ij} I_i$ for all time.

Be familiar with the derivation and consequences.

- If $I_1 = I_2 \neq I_3$, then $\vec{\omega}$ and $\vec{L}$ precess around $\hat{u}_3$ in the body frame while $\hat{u}_3$ and $\vec{\omega}$ precess around $\vec{L}$ in an inertial frame.
- If all the eigenvalues are different, the body can precess around the axis with the highest or lowest eigenvalue, but not the middle one (you showed this on the homework).

4.5.2 Euler Angles

- Be able to do rotational manipulations
- Be acquainted with the symmetrical top as an application

A Appendix: Correspondence to Class Lectures

Date	Sections	Pages	Topics
2004 Apr 27	1	2–5	Euler's Equation; Free Precession
2004 Apr 27	2	5–7	Euler Angles
2004 Apr 29	3	7–8	The Symmetric Top
2004 May 4	4	8–10	Course Summary