

# Physics A301: Classical Mechanics II

## Problem Set 1

Assigned 2004 January 15  
Due 2004 January 22

**Show your work on all problems!** Be sure to give credit to any collaborators, or outside sources used in solving the problems.

## 1 Position-Dependent Mass Distribution

Consider a spherically symmetric distribution of mass with density

$$\rho(\vec{r}') = \frac{Ma^2}{2\pi r'(r'^2 + a^2)^2} \quad 0 \leq r' < \infty \quad (1.1)$$

- Show by explicit calculation that the parameter  $M$  appearing in (1.1) is indeed the total mass of the distribution.
- Calculate the gravitational potential  $\varphi(\vec{r})$  due to this mass distribution.
- From your result to part b), calculate the gravitational field  $\vec{g}(\vec{r})$  of this mass distribution, taking the gradient ( $\vec{g} = -\vec{\nabla}\varphi$ ) in spherical coordinates.

## 2 Flat Earth Society

Consider an infinite sheet of mass of thickness  $L$  and uniform density  $\rho$ . Use symmetry properties of the mass distribution, along with Gauss's law for gravitation (which says that the flux of the gravitational field through any closed surface is equal to  $-4\pi G$  times the mass enclosed within the surface) to deduce the gravitational field as follows:

- By considering a surface lying entirely above or below the sheet, show that the gravitational field is constant outside the sheet. Be sure to explain clearly and carefully your reasoning in each step.
- By considering a surface whose top is above and whose bottom is below the sheet, find the magnitude of the gravitational field outside the sheet.
- If the Earth were an infinite sheet of density  $5.5 \text{ g/cm}^3$ , how thick would it need to be to produce the observed gravitational acceleration of  $9.8 \text{ m/s}^2$ ?

### 3 Dig a Hole to China

Assume the Earth is a uniform-density sphere of mass  $M$  and radius  $R$ .

- a) What is the density  $\rho$  inside the Earth in terms of  $M$  and  $R$ ? For the rest of the problem, express your final answers in terms of  $M$  and  $R$ , without referring to  $\rho$ .
- b) Defining spherical coordinates with their center at the center of the Earth, find the gravitational field  $\vec{g}(\vec{r})$  at any point, inside or outside the Earth. (To make this as easy as possible, you should use the physical result concerning the gravitational influence of a spherical shell of mass from section 6.2 of Symon, also derived in class January 20th.<sup>1</sup>)
- c) By solving the differential equation  $\vec{g} = -\vec{\nabla}\varphi$ , find the gravitational potential  $\varphi(\vec{r})$  both inside and outside the Earth.
- d) Consider a tunnel of negligible width (so we can ignore the mass removed to drill it) through the Earth along the  $z$  axis.
  - i) Show that the equation of motion for a particle dropped into the tunnel is that of a simple harmonic oscillator and write the oscillation frequency in terms of  $M$  and  $R$  (and  $G$ ).
  - ii) Supposing the particle is dropped from rest at the surface of the Earth (so that  $\vec{r}(0) = R\hat{z}$  and  $\vec{v}(0) = \vec{0}$ ), what is its speed as it passes the center of the Earth?
  - iii) Look up the actual mass and radius of the Earth and express the results to parts i) and ii) in reasonable physical units for those values of  $M$  and  $R$ . Make sure your results are expressed to an appropriate number of significant figures for the values you quote.

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<sup>1</sup>Succinctly stated, this is that the gravitational field due to a spherical shell vanishes if you're inside the shell and is the same as that of a point mass at the center if you're outside the shell.