

Physics A301: Classical Mechanics II

Problem Set 3

Assigned 2004 January 29
Due 2004 February 5

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems.

1 Periodically Accelerating Reference Frame

Symon Chapter Seven, Problem Two.

2 Rotational Oblateness of the Earth

Consider the Earth, rotating at a fixed angular velocity $\vec{\omega}$. Define two sets of Cartesian coordinates, (x, y, z) and (x^*, y^*, z^*) , each with its origin at the center of the Earth and its positive z (or z^*) axis pointed towards the North Pole along the Earth's rotation axis, so that $\vec{\omega} = \omega \hat{z} = \omega \hat{z}^*$. Let the unstarred coordinate axes be fixed in space while the starred ones rotate along with the Earth.

- Define spherical coordinates, (r, θ, ϕ) and (r^*, θ^*, ϕ^*) , respectively, corresponding to the non-rotating and rotating Cartesian coordinate systems. Explain (or show) why $r^* = r$ and $\theta^* = \theta$, and also $\hat{r}^* = \hat{r}$ and $\hat{\theta}^* = \hat{\theta}$.
- Assume that most of the matter in the Earth is spherically symmetric about its center, so that the gravitational field is well approximated by

$$\vec{g} = -g(r) \hat{r} \quad (2.1)$$

where the magnitude $g(r)$ is a function of r alone. Find the effective gravitational field \vec{g}^{eff} including centrifugal effects in the co-rotating coordinate system, in terms of $g(r)$, ω , r , θ , and the basis vectors \hat{r} and $\hat{\theta}$.

- In the limit $\omega \rightarrow 0$, the Earth is just a sphere of radius R and mass M , and the gravitational acceleration at its surface is

$$g_0 = \frac{GM}{R^2}. \quad (2.2)$$

Construct the dimensionless combination of ω , g_0 , and R which is proportional to ω^2 , and call this ε .

d) Let the actual shape of the surface of the rotating Earth be given by

$$r = R + \delta R(\theta) \quad (2.3)$$

where $\delta R(\theta)$ is a small correction of order ε . If \vec{n} is a vector normal (perpendicular) to this surface, what is the ratio of components n_θ/n_r ?

- e) Using the results of part b), construct the ratio $g_\theta^{\text{eff}}/g_r^{\text{eff}}$. This should be proportional to ω^2 and therefore also of order ε .
- f) In your answer to part e), set r to R and neglect the ω^2 term appearing in the *denominator*. (These approximations are justified because anything more accurate would just add a correction of order ε^2 .) Express this approximate ratio in terms of g_0 , R , ω , and θ . Verify that you get sensible results at the North Pole ($\theta = 0$), the Equator ($\theta = \pi/2$) and the South Pole ($\theta = \pi$).
- g) By requiring \vec{g}^{eff} to be normal to the surface of the Earth, obtain and solve a differential equation for $\delta R(\theta)$. (Don't forget to include the integration constant in the solution.) What is the difference between the polar and equatorial radii, in terms of g_0 , R , and ω ?
- h) Using the actual values for the Earth ($g_0 = 9.8 \text{ m/s}^2$, $\omega = 2\pi/(24 \text{ hr})$, and $R = 6.4 \times 10^3 \text{ km}$), evaluate the following ratios to two significant figures:
- i) ε , the "small" parameter defined in part c);
 - ii) The fractional decrease of the magnitude of \vec{g}^{eff} at the equator relative to the poles;
 - iii) $\frac{\delta R(\pi/2) - \delta R(0)}{R}$, the size of the equatorial bulge as a fraction of the Earth's radius.

3 Deflection of a Falling Object Due to Coriolis Force

Do Symon Chapter Seven, Problem Seven. Also evaluate your expression for the displacement, for an object dropped from a height of 20 meters at a latitude of 30°N , and specify the direction of the deflection.