Physics A300: Classical Mechanics I

Problem Set 1

Assigned 2006 January 12
Due 2006 January 20

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that the answers to the some problems may be in the appendix of Symon, but they should only be used to check your work, since the answers themselves are an insufficient solution to the problem.

1 Drill Problem on Dimensional Analysis

1.1 Dimensionally Meaningful Expressions

Which of the following expressions or relations are sensible from a dimensional point of view? For the ones which aren't, state the reason why not.

a) $40\,\text{kg} + 15\,\text{N}$

b) $5\,\text{ft} + 1\,\text{km}$

c) $F > 5$ where F is a force

d) $F = mx$ where F is a force, m is a mass, and x is a length

e) $v^2 - 5G\frac{M}{r}$ where M is a mass, r is a length, and $G = 6.67 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$

f) $\ddot{x} = ge^t$ where x is a coördinate distance, $g = 9.8\,\text{m/s}^2$, and t is a time

1.2 Conversion of Units

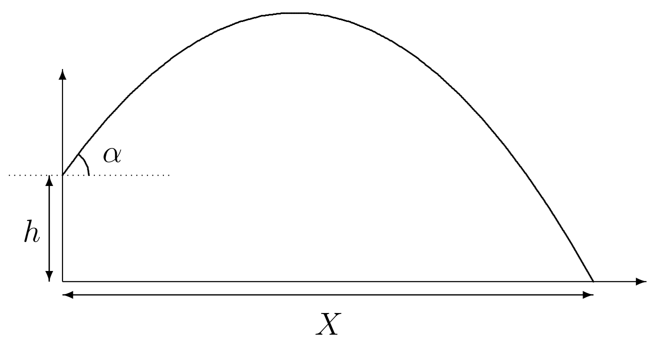
Convert the following expressions into the units requested

a) $\frac{18\,\text{cm}+24\,\text{m}}{3\times10^8\,\text{m/s}}$ expressed in nanoseconds ($1\,\text{s} = 10^9\,\text{ns}$) (Your answer should be exact)

b) $5.12\,\text{in/yr}$ expressed in centimeters per second. (Your answer should be written to three significant figures.)

2 Projectile Motion

An Olympic athlete throws a shotput, releasing it from a height of 2.00 meters with an initial speed of 10.0 meters per second. The shotput lands 10.0 meters from where the athlete was standing. Neglecting air resistance and taking the acceleration of gravity to be $9.80\,\mathrm{m/s^2}$, at what angle (in degrees) to the horizontal was the shotput thrown? (There are two possible values for this angle; give both of them.)



You may find it useful to break up the problem into the following steps:

First, it's usually best to wait until the end of a problem before inserting numerical values, so define $h = 2.00\,\mathrm{m}$, $v_0 = 10.0\,\mathrm{m/s}$, $X = 10.0\,\mathrm{m}$, and $g = 9.80\,\mathrm{m/s^2}$. Call the unknown angle, for which we're trying to solve, α. Define a coördinate system in which the origin is at the athlete's feet, the y axis points straight up, and the x axis points along the ground in the direction the shotput was thrown. The trajectory of the shotput is then defined by $x(t)$ and $y(t)$.

a) Consider the forces on the shotput while it's in flight and find the components $\ddot{x}(t)$ and $\ddot{y}(t)$ of the acceleration it experiences at an arbitrary time.

b) Let $t = 0$ be the time when the shotput is released, and work out the components $\dot{x}(0)$ and $\dot{y}(0)$ of the initial velocity, i.e., the velocity at $t = 0$.

c) Write down the coördinates $x(0)$ and $y(0)$ of the initial position of the shotput. The answers to this part and the previous one make up the *initial conditions* for the differential equation given in part a).

d) Integrate the equations of motion from part a) to obtain expressions for $\dot{x}(t)$ and $\dot{y}(t)$. (You may want to refer to the Calculus handout, but note that you'll need to use a name other than t for the integration variable if you use the definite integral method.)

e) Integrate a second time to obtain expressions for $x(t)$ and $y(t)$. (Same advice as before applies.)

f) You now have the general trajectory in terms of a bunch of known constants and the unknown angle α. Now you need to use the fact that the shotput lands a distance X from the athlete's feet. Define T to be the (unknown) time when this happens, and write expressions for $x(T)$ and $y(T)$ both from the trajectory you found in part e) and the fact that T is the time when the shotput lands. Equating the two expressions for $x(T)$ gives you one equation and equating the two expressions for $y(T)$ gives you another.

g) From part f) you have two equations in the two unknowns T and α. Since you don't care about the value of T, use one equation to eliminate T from the other and obtain a single equation in α.

h) Your equation from part g) will probably contain more than one trigonometric function of α. However, by starting from the identity $\sin^2\alpha + \cos^2\alpha = 1$ (one of the only three trig identities you really need to memorize, as all the others can be quickly derived from those three), you should be able to replace one of the trig functions of α with an expression containing only the other one. The resulting equation will be a quadratic equation in that remaining trig function.

i) Now is a good time to put in the actual numerical values of g, v_0, h, and X. It's a good idea to divide by the coëfficient of the quadratic term first, though, and verify that all the terms in the quadratic equation are dimensionless.

j) Use the quadratic formula to find the two roots and take the appropriate inverse trig function (with a calculator!) to obtain the two solutions for α. Convert these into degrees, and you're done.

3 Orbits and Gravity

The Sun orbits the center of our Milky Way galaxy in a circle of radius approximately 2.5×10^4 light years[1], at about 9.4×10^{-4} times the speed of light. Assuming all of the mass of the galaxy is concentrated at its center, work out the ratio of our galaxy's mass to the Sun's mass (i.e., the mass of the Milky Way in solar masses). Do this by comparing the Sun's orbit around the galactic center with the Earth's orbit (in a circle of radius 1.5×10^8 km) around the Sun, without looking up any other quantities such as the mass of the Sun or the value of G.

[1] A light year is the distance light travels in one year, i.e., 9.46×10^{12} km