WS for December 17th : 1016-351: Coverage –3.3, 3.4: Dr. Chulmin Kim

(Useful formulas are provided on the back.)

1. Wegman's gives its customers cards that may win them a prize when matched with other cards. The back of the card announces the following probabilities of winning various amounts:

Amount	\$100	\$20	\$6	\$0
Probability	1/100	1/10	1/2	?

   a. What is the probability of winning nothing, i.e., getting \$0?

   b. What is the mean amount won?

   c. What is the standard deviation of the amount won?

   d. Find the variance of $(3X\text{-}100)$ using the results in part b and c.

2. Let $E[X(X+1)]=16$ and $Var(X)=10$. Find $E(X^2)$ if $E(X)<0$.

3. Corinne is a basketball player who makes 80% of her free throws over the course of a season. In a key game, she shoots 4 free throws and misses 2 of them. What is the probability she misses 2 or more out of 4?

4. Children inherit their blood type from their parents, with probabilities that reflect the parents' generic makeup. Children of Juan and Maria each have probability 1/4 of having type A and inherit independently of each other. Juan and Maria plan to have 3 children. Let *X* be the number of children who have blood type A.
[**Hint**: At first find the most appropriate distribution for *X*.]

   a. Make the probability distribution table of X.

x	0	1	2	3	Total
P(*X*=*x*)					1

   b. Find the mean number of children with type A blood, and standard deviation.

- $\mu_X = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k$

  $$= \sum_{i=1}^{k} x_i p_i \cdot$$

- Var(aX +b) = a^2Var(X)
- Var(X) = $E(X^2)$-[$E(X)$]^2
- If a count X has the binomial distribution B(n,p), then

  $\mu_X = n \times p$ $\qquad$ $\sigma_X = \sqrt{n \times p \times (1-p)}$

VARIANCE OF A DISCRETE RANDOM VARIABLE

Suppose that X is a discrete random variable whose distribution is

Value of X	x_1	x_2	x_3	$\cdots$	x_k
Probability	p_1	p_2	p_3	$\cdots$	p_k

and that μ_X is the mean of X. The **variance** of X is

$$\sigma_X^2 = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_k - \mu_X)^2 p_k$$
$$= \sum (x_i - \mu_X)^2 p_i$$

The **standard deviation** σ_X of X is the square root of the variance.

Definition, pg 300
Introduction to the Practice of Statistics, Fifth Edition

BINOMIAL PROBABILITY

If X has the binomial distribution $B(n, p)$ with n observations and probability p of success on each observation, the possible values of X are 0, 1, 2, . . . , n. If k is any one of these values, the **binomial probability** is

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Definition, pg 349b
Introduction to the Practice of Statistics, Fifth Edition