CLASS #6

INDEPENDENT EVENTS AND CONDITIONAL PROBABILITIES

Textbook's Sections 2.4 and 2.5

Probability, 1016-351

Rochester Institute of Technology

A Rule of Probability

P(A and B) = P(A)P(B)   [page 77]

only for A and B independent events.

This is called the multiplication rule.

example: P(rain in London tomorrow
and Buffalo Bills will win the next
Super Bowl)

Example

Given that we know that P(rain in London tomorrow) = 0.7 and P(Buffalo Bills will win the next Super Bowl) = 0.03,

P(rain in London tomorrow
      <u>and</u> Buffalo Bills will win the next
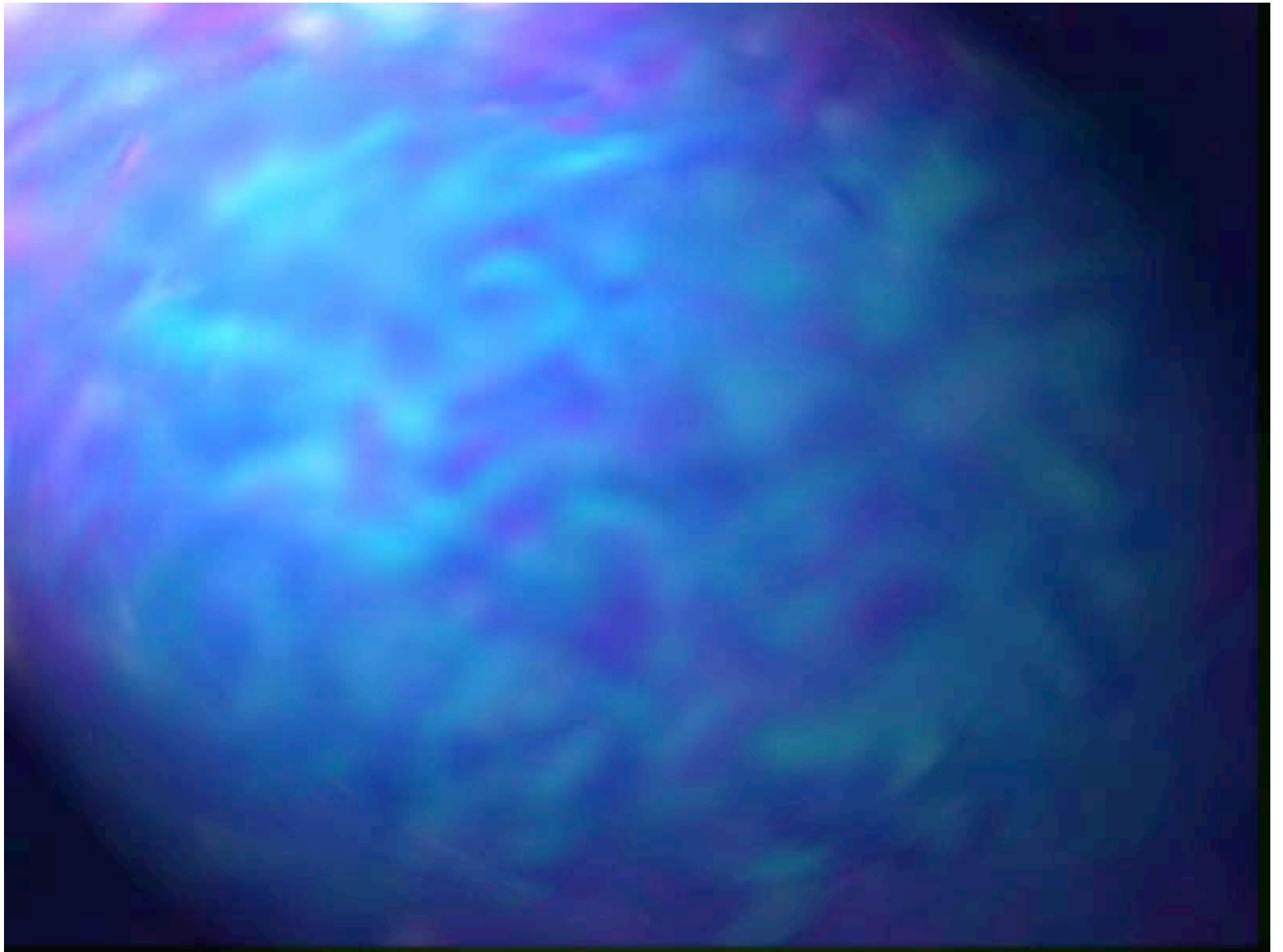   Super Bowl)

      = (0.7)(0.03) = 0.021

      the probability that <u>both</u> will occur

<u>Independent events</u> are events such that knowing about the occurrence or non-occurrence of one of them tells you nothing about the probability that the other event will occur.

A different type of example: Are "getting an ace" and "getting a spade" independent events when randomly drawing a single card from a standard deck of cards?

YES _____ , NO _____

A Previous Example Revisited

We are given the three probabilities below

P(A) = P(engineering major) = 0.071

P(B) = P(female) = 0.320

P(A and B) = P(engineering major and

female) = 0.005

P(A or B) = P(A) + P(B) - P(A and B)

Example continued:

P(A or B)

= P(engineering major or female)

= 0.071 + 0.320 - 0.005 = 0.386

Are being a female and being an engineering major independent?

No.

P(female)P(engineer) = (0.320)(0.071) = 0.02272

P(being both) = 0.005

These would be the same values if they were independent events.

Conditional Probabilities

Example on the board:

Sample from a collection of coins containing 60 showing Phillip the Good and 40 showing Charles the Bold.

When selecting two coins, place the first coin aside before selecting the second coin. This is called sampling without replacement.

The following is an example from another textbook:

In a sample of 150 residents, each person was asked if he or she favored the concept of having a single countywide police agency. The county is composed of one large city and many suburban townships. The residence (city or outside the city) and the responses of the residents are summarized in Table 4.3. If one of these residents was to be selected at random, what is the probability that the person will (a) favor the concept? (b) favor the concept if the person selected is a city resident? (c) favor the concept if the person selected is a resident from outside the city? (d) Are the events F (favor the concept) and C (reside in city) independent?

| | Opinion | | |
Residence	Favor (F)	Oppose ($\overline{F}$)	Total
In city (C)	80	40	120
Outside of city ($\overline{C}$)	20	10	30
Total	100	50	150

Note that this textbook uses $\overline{F}$ for "not F."

The entries in the table could be written as probabilities:

Opinion

	F	not F	
City	80/150	40/150	120/150
Residence			
Not City	20/150	10/150	30/150
	100/150	50/150	1

(a)   P(Favor) = 100/150

(b)   P(Favor <u>given that</u> City) = 80/120

(c)   P(Favor given that Outside the City)

       = 20/30

(d)   Are Favoring and being in the City

          independent? Answer: Yes,

    since P(F) P(C) = (100/150) (120/150)

          = 8/15 = 0.5333…

    and P(F and C) = 80/150 = 8/15

          = 0.5333...

for Part (d), independence can be shown at least two other ways:

(1) P(F) = 100/150 = 2/3 and P(F I C) = 80/120 = 2/3 from Parts (a) and (b), so P(F) = P(F I C). That is, the probability of F occurring is the same as the probability of F occurring if C has occurred.

(2) The rows of the table are proportional. That is 80 is to 40 as 20 is to 10.

Always (see page 69):
$P(A \text{ and } B) = P(A) \, P(B \mid A)$

Independent events (see page 77):
$P(A \text{ and } B) = P(A) \, P(B)$

A criteria for independence is
$P(B \mid A) = P(B)$

Some words that have been used

conditional probability
row
column
matrix notation
contingency table
cross classify
marginal
marginal probability
marginal sum   more –>

Some words that have been used continued

sample size

two way table

bivariate distribution

variables (Opinion and Residence)

r by c table (This example is 2 by 2.)

Some formulas that have been used

For any events E and F,
$$P(F \mid E) = P(E \text{ and } F)/P(E)$$
$$P(E \text{ and } F) = P(E) \, P(F \mid E)$$

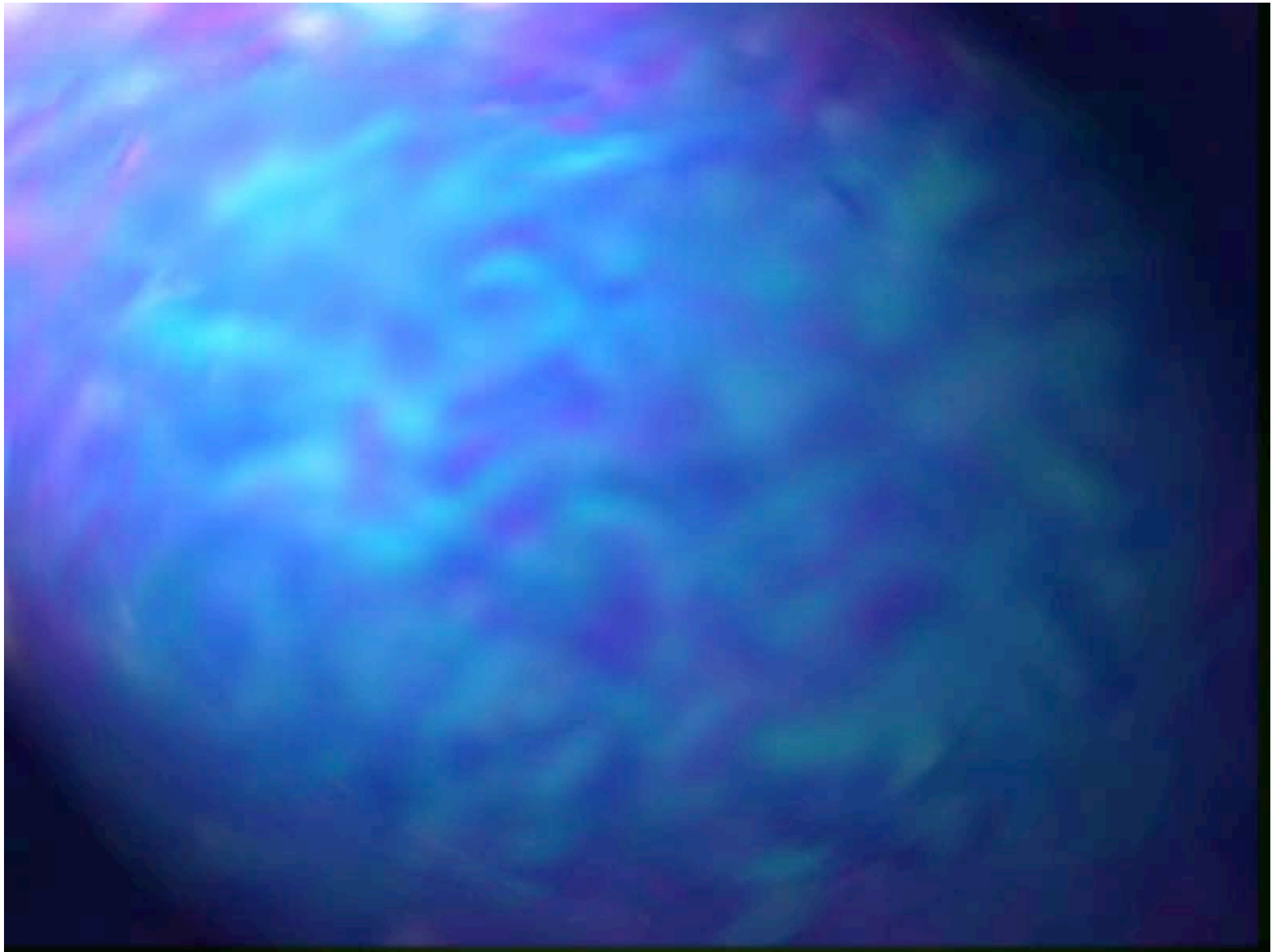For independent events E and F,
$$P(E \text{ and } F) = P(E) \, P(F)$$
$$P(F \mid E) = P(F)$$

Consider events E and F that have positive probability

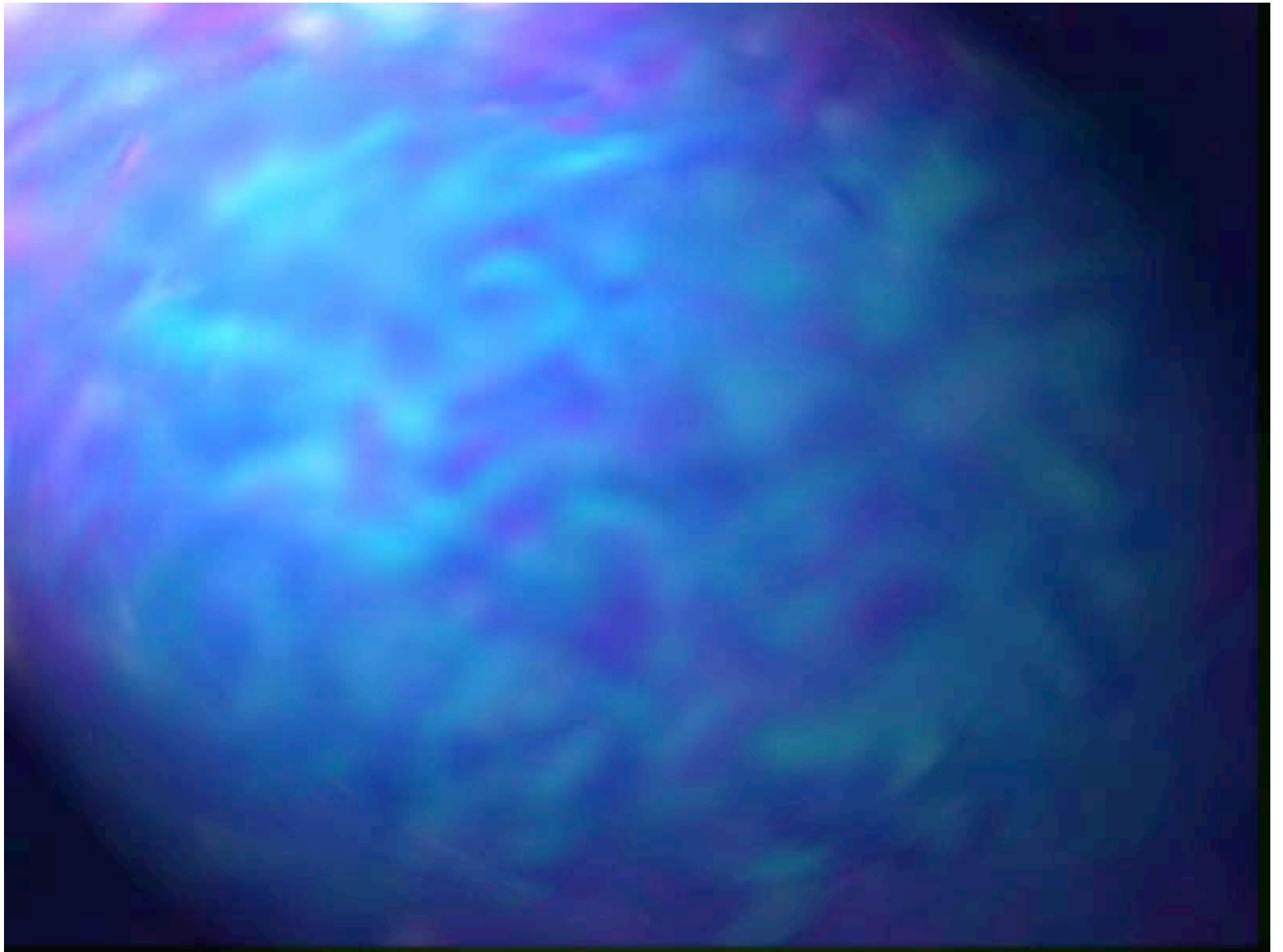If E and F are mutually exclusive, can they be independent?

Yes (always) ___   No (never) ___   Maybe ___

Consider events E and F that have positive probability

If E and F are independent, can they be mutually exclusive?

Yes (always) ___   No (never) ___   Maybe ___

LAST SLIDE FOR CLASS #6