March 18th Dr. Kim's note (Ch2.4, Ch2.5)

Conditional Probability

The new notation $P(B|A)$ is a conditional probability. That is, it gives the probability of one event under the condition that we know another event. You can read the bar "|" as "given the information that."

Definition of Conditional Probability

For any two events A and B with $P(A)>0$, the conditional probability of B given A is

$$P(B|A) = \frac{P(A\,and\,B)}{P(A)}.$$

TABLE 4.1 Age and marital status of women (thousands of women)

	Age			
	18 to 29	30 to 64	65 and over	Total
Married	7,842	43,808	8,270	59,920
Never married	13,930	7,184	751	21,865
Widowed	36	2,523	8,385	10,944
Divorced	704	9,174	1,263	11,141
Total	22,512	62,689	18,669	103,870

Example Let's define two events:

A = the woman chosen is young (18 to 29)

B = the woman chosen is married

Probability of choosing a young woman is

$$P(A) = \frac{22{,}512}{103{,}870} = 0.217.$$

The probability that we choose a woman who is both young and married is

$$P(A \, and \; B) = \frac{7{,}842}{103{,}870} = 0.075.$$

The conditional probability that a woman is married when we know she is under 30 is

$$P(B \mid A) = \frac{P(A \, and \; B)}{P(A)} = \frac{7{,}842}{22{,}512} = 0.348.$$

The Multiplication Rule

The probability that both of two events A and B happen together can be found by

$$P(A \, and \; B) = P(A) \times P(B \mid A).$$

Example Slim is still at the poker table. At the moment, he wants very much to draw two diamonds in a row. As he sits at the table looking at his hand and at the upturned cards on the table, Slim sees 11 cards. Of these, 4 are diamonds. The full deck contains 13 diamonds among its 52 cards, so 9 of the 41 unseen cards are diamonds. To find Slim's probability of drawing two diamonds, first calculate

P(1st card diamond) = 9/41

P(2nd card diamond | 1st diamond) = 8/40

Multiplication rule $P(\text{A and B}) = P(\text{A}) \times P(\text{B} \mid \text{A})$,

P(both cards diamonds)=(9/41)x(8/40)=.044.

What's P(2nd card diamond)?

The Law of Total Probability

Let $A_1, \ldots, A_k$ be mutually exclusive and exhaustive events. Then for another event B,

$$P(B) = P(B \mid A_1)P(A_1) + \ldots + P(B \mid A_k)P(A_k)$$

$$= \sum_{i=1}^{k} P(B \mid A_i)P(A_i)$$

Bayes' Theorem

Let $A_1, \ldots, A_k$ be mutually exclusive and exhaustive events with *prior* probabilities $P(A_i)$ ($i=1, \ldots, k$). Then for any other event B for which $P(B)>0$, the *posterior* probability of A_j given that B has occurred is

$$P(A_j \mid B) = P(A_j \cap B)/P(B)$$

$$= P(B \mid A_j)P(A_j) / \sum_{i=1}^{k} P(B \mid A_i)P(A_i).$$

If A and B are any events whose probabilities are not 0 or 1,

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B \mid A) \times P(A) + P(B \mid A^c) \times P(A^c)}.$$

Independent Events

Two events A and B are **independent** if knowing the results of A does not help to predict the results of B.

Two events A and B that both have positive probability are independent if $P(A \mid B)=P(A)$. A and B are independent if and only if $P(A \cap B)=P(A) \times P(B)$.