

1016-420-02

Complex Variables

In-Class Exercise Solutions

2012 November 27

NAME:

Using what you know about algebra, and the fact that $i^2 = -1$, write the following expressions in the form $a + bi$, where a and b are ordinary real numbers, i.e., write a real number (possibly zero or negative) in each box.

1.

$$(1 + 2i) + (3 + i) = (1 + 3) + (2 + 1)i = \left(\begin{array}{|c|} \hline 4 \\ \hline \end{array} \right) + \left(\begin{array}{|c|} \hline 3 \\ \hline \end{array} \right) i$$

2.

$$(1 + 2i) - (3 + i) = (1 - 3) + (2 - 1)i = \left(\begin{array}{|c|} \hline -2 \\ \hline \end{array} \right) + \left(\begin{array}{|c|} \hline 1 \\ \hline \end{array} \right) i$$

3.

$$2(2 + i) = \left(\begin{array}{|c|} \hline 4 \\ \hline \end{array} \right) + \left(\begin{array}{|c|} \hline 2 \\ \hline \end{array} \right) i$$

4.

$$(-3i)(2 + i) = -6i - 3i^2 = -6i - 3(-1) = \left(\begin{array}{|c|} \hline 3 \\ \hline \end{array} \right) + \left(\begin{array}{|c|} \hline -6 \\ \hline \end{array} \right) i$$

5.

$$(2 - 3i)(2 + i) = (4 + 2i) + (3 - 6i) = (4 + 3) + (2 - 6)i = \left(\begin{array}{|c|} \hline 7 \\ \hline \end{array} \right) + \left(\begin{array}{|c|} \hline -4 \\ \hline \end{array} \right) i$$

6.

$$(1 - 3i)(1 + 3i) = 1 + 3i - 3i - 9i^2 = 1 + 9 = \begin{pmatrix} \boxed{10} \\ \end{pmatrix} + \begin{pmatrix} \boxed{0} \\ \end{pmatrix} i$$

7.

$$\frac{1 + 3i}{(1 - 3i)(1 + 3i)} = \frac{1 + 3i}{10} = \begin{pmatrix} \boxed{0.1} \\ \end{pmatrix} + \begin{pmatrix} \boxed{0.3} \\ \end{pmatrix} i$$

8.

$$\frac{1 + 3i}{1 + 3i} = \begin{pmatrix} \boxed{1} \\ \end{pmatrix} + \begin{pmatrix} \boxed{0} \\ \end{pmatrix} i$$

9.

$$\frac{1}{1 - 3i} = \frac{1}{1 - 3i} \cdot \frac{1 + 3i}{1 + 3i} = \begin{pmatrix} \boxed{0.1} \\ \end{pmatrix} + \begin{pmatrix} \boxed{0.3} \\ \end{pmatrix} i$$