1016-420-02
Complex Variables

In-Class Exercise

2012 December 13

NAME:

Consider the function

$$f(z) = e^{z^2}$$

1. Recalling that $(x + iy)^2 = (x^2 - y^2) + i2xy$, write $f(x + iy) = \rho(x, y)e^{i\phi(x,y)}$ where $\rho(x, y)$ and $\phi(x, y)$ are real functions of x and y.

$\rho(x, y) = \boxed{}$ $\qquad$ $\phi(x, y) = \boxed{}$

2. Use the Euler relation $e^{i\alpha} = \cos\alpha + i\sin\alpha$ and the results of part 1 to write $f(x + iy) = u(x, y) + iv(x, y)$, where $u(x, y)$ and $v(x, y)$ are real functions of x and y.

$u(x, y) = \boxed{}$ $\qquad$ $v(x, y) = \boxed{}$

3. Take the partial derivatives of the $u(x, y)$ and $v(x, y)$ you found in part 2.

$$\frac{\partial u}{\partial x} = \boxed{} \qquad \frac{\partial v}{\partial x} = \boxed{}$$

$$\frac{\partial u}{\partial y} = \boxed{} \qquad \frac{\partial v}{\partial y} = \boxed{}$$

4. Use the results of part 3 to show $f(z) = e^{z^2}$ is analytic everywhere.