# 1016-345-01
# Probability and Statistics for Engineers

Problem Set 2

Assigned 2013 March 12
Due 2013 March 19

**Show your work on all problems! If you use a computer to assist with numerical computations, turn in your source code as well.**

## 1 Devore Chapter 3, Problem 12

## 2 Devore Chapter 3, Problem 18

## 3 Devore Chapter 3, Problem 32

## 4 Devore Chapter 3, Problem 46

## 5 Computational Exercise (Extra Credit)

This exercise lets you apply the binomial distribution and Bayes's theorem to consider the interpretation of a (somewhat) realistic experiment.

Suppose that you have a box containing ten six-sided dice. Nine of them are fair (1/6 chance of rolling each number), and one is loaded so that it has a 50% chance of rolling a six. Suppose you pick up one of the dice, roll it $n = 30$ times, and count how many sixes you get.

**a.** If you choose the fair die, the random variable $X$ representing the number of sixes will obey a binomial distribution

$$p(x|\text{fair}) = \binom{n}{x}\left(\frac{1}{6}\right)^x\left(\frac{5}{6}\right)^{n-x}$$

Use a computer to plot $p(x|\text{fair})$ versus $x$ for all of the possible values of $x$. (Hint: if you're using python, the binomial coëfficient $\binom{n}{x}$ can be calculated with the scipy function `comb(n,x)`, so you need

```
from scipy.misc import comb
```

Also, it's a good idea to use `1./6.` rather than `1/6` to avoid the gotchas of integer division.

**b.** If you choose the loaded die, $X$ will obey

$$p(x|\text{loaded}) = \binom{n}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{n-x}$$

Use a computer to plot $p(x|\text{loaded})$ versus $x$ for all of the possible values of $x$.

**c.** If you choose a die at random, the a priori probability of choosing a fair die is $p(\text{fair}) = .9$ while $p(\text{loaded}) = .1$. Using the law of total probabilty, you can find

$$p(x) = p(x|\text{fair})p(\text{fair}) + p(x|\text{loaded})p(\text{loaded})$$

use a computer to plot $p(x)$ vs $x$.

**d.** You can now use Bayes's theorem to calculate

$$p(\text{fair}|x) = \frac{p(x|\text{fair})p(\text{fair})}{p(x)}$$

for each possible value of $x$. Use a computer to plot $p(\text{fair}|x)$ vs $x$.

**e.** Find the explicit value of $p(\text{fair}|15)$, the probability that you chose the fair die, given that 15 out of 30 rolls were sixes.