# STAT 405-01: Mathematical Statistics I

Problem Set 5

Assigned 2015 October 1
Due 2015 October 8

**Show your work on all problems!** Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that if using an outside source to do a calculation, you should use it as a reference for the method, and actually carry out the calculation yourself; it's not sufficient to quote the results of a calculation contained in an outside source.

## 1 Hogg 2.6.1

## 2 Hogg 2.6.2

## 3 Hogg 2.6.8

## 4 Hogg 2.6.5

## 5 Hogg 2.6.9

## 6 Correlation Coëfficient

Consider two random variables $X_1$ and $X_2$ with variances $\operatorname{Var}(X_i) = \sigma_i^2$ and covariance $\operatorname{Cov}(X_1, X_2) = \rho\sigma_1\sigma_2$. You showed on a previous problem set, using algebra, that $-1 \leq \rho \leq 1$. In this problem, you will give an alternate proof, following from the fact that the variance-covariance matrix $\mathbf{\Sigma}$ is positive definite.

**(a)** Show that $\mathbf{\Sigma} = \mathbf{ASA}$ where $\mathbf{A}$ is the positive definite matrix $\operatorname{diag}(\sigma_1^{-1}, \sigma_2^{-1})$ and $\mathbf{S} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$

**(b)** Find the eigenvalues of $\mathbf{S}$ and show that the condition that they both be non-negative is $-1 \leq \rho \leq 1$.

**(c)** Use this method to obtain a condition that must be satisfied by the three correlation coëfficients $\rho_{12}$, $\rho_{13}$, $\rho_{23}$ for three random variables $\{X_i | i = 1, 2, 3\}$.