STAT 406-01: Mathematical Statistics II

Problem Set 5

Assigned 2016 March 3
Due 2016 March 10

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that if using an outside source to do a calculation, you should use it as a reference for the method, and actually carry out the calculation yourself; it's not sufficient to quote the results of a calculation contained in an outside source.

1 Hogg 6.3.5

2 Hogg 6.3.6

[I will assume that "computational facilties are available"–JTW]

3 Hogg 6.3.10

4 Hogg 6.3.15

Extra Credit: Construct the Bayes factor $\mathcal{B}_{01}$, assuming that H_1 assigns a uniform prior to θ. Construct a test which rejects H_0 if $P(H_0|\mathbf{X}) < \alpha$, assuming prior probabilities of $P(H_0) = P(H_1) = \frac{1}{2}$.

5 Hogg 6.4.1

6 Hogg 6.4.3

7 Jeffreys Prior: Multiparameter Case

Consider a probability distribution $f_{X|\boldsymbol{\Theta}}(x|\theta_1, \ldots, \theta_p)$ characterized by p parameters, which has Fisher information matrix $\mathbf{I}$ with elements

$$I_{\alpha\beta}(\boldsymbol{\theta}) = E\left(\left[\frac{\partial \ln f_{X|\boldsymbol{\Theta}}(X|\boldsymbol{\theta})}{\partial\theta_\alpha}\right]\left[\frac{\partial \ln f_{X|\boldsymbol{\Theta}}(X|\boldsymbol{\theta})}{\partial\theta_\beta}\right]\right) \tag{7.1}$$

(a) Let $\boldsymbol{\lambda} = \boldsymbol{\lambda}(\boldsymbol{\theta})$ be an invertable transformation from the p parameters $\{\theta_\alpha\}$ to a different set of p parameters $\{\lambda_\gamma\}$, which has a Jacobian matrix $\mathbf{J}$ with elements

$$J_{\alpha\gamma} = \frac{\partial\theta_\alpha}{\partial\lambda_\gamma} \tag{7.2}$$

Show that the Fisher information matrix for the new parameters is given by $\mathbf{I}(\boldsymbol{\lambda}) = \mathbf{J}^{\mathrm{T}}\,\mathbf{I}(\boldsymbol{\theta})\,\mathbf{J}$, i.e., it has elements

$$I_{\gamma\delta}(\boldsymbol{\lambda}) = \sum_{\alpha=1}^{p}\sum_{\beta=1}^{p} J_{\alpha\gamma} I_{\alpha\beta} J_{\beta\delta} \tag{7.3}$$

(b) Consider the Jeffreys prior on $\boldsymbol{\theta}$, defined by $f_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) \propto \sqrt{\det \mathbf{I}(\boldsymbol{\theta})}$. Show that under a change of variables in the pdf, this prior becomes $f_{\boldsymbol{\Lambda}}(\boldsymbol{\lambda}) \propto \sqrt{\det \mathbf{I}(\boldsymbol{\lambda})}$