STAT 406-01: Mathematical Statistics II

Problem Set 6

Assigned 2016 March 10
Due 2016 March 17

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that if using an outside source to do a calculation, you should use it as a reference for the method, and actually carry out the calculation yourself; it's not sufficient to quote the results of a calculation contained in an outside source.

1 Hogg 6.3.18

2 Hogg 6.4.4

[Note the function given is the *cumulative* distribution function – JTW]

3 Hogg 6.5.2

4 Hogg 6.5.5

5 Hogg 6.5.8

6 Fisher Information Matrix

Suppose that the random variables X and Y are drawn from a bivariate normal distribution with mean $\boldsymbol{\theta} = \begin{pmatrix}\theta_1 \\ \theta_2\end{pmatrix}$ and invertible variance-covariance matrix $\boldsymbol{\Sigma} = \begin{pmatrix}1/2 & 1/4 \\ 1/4 & 3/8\end{pmatrix}$.

- **(a)** Work out the Fisher information matrix $\mathbf{I}(\boldsymbol{\theta})$ for this distribution.
- **(b)** Consider a sample of size n and show that the maximum likelihood estimators $\widehat{\theta}_1$ and $\widehat{\theta}_2$ are $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$, respectively.
- **(c)** Assume that $n = 8$. Calculate $\operatorname{Var}(\overline{X})$, $\operatorname{Var}(\overline{Y})$ and $\operatorname{Cov}(\overline{X}, \overline{Y})$.
- **(d)** Evaluate $\frac{1}{nI_{11}(\boldsymbol{\theta})}$, $\frac{1}{nI_{22}(\boldsymbol{\theta})}$, $n^{-1}I_{11}^{-1}(\boldsymbol{\theta})$, and $n^{-1}I_{22}^{-1}(\boldsymbol{\theta})$, again taking $n = 8$. Comment on the relationship of these results to those of the previous part.
- **(e)** Work out the joint distribution satisfied by $\overline{X}$ and $\overline{Y}$. (Hint: use Theorem 3.5.1)
- **(f)** **Extra Credit**: Numerically simulate 1000 pairs of values for $\overline{X} - \theta_1$ and $\overline{Y} - \theta_2$ and make a scatter plot. Put error bars centered at the origin corresponding to $\frac{1}{\sqrt{nI_{11}(\boldsymbol{\theta})}}$ and $\sqrt{n^{-1}I_{11}^{-1}(\boldsymbol{\theta})}$ in the x direction and $\frac{1}{\sqrt{nI_{22}(\boldsymbol{\theta})}}$ and $\sqrt{n^{-1}I_{22}^{-1}(\boldsymbol{\theta})}$ in the y direction.