STAT 406-01: Mathematical Statistics II

Problem Set 10

Assigned 2016 April 19
Due 2016 April 26

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that if using an outside source to do a calculation, you should use it as a reference for the method, and actually carry out the calculation yourself; it's not sufficient to quote the results of a calculation contained in an outside source.

1   Hogg 7.8.2

2   Hogg 7.8.3

3   Sufficient and Ancillary Statistics

Consider a sample (X_1, Y_1), (X_2, Y_2), ..., (X_n, Y_n) drawn from the bivariate distribution

$$f(x, y; \theta) = \exp\left(-\theta x - \frac{y}{\theta}\right) \qquad 0 < x < \infty, \quad 0 < y < \infty \qquad (3.1)$$

where $0 < \theta < \infty$.

  (a) Show that $V = \sum_{i=1}^{n} X_i$ and $W = \sum_{i=1}^{n} Y_i$ are joint sufficient statistics for θ.
  (b) Show that $T = \sqrt{W/V}$ is the maximum likelihood estimator for θ.
  (c) Show that $U = \sqrt{VW}$ is an ancillary statistic for θ.
  (d) Show that T is *not* a sufficient statistic for θ.
  (e) Explain why this means T and U (or V and W) are the minimal sufficient statistics for θ.

4   Hogg 7.9.4

5   Hogg 7.9.9

6   Hogg 7.9.11