STAT 489-01: Bayesian Methods of Data Analysis

Problem Set 7

Assigned 2017 March 23
Due 2017 March 30

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that if using an outside source to do a calculation, you should use it as a reference for the method, and actually carry out the calculation yourself; it's not sufficient to quote the results of a calculation contained in an outside source.

1 Gelman Chapter 10, Exercise 6

2 Gelman Chapter 10, Exercise 7

3 Gelman Chapter 11, Exercise 1

4 Markov Chain Monte Carlo

Consider a model with a likelihood function

$$p(\mathbf{y}|\boldsymbol{\theta}) \propto (1+e^{-\theta_1})^{-y_1}(1+e^{-\theta_2})^{-y_2}(1+e^{-\theta_1-\theta_2})^{-y_3}(1+e^{\theta_1})^{-y_4}(1+e^{\theta_2})^{-y_5}(1+e^{\theta_1+\theta_2})^{-y_6} \quad (4.1)$$

and an improper uniform prior on θ_1, θ_2. Let $y_1 = 2$, $y_2 = 2$, $y_3 = 1$, $y_4 = 0$, $y_5 = 1$, $y_6 = 1$. Perform a Markov Chain Monte Carlo using the Metropolis method with a multivariate normal proposal distribution $\boldsymbol{\theta}^* \sim N_2(\boldsymbol{\theta}^{t-1}, \boldsymbol{\Sigma})$ where $\boldsymbol{\theta}^{t-1} = \begin{pmatrix}\theta_1^{t-1} & \theta_2^{t-1}\end{pmatrix}$ and $\boldsymbol{\Sigma} = \sigma^2 \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$. (Multivariate normal random draws can be generated either the `rmvnorm()` function from the `mvtnorm` library or the `mvrnorm()` function from the `MASS` library.)

(a) Let $\sigma = 1$ in the proposal distribution, and construct two 20000-step chains, one starting at $(\theta_1, \theta_2) = (0, 0)$ and one at $(\theta_1, \theta_2) = (5, 5)$. (Please code this by hand rather than using an MCMC library.)

- **(i)** Plot, separately for each chain, in the θ_1, θ_2 plane, with appropriate jitter: the path of the first 100 steps; the path of the first 1000 steps; the second half of the chain (no need to connect the points). Use the same axis ranges for all plots.
- **(ii)** Check convergence between chains by plotting, on the same axes, θ_1 versus step number for the second half of each chain. Repeat this for θ_2.
- **(iii)** Estimate the expectation values $E(\theta_1|\mathbf{y})$ and $E(\theta_2|\mathbf{y})$ from the second half each of the two chains. Plot the evolution of each of these quantities with step number.

(b) Repeat the previous part for $\sigma = 10$.

(c) Repeat the previous part for $\sigma = 0.01$.