STAT 489-01: Bayesian Methods of Data Analysis

Problem Set 8

Assigned 2017 March 30
Due 2017 April 6

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that if using an outside source to do a calculation, you should use it as a reference for the method, and actually carry out the calculation yourself; it's not sufficient to quote the results of a calculation contained in an outside source.

1 Gibbs Sampler

Consider the example in class, applying the Gibbs sampler to the Dirichlet posterior $p(\theta_1, \theta_2|\mathbf{y}, I) \propto \theta_1^{\alpha_1}\theta_2^{\alpha_2}(1-\theta_1-\theta_2)^{\alpha_3}$ where $0 < \theta_1, \theta_2$ and $\theta_1 + \theta_2 < 1$. Writing the Dirichlet distribution in the more general form

$$p(\theta_1, \theta_2, \theta_3|\mathbf{y}, I) \propto \theta_1^{\alpha_1}\theta_2^{\alpha_2}\theta_3^{\alpha_3}\delta(\theta_1+\theta_2+\theta_3-1), \qquad 0 < \theta_1, \theta_2, \theta_3 \tag{1.1}$$

allows us to consider different combinations of the three parameters.

(a) Construct the conditional distributions $p(\theta_1|\theta_3, \mathbf{y}, I)$, $p(\theta_3|\theta_1, \mathbf{y}, I)$, $p(\theta_2|\theta_3, \mathbf{y}, I)$, and $p(\theta_3|\theta_2, \mathbf{y}, I)$. (We constructed $p(\theta_1|\theta_2, \mathbf{y}, I)$ and $p(\theta_2|\theta_1, \mathbf{y}, I)$ in class.)

(b) Show that the procedure to perform a Gibbs step using $p(\theta_3|\theta_2, \mathbf{y}, I)$ is identical to that for a Gibbs step using $p(\theta_1|\theta_2, \mathbf{y}, I)$. What other pairs of conditionals have this property?

(c) Write a program which cycles through the three unique types of Gibbs steps, and apply it to the case $\boldsymbol{\alpha} \equiv (3, 6, 4)$. Plot the first 30 steps of the chain on a ternary plot.

2 Hierarchical Normal Model Using JAGS

(a) The file `http://ccrg.rit.edu/~whelan/courses/2017_1sp_STAT_489/data/ps08_prob2_model.txt` contains a JAGS model for the hierarchical normal model considered in Example 11.6 of Gelman. Use it to run an MCMC using the data in Table 11.2 and estimate the marginal posterior distributions of each θ_i, μ, σ and τ. Calculate the 2.5th, 25th, 50th, 75th and 97.5th percentile of each distribution and compare the values to those in Table 11.3

3 Gelman Chapter 11, Exercise 3