

STAT 489-01: Bayesian Methods of Data Analysis

Problem Set 11

Assigned 2017 May 4
Due 2017 May 11

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that if using an outside source to do a calculation, you should use it as a reference for the method, and actually carry out the calculation yourself; it's not sufficient to quote the results of a calculation contained in an outside source.

1 Logistic Regression

Recall the logistic regression problem considered in class, which assumed the model

$$y_i \sim \text{Bin}(1, p_i) ; \quad \lambda_i = \log \frac{p_i}{1 - p_i} = \alpha + \beta(x_i - \bar{x}) = \beta(x_i - x_{50}) \quad (1.1)$$

- (a) In class, we implicitly used an improper prior $p(\alpha, \beta|I) = \text{const}$. Convert this into a prior $p(x_{50}, \beta|I)$.
- (b) Consider the prior $p(x_{50}, \beta|I)$ above, along with alternative priors $p(x_{50}, \beta|I') = \text{const}$ and $p(x_{50}, \beta|I'') \propto \beta^{-1}$. Using the data¹ http://ccrg.rit.edu/~whelan/courses/2017_1sp_STAT_489/data/found_rm.dat to perform an MCMC with each of these priors and compare the posterior samples drawn in each case. Specifically, scatter-plot each sample in x_{50}, β space, using the same axes, and produce overlay plots of estimates of the marginal posteriors for x_{50} and β . (I.e., plot $p(x_{50}|\{y_i\}, \{x_i\}, I)$, $p(x_{50}|\{y_i\}, \{x_i\}, I')$, and $p(x_{50}|\{y_i\}, \{x_i\}, I'')$ on a single plot, and likewise for the marginal posteriors of β .)
- (c) Using the prior $p(x_{50}, \beta|I') = \text{const}$, generate posterior samples using as data the files `found_ts.dat`, `found_rm.dat`, `found_sb.dat`, and `found_pn.dat` in the directory http://ccrg.rit.edu/~whelan/courses/2017_1sp_STAT_489/data/. For each search, generate the 25th, 50th and 75th percentiles of $p(x; x_{50}, \beta)$ versus 10^x for 100 evenly spaced values of 10^x , and plot these on the same set of axes. Compare to the top panel of figure 3 of Messenger et al, *Phys. Rev. D* **92**, 023006 (2015).
- (d) Try to do the same thing with http://ccrg.rit.edu/~whelan/courses/2017_1sp_STAT_489/data/found_cc.dat. What goes wrong?

2 Project Report (one per team)

Submit a final written report on your project, detailing your methods and results.

¹*Phys. Rev. D* **92**, 023006 (2015); <https://dcc.ligo.org/LIGO-G1500977/public>