

ASTP 611-01: Statistical Methods for Astrophysics

Problem Set 3

Assigned 2017 September 12
Due 2017 September 19

Show your work on all problems! Be sure to give credit to any collaborators, or outside sources used in solving the problems. Note that if using an outside source to do a calculation, you should use it as a reference for the method, and actually carry out the calculation yourself; it's not sufficient to quote the results of a calculation contained in an outside source.

1 Phases of Colored Noise

Carry out the exercises in the ipython notebook at
http://ccrg.rit.edu/~whelan/courses/2017_3fa_ASTP_611/data/ps03.ipynb

2 Exercises in Logic and Probability

Do all of the problems at the end of Chapter 2 of Gregory.

3 Cumulant Generating Function

Given the moment generating function $M(t) = E[e^{tX}]$ from which you can calculate $E[X^k] = M^{(k)}(0)$, and in particular $M(0) = 1$, $M'(0) = E[X] = \mu_X$, and $M''(0) = E[X^2]$, it is often convenient to define the cumulant generating function $\psi(t) = \ln M(t)$.

- Show that $\psi(0) = 0$ and $\psi'(0) = E[X]$.
- Show that $\psi''(0) = \text{Var}(X)$.

4 Change of Variables

Let X be a Cauchy random variable with pdf

$$f_X(x) = \frac{1}{\pi} \frac{1}{1+x^2} \quad -\infty < x < \infty \quad (4.1)$$

Define Y such that $X = \tan Y$ and $-\frac{\pi}{2} < Y < \frac{\pi}{2}$ and work out the pdf $f_Y(y)$.